1. Introduction

During explosive volcanic eruptions, a mixture of hot pyroclasts (volcanic ash) and volcanic gas is released from the vent into the atmosphere. The mixture generally has an initial density several times larger than atmospheric density at the volcanic vent. As the ejected material entrains ambient air, the density of the mixture decreases because the entrained air expands by heating from the pyroclasts. If the density of the mixture becomes less than the atmospheric density before the eruption cloud loses its upward momentum, a buoyant plume forms a plinian eruption column. On the other hand, if the mixture loses its upward momentum before it becomes buoyant, the eruption column collapses to generate a pyroclastic flow. Because the impact and type of volcanic hazards are largely different between the two eruption styles, it has been a central subject of volcanology to quantitatively predict the condition where an eruption column collapses to generate a pyroclastic flow; we refer to this condition as “the column collapse condition”.

When the buoyant plume develops in the atmosphere, the plume interacts with the wind. If the plume has much greater vertical velocity than the cross-wind speed, its trajectory is not significantly controlled by the wind and rises vertically (referred to as “strong plume” [1]). On the other hand, if the vertical velocity is much smaller than the cross-wind speed, the plume is largely distorted by the cross-wind and shows a bent-over trajectory (“weak plume”); this effect can modify the maximum eruption column height. Because the height of eruption column is one of the few available data for estimating the eruption conditions, it is of critical importance to clarify the relationship between the eruption condition and column height under the cross-wind conditions.

In this report, we aim to investigate the effect of decompression and/or compression in/above the crater on the column collapse condition (Section 2), and the effect of the cross-wind on the column height (Section 3).
2. Effects of Decompression and/or Compression on the Column Collapse Condition

When magma properties (e.g., water content and temperature) are fixed, the column collapse condition has been considered to depend primarily on magma discharge rate [e.g., 2]. Here, we show that the column collapse condition also strongly depends on the crater shape.

According to Koyaguchi et al. [3], the flow in/above the crater is divided into 4 regimes in the parameter space of mass flow rate at the crater base \((q_t/q^*_t)\) in Fig. 1) and the ratio of sectional area between the top and base of the crater \((A_t/A_b)\) in Fig. 1). Here \(q_t\) is the mass flow rate at the crater base, \(q^*_t\) is the mass flow rate for the reference flow (sonic flow at 1 atm), and \(A_t\) and \(A_b\) are the sectional areas at the crater top and base, respectively. When \(q_t\) is small, the magma ascends as a subsonic flow through both the conduit and the crater. As \(q_t\) increases, the flow reaches the choking condition at the base or top of the crater. When \(A_t/A_b\) is around unity, the flow is choked at the crater top. It has an exit pressure \((p_e)\) greater than the atmospheric pressure \((p_a)\) and is freely decompressed into the atmosphere. We refer to this type of flow as “free decompression flow”. When \(A_t/A_b\) exceeds a certain value, the flow reaches the choking condition and changes from a subsonic to a supersonic flow at the crater base. Flow that is choked at the crater base is subdivided into four types. When \(A_t/A_b\) is relatively small or \(q_t\) is large, the gas-pyroclast mixture issues from the crater top as a supersonic flow with \(p_t > p_a\). The flow is decompressed and generates rarefaction waves just above the crater. We refer to this type as “underexpanded flow”. As \(A_t/A_b\) increases or \(q_t\) decreases, the exit pressure \((p_e)\) decreases. When \(p_e = p_a\), the expansion of the gas-pyroclast mixture in the crater is most efficiently transferred to upward momentum. This flow type is referred to as “correctly expanded flow”. For greater \(A_t/A_b\) or smaller \(q_t\), the gas-pyroclast mixture erupts as a supersonic flow with \(p_t < p_a\), and is then compressed and decelerated by oblique shocks just above the crater. This flow type is referred to as “overexpanded flow”. As \(A_t/A_b\) further increases or \(q_t\) decreases, a shock forms inside the crater. In this instance, the gas-pyroclast mixture is compressed and decelerated by the shock and issues from the crater top as a subsonic flow.

