Parameterizing ocean eddy transports from surface to bottom

Hidenori Aiki,[1] Tivon Jacobson,[2] and Toshio Yamagata[1,2]

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[1] We consider new parameterization schemes for the extra transport velocity by eddies in a stratified fluid, with particular emphasis on baroclinic instability. These come in the form of elliptic equations, previously unmentioned, which we derive for the eddy-induced overturning stream function. They guarantee decrease of the mean field potential energy. Our principal example gives a relationship between the vertical shear of the overturning velocity and the buoyancy torque of the main geostrophic current. The parameterized velocity is nonsingular at the bottom and the sea surface, contrasting with previous constant-coefficient theories based on depth diffusion. Idealized two-dimensional numerical experiments successfully reproduce meridional overturning circulation even when the background density gradient is uniform everywhere (the Eady problem) or when the bottom is steeply sloped. We further demonstrate that adding an eddy form drag (wave stress) term in the TRM momentum equations yields overturning of the velocity field. *INDEX TERMS:* 1635 Global Change: Oceans (4203); 4255 Oceanography: General: Numerical modeling; 4520 Oceanography: Physical: Eddies and mesoscale processes; 4532 Oceanography: Physical: General circulation. **Citation:** Aiki, H., T. Jacobson, and T. Yamagata (2004), Parameterizing ocean eddy transports from surface to bottom, *Geophys. Res. Lett.*, *31*, L19302, doi:10.1029/2004GL020703.

1. Introduction

[2] Many climate ocean models currently use subgrid-scale eddy fluxes to maintain deep water formation in the polar regions and meridional overturning circulation in the world's oceans. The *Gent and McWilliams* [1990] scheme (hereafter GM) is widely adopted in coarse-resolution ocean models [*Bryan et al.*, 1999; *Griffies et al.*, 2000]; one of several parameterization theories concerning the extraction of potential energy by baroclinic instability [*Treguier et al.*, 1997; *Killworth*, 1997; *Canuto and Dubovikov*, 2005].

[3] As baroclinic instability is an adiabatic process, eddy roles can be expressed by an effective advection in a density equation [*Gent et al.*, 1995]. A logical approach in z-coordinates is the temporal-residual-mean (TRM) theory that is based on temporal low-pass averaging performed in density coordinates to eliminate unphysical diffusion [*McDougall and McIntosh*, 1996]. The inversely mapped density in z-coordinates (termed the TRM density $\tilde{\gamma}$), which differs slightly from the Eulerian mean density $\bar{\gamma}$, is advected by the Eulerian mean flow $\bar{\mathbf{U}} = (\bar{u}, \bar{v}, \bar{w})$ and by $\mathrm{U}^+ = (u^+, v^+, w^+)$:

$$\tilde{\gamma}_t + (\bar{\mathbf{U}} + \mathbf{U}^+) \cdot \nabla\tilde{\gamma} = 0. \tag{1}$$

For example, the quasi-Stokes velocity associated with the GM flux is written as $\mathbf{V}^+ = (u^+, v^+) = (\kappa\nabla_H\tilde{\gamma}/\tilde{\gamma}_z)_z$ [*Gent et al.*, 1995]. (This yields an isopycnic diffusion of the isopycnal depth: $z_t + \bar{\mathbf{V}} \cdot \nabla_\gamma z - \bar{w} = \nabla_\gamma \cdot (\kappa\nabla_\gamma z)$.) If the background density gradient is uniform everywhere (i.e., the Eady problem) however, this velocity becomes zero all over the domain except for delta functions at the vertical and lateral boundaries [*Killworth*, 2001]. In most applications, the GM parameterization is turned off immediately adjacent to boundaries and in the mixed layer at the ocean surface.

[4] Another approach to represent baroclinic instability is to apply a layer thickness form drag in the momentum equations [*Greatbatch*, 1998]. *Wardle and Marshall* [2000] considered an Eliassen-Palm flux whose resulting forcing does not generate any net momentum in the channel but only acts to redistribute it.

[5] In Section 2 we will formulate and discuss a new parameterization for a mean-potential-energy-decreasing eddy-induced velocity. Its performance is examined in Section 3 using numerical experiments, followed by a summary in Section 4.