Koyaguchi et al. [3] proposed that the column collapse condition for these flow regimes can be roughly estimated from the 1-D steady decompression model above the crater [4, 5] and the 1-D steady eruption column dynamics model [e.g., 6]; however, the two 1-D steady decompression models provide different results of column collapse condition for the free decompression flow. Woods and Bower [4] proposed that the free decompression flow is efficiently accelerated because of expansion above the vent so that it tends to generate stable buoyant column. On the other hand, Ogden et al. [5] proposed that the vertical acceleration is reduced because of the effect of radial expansion in the free decompression flow so that the generation of pyroclastic flow is more likely to occur than the prediction by Woods and Bower [4]. In this study we test this point on the basis of a series of 3-D simulations.

2.1 Model Description

The numerical model is designed to describe the injection of a gas-pyroclasts mixture from a circular vent above a flat surface with a temperature gradient typical of the tropical atmosphere.

The vent is located in the center of the ground surface. The physical domain involves a vertical and horizontal extent of several tens of kilometers. At the ground boundary, the free-slip condition is assumed for the velocities of the ejected material and air. At the upper and other boundaries of computational domain, the fluxes of mass, momentum, and energy are assumed to be continuous. We assume steady conditions; for each run, vent radius and exit velocity are fixed. The initial temperature and water content are set to be \(T_0=1053\) K and \(n_{w0}=0.06\), respectively.

We apply a pseudo-gas model; we ignore the separation of solid pyroclasts from the eruption cloud and treat an eruption cloud as a single gas whose density is calculated from mixing ratio of the ejected material and entrained air. The fluid dynamics model solves a set of partial differential equations describing the conservation of mass, momentum, and energy, and constitutive equations describing the thermodynamics state of the mixture of pyroclasts, volcanic gas, and air. These equations are solved numerically by a general scheme for

![Fig. 1 Regime map for flows inside craters and eruption column dynamics in the \(q_t/q^*_t-A_t/A_b\) space. The boundary of different flow regimes inside craters are based on Koyaguchi et al. [3]. Flow regimes for eruption column dynamics are based on the 3-D simulations in this study. Column collapse conditions based on Woods and Bower [4] and Ogden et al. [5] are also shown in this diagram.](image)
compressible flow. Details of the numerical procedures are described in Suzuki et al. [7].

2.2 Numerical Simulations

Fig. 2 shows a representative result of our 3-D simulations of free decompression flow. The free decompression flow accelerates because of expansion and becomes a supersonic flow with high Mach number. Along the jet axis, the pressure continuously decreases to values below the ambient pressure, which discontinuously increases by passing through the shock wave, so-called Mach disk; the flow along the jet axis decelerates by this shock and becomes subsonic above it. The free decompression flow is also expanded radially at the exit, and the flow boundary forms an expanded shape. Expansion waves originating at the exit corner are reflected at the jet boundary as compression waves; these compression waves coalesce to form the so-called barrel shock. The flow passing through the barrel shock is compressed by the reflected shock. Because the barrel shock and the reflected shock are oblique to the flow, the deceleration by these shocks is limited; the flow passing through these shocks is still supersonic. The annular supersonic flow is separated from the inner subsonic flow by the slip line.

Our results are consistent with the conclusion by Ogden et al. [5] in the sense that the effects of the radial expansion and the deceleration by Mach disk play an important role in the column dynamics. However, as far as the column collapse condition is concerned, our results support the prediction by Woods and Bower [4]. In our simulations the high speed annular flow is highly turbulent, and mixes with ambient atmosphere. It is suggested that the instability in the annular supersonic up-flow enhances mixing between ejected material and ambient air, and hence, stabilizes the eruption column.

3. Effects of Cross-Wind on Column Height

Because the plume height is principally determined by the balance between the thermal energy ejected from the vent and the work done in transporting the ejected material plus entrained air through the atmospheric stratification, it is controlled by the efficiency of turbulent mixing; as the amount of entrained air increases, the plume height decreases. Bursik [8] presented a theoretical model of steady eruption column in a cross-wind and demonstrated that cross-wind causes enhanced entrainment of ambient air and the plume height decreases as the wind speed increases. In his model, the entrainment velocity is given as a simple function of the relative velocity between the eruption column and cross-wind. However, the laboratory experiments of turbulent jet/plumes indicated that in the bent-over jet/plumes the cross-wind generates complex 3-D vortical structures which control the turbulent mixing [e.g., 9].

In this study, we aim to develop a numerical model which can reproduce the 3-D vortical structures in the bent-over plumes of eruption clouds, and investigate the effects of cross-wind on the column height.

3.1 Model Description

The numerical model is based on the one described in Section 2.1. We assume that the cross-wind speed increases linearly with height, $z$ [km], as $U_w=0.8z$ [m/s]. The mid-latitude atmosphere is applied to the atmospheric condition. The computational domain of the numerical model extends 12 km vertically, 1 km upstream and 20 km downstream from the volcanic vent. It also extends 7 km from the vent in the horizontal direction perpendicular to the wind. In this study, the calculations are performed on a...
3-D domain with a non-uniform grid with $1014 \times 512 \times 512$ grid points. The grid size is set to be sufficiently smaller than $L_0/8$ near the vent, and to increase at a constant rate (by a factor of 1.02) with the distance from the vent up to $L_0$, where $L_0$ is the vent radius.

Our primary concern is how the maximum height of bent-over plume deviates from those in a still atmosphere. For this purpose, it is important to set vent radii sufficiently smaller than the length scale at which the initial upward momentum is lost in the simulations. When the vent radius is greater than a few hundred meters, the eruption cloud loses its momentum before turbulent mixing fully develops from the edge to the core of the flow; as a result, the heavy unmixed core forms a radially suspended flow and another type of vortical structure develops [7]. In order to avoid additional effects caused by this vortical structure, we set vent radii to be narrow (26.7 m). An initial temperature of $T_0=1000 \text{ K}$ and water content of $n_0=0.0284$ are assumed. The exit velocity is set to be the sound velocity of the gas-pyroclasts mixture. We assume that the pressure at the vent is same as the atmospheric pressure for simplicity.

### 3.2 Numerical Simulations

Our simulation has successfully reproduced the bent-over plume of eruption cloud (Figs. 3 and 4a). Near the vent, the eruption cloud rises vertically. As it rises, the plume is highly unstable and undergoes a meandering instability that induces efficient mixing so that its radial scale gradually increases with height. Above 3 km, the cloud is largely distorted by the cross-

![Fig. 4 Simulation results of (a) the bent-over plume in the cross-wind and (b) the vertical plume in the still atmosphere at 406 s. Cross-sectional distributions of the mass fraction of the ejected material in $x-z$ space ($y=0 \text{ km}$). Red arrow in (a) represents the wind direction. Green line indicates the position of the slice in Fig. 6.](image)

![Fig. 5 Vortical structures in the bent-over plume. Iso-surface of $Q$-value (0.002) at 232 s. Red arrow represents the wind direction.](image)

![Fig. 6 Simulation result of the bent-over plume. Cross-sectional distribution of the mass fraction of the ejected material in $x-y$ space ($z=4 \text{ km}$ as indicated in Fig. 4a) at 406 s. Red arrow represents the wind direction.](image)
wind. Then, the eruption cloud stops to rise and eventually spreads horizontally along the wind direction around the height of 5-8 km.

Some characteristic structures are observed in the bent-over plume. First, a spatially periodic structure forms in the upper part of the bent-over plume (Fig. 4a). This coherent structure is generated just after the flow is distorted by the cross-wind ($\xi > 3$ km) and is similar to “the shear layer vortices” observed in the laboratory experiments of bent-over jet/plumes, which result from the Kelvin-Helmholtz instability in the shear layer when the two streams meet [9]. Secondly, the spiral structure develops at the edge of the plume (Fig. 5). This structure is also generated after the flow shows a bent-over trajectory and is similar to “the counter-rotating vortex” observed in the laboratory experiments [9], which occurs as a result of process of roll-up, tilting and folding of the shear layer vortices. Finally, the plume develops an inlet at the front and splits into two distinct lobes (Fig. 6). These structures were also reported by the experimental studies [9]. Ernst et al. [10] suggested that the pressure distribution around the counter-rotating vortex initially triggers such bifurcation.