2. Formulation

[6] In addition to deriving equation (1), the TRM theory has shown that the nondivergent quasi-Stokes velocity $\mathbf{U}^+$ can always be written as $(\mathbf{V}^+, w^+) = (\mathbf{\Psi}^+_z, -\nabla_H \cdot \mathbf{\Psi}^+)$ with $\nabla_H = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$. The vector $\mathbf{\Psi}^+ = (\Psi^{x+}, \Psi^{y+})$ is called the quasi-Stokes stream function. It has a definite physical interpretation discussed by *McDougall and McIntosh* [2001], who also give arguments that it must vanish at boundaries and free surfaces. The zero boundary condition on $\mathbf{\Psi}^+$ guarantees that the integral of the horizontal velocity $\mathbf{V}^+$ over a fluid column is zero. We will examine the potential energy equation to find some $\mathbf{\Psi}^+$ which differ from the GM prescription but which guarantee decrease of the mean potential energy.

[7] The equation for mean potential energy in a domain Ω is

$$\frac{\partial}{\partial t}\int_\Omega g\tilde{\gamma}z d^3x = -\int_\Omega gz\nabla \cdot \left(\left(\bar{\mathbf{U}} + \mathbf{U}^+\right)\tilde{\gamma}\right) d^3x. \tag{2}$$

[1] Frontier Research Center for Global Change, Japan Agency for Marine-Earth Science and Technology, Yokohama, Japan.

[2] Department of Earth and Planetary Science, University of Tokyo, Tokyo, Japan.

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Integrating by parts, using the no-normal-flow condition, and considering only the effects of the quasi-Stokes field yields after a short calculation

$$\frac{\partial}{\partial t}\int_{\Omega} g\tilde{\gamma} z d^3x = g\int_{\Omega}\mathbf{\Psi}^{+}\cdot\nabla_H\tilde{\gamma} d^3x. \tag{3}$$

Decrease of mean potential energy can be made to hold by asserting that $\mathbf{\Psi}^{+}\propto -\nabla_H\tilde{\gamma}$. This was essentially the choice of GM. However, we notice that there are other possibilities. Our main example is to set

$$\left(\frac{\mathbf{\Psi}_z^{+}}{C(x,y,z)}\right)_z \equiv \left(\frac{\mathbf{V}^{+}}{C(x,y,z)}\right)_z = \nabla_H\tilde{\gamma}. \tag{4}$$

The energy equation becomes, integrating by parts and remembering the boundary conditions on $\mathbf{\Psi}^{+}$,

$$\frac{\partial}{\partial t}\int_{\Omega} g\tilde{\gamma} z d^3x = -g\int_{\Omega}\frac{1}{C}\left|\mathbf{\Psi}_z^{+}\right|^2 d^3x. \tag{5}$$

Provided that C is positive and that equation (4) can be solved in a way which respects the boundary conditions, mean potential energy will be consumed by the parameterization. This can in fact be done; the explicit solution is $\mathbf{V}^{+} = C(x, y, z)\,[\mathbf{D}(x, y) + \int_{-h(x,y)}^{z}\nabla_H\tilde{\gamma}dz']$. The function $\mathbf{D}(x, y)$ enforces the top and bottom boundary conditions on $\mathbf{\Psi}^{+}$. In addition, in order that the side boundary conditions be satisfied, C must tend to zero at side walls.

[8] A second possibility which guarantees consumption of mean potential energy is the Poisson equation for $\mathbf{\Psi}^{+}$:

$$\nabla\cdot\left(\frac{1}{C}\nabla\mathbf{\Psi}^{+}\right) = \nabla_H\tilde{\gamma}. \tag{6}$$

Here, $\nabla\mathbf{\Psi}^{+}$ should be interpreted componentwise. Other elliptic equations are also possible. For reasonable C and Ω, these equations may be solved with the required boundary conditions. In contrast with GM (which is locally defined based on the curvature of density surfaces), parameterization equations (4) and (6) will operate for any nonzero $\nabla_H\tilde{\gamma}$. They suggest a certain amount of non-locality, which will depend on the equation and the way C depends on the mean field and the spatial coordinates. This is at least plausible: the quasi-Stokes stream function is an average over time so that influences originating in some other region of the flow field may contribute to the stream function at a point.