For comparison, we also carried out a simulation of eruption column in a still atmosphere in which eruption conditions at the vent are same as those for the above simulation with cross-wind. In the simulation in the still atmosphere, the eruption plume reaches up to 12 km (Fig. 4b), which is substantially greater than the maximum height of the bent-over plume (8 km; see Fig. 4a). This difference in the maximum column height between the cases with and without the cross-wind can be accounted for by the fact that the efficiency of entrainment is enhanced by the cross-wind. It is suggested that the shear layer vortices and counter-rotating vortices cause more efficient turbulent mixing.

References
火山噴煙の3次元数值シミュレーション：火口クレーターの形状と風の影響

プロジェクト責任者
小屋口剛博 東京大学 地震研究所
著者
鈴木雄治郎 東京大学 地震研究所

本プロジェクトでは、固体地球と地球表層・大気にまたがる火山現象について、大規模数値シミュレーションを用いた物理過程の理解と計算結果の防災への応用を目指している。平成23年度は特に、火山噴煙において（1）火口クレーターでの加圧・減圧過程が火砕流発生条件に与える影響、（2）大気の風が噴煙高度に与える影響について研究を進めた。

（1）火口クレーターでの加圧・減圧過程が火砕流発生条件に与える影響
爆発的火山噴火では一般に、噴火花と火砕流という二つの特徴的な噴火レジームが見られる。これら二つのレジームの境界を火砕流発生条件と呼ぶ。マグマの性質（例えば含水量・温度）が与えられた場合、火砕流発生条件は、主に噴出圧力によって決定されると考えられている（例えば、Carazzo et al., 2008）。但し、火口の形状（例えば火口上端の径）が変化した場合、一定の噴出圧力の下で噴出圧力、噴出速度が変化し、それが火砕流発生条件に影響を与えられる。Koyaguchi et al.（2010）は、火口直上での減圧過程について定常1次元モデル（Woods and Bower, 1995）と非定常2次元モデル（Ogden et al., 2008）に基づいて火砕流発生条件を求め、自由膨張流として噴出する場合に、両モデルに基づく火砕流発生条件が大きく食い違うことを指摘した。そこで、本研究では、非定常3次元噴火モデル（Suzuki et al., 2005）によるシミュレーションを行い、同条件下における火砕流発生条件を求めた。

シミュレーションの結果、3次元計算による火砕流発生条件は Woods and Bower（1995）の予測と半定量的に一致した。自由膨張領域の計算条件下でシミュレーションを行ったところ、広い計算条件で安定な噴火花を形成する結果が得られた。この結果は、Ogden et al.（2008）のモデルよりも Woods and Bower（1995）のモデルに基づく火砕流発生条件を支持する。このような自由膨張流では、パラレルショックとマッハディスクが形成され、マッハディスク直上で中心部衝突流と外縁部超音速流に分離する。外縁部超音速流では、圧力変動を伴う不安定な流れが発生し、それが大気と噴火花の効率的な混合を促すことによって、安定な噴火花を形成しやすいことが新たに分かった。

（2）大気の風が噴煙高度に与える影響
噴煙高度は単位時間がにマグマから大気に供給される熱エネルギーの直接的指標であり、爆発的噴火過程や喷火強度を推定する上で貴重な情報源となる。これまでに、定常1次元噴火モデル（Woods, 1988）によって、喷火山火は主に火口での噴火率で決定されることと予想されてきた。Bursik（2001）は、大気中に風が存在する場合、噴火と大気の混合が促進され、その結果、噴火高度が低下する可能性を示した。しかし、Bursik（2001）のモデルでは風と大気の混合効率について単純な関数が仮定されており、横風を受けるジェットやブームで見られるような複雑な3次元的渦構造が考慮されていない。そこで、本研究では、3次元シミュレーションで渦構造を含めた噴火流動を直接的に再現することを試みた。

シミュレーションの結果、風によってその軌跡が大きく曲がる噴火を、3次元的に実現に再現することに成功した。噴火率が4x10^6 kg/sの場合、横風を受けた噴火の到達高度は、風がない場合に比べて4km低下した。噴火内部の渦構造を可視化したところ、風で曲がり始める領域で噴火側部から中心に向かって巻き込むような渦構造が観察された。この現象は、風によって形成される渦構造が噴火と大気の混合を促進し、その結果として噴火高度を低下させることを合理的に説明する。

キーワード：火山噴火、火砕流、擬似ガスモデル、乱流混合、火山災害