[9] Although we do not know how to prescribe C in general, we briefly consider its scaling and specialize to the case of mesoscale eddies away from the equator. Thermal wind balance for $\nabla_H\tilde{\gamma}$ in equation (4) gives

$$C \sim \frac{g}{\gamma_0 f}\frac{[\mathbf{V}_z^{+}]}{[\overline{\mathbf{U}}_z]} \equiv \frac{g}{\gamma_0 f}C'. \tag{7}$$

Here the brackets are used to indicate scales and the nondimensional $C'(x, y, z)$ compares the vertical shear of the eddy-induced overturning velocity and the main geostrophic velocity. It seems reasonable to suppose that C' is some function of the nondimensional parameters of the baroclinic instability problem such as f^2L^2/N^2H^2 and $\beta L^2/U$. An inverse timescale for relaxation of the density field comes from scaling the density equation itself; it is

$$\frac{\delta\gamma}{T} = \frac{[\mathbf{V}^{+}]}{[\overline{\mathbf{U}}]}[\overline{\mathbf{U}}]\frac{\delta\gamma}{L} = C'\frac{gH[\nabla_H\tilde{\gamma}]}{\gamma_0 f}\frac{\delta\gamma}{L}. \tag{8}$$

The ratio H/L gives the approximate slope S of the isopycnals. Simplifying, one has

$$\frac{1}{T} = C'\frac{g[\nabla_H\tilde{\gamma}]}{\gamma_0 f}S. \tag{9}$$

3. Two-Dimensional Experiments

[10] In this section, we show the results of some numerical experiments which demonstrate, in a idealized mid-latitude context, the viability of the parameterization equation (4). We have taken a simple reading of equation (7) and set the coefficient C in the interior of the numerical domain to be spatially constant; equation (4) is now $\mathbf{\Psi}_{zz}^{+} = \mathbf{V}_z^{+} = (C'/|f|)g\nabla_H\tilde{\gamma}/\gamma_0$. Since C' must approach zero at the side boundaries, the numerical implementation has C' tapering to zero over the grid cell adjacent to the boundary. A legacy of this tapering can be seen in the tightly packed stream function contours adjacent to the boundary in some of the simulations.

[11] The domain is two-dimensional; in this special case, the quasi-Stokes stream function is a scalar. We used a 2000 km wide and 4000 m deep zonally uniform channel on a f-plane at 45°N. The experiments were conducted on 20 × 40 mesh in yz section with a grid spacing of Δy = 100 km and Δz = 100 m. This mimics a coarse resolution ocean model which is unable to reproduce baroclinic instability and thereby requires subgrid-scale parameterization. The initial condition is one of no motion with uniform meridional and vertical density gradients ($\bar{\mathbf{U}} = 0$, $g\tilde{\gamma}_y/\gamma_0 = f^2$, $g\tilde{\gamma}_z/\gamma_0 = -400f^2$). Slip is allowed at the lateral walls and at the bottom, while the top boundary is a free surface, initially level.

3.1. Without Momentum Equations

[12] Experiments are first conducted without Eulerian mean velocities (i.e., $\bar{v} = \bar{w} = 0$) to check whether our new velocity is able to reproduce overturning. With the given parameters, the time scale for the fastest growing Eady mode is on the order of 10^6 s [*Eady*, 1949]. We set C' to be 0.3, which, from equation (9), gives a time scale about one order of magnitude larger. Superimposed with density surfaces in Figure 1 is the overturning stream function. The parameterized advective velocity is purely northward (southward) at the initial instant in the upper (lower) half of the section (Figure 1a), while vertical velocities are found near the northern and southern sidewalls. After 100 days (Figure 1b), the density slope has flattened substantially as the result of advection by the quasi-Stokes velocity. The overturning transport has decreased to 45 m^2/s from its initial magnitude of 60 m^2/s. The stream function virtually disappears after 500 days (Figure 1c) when the density surfaces are almost horizontal. Our scheme shows no singular values at the sea surface and the bottom, a contrast with the constant coefficient GM parameterization.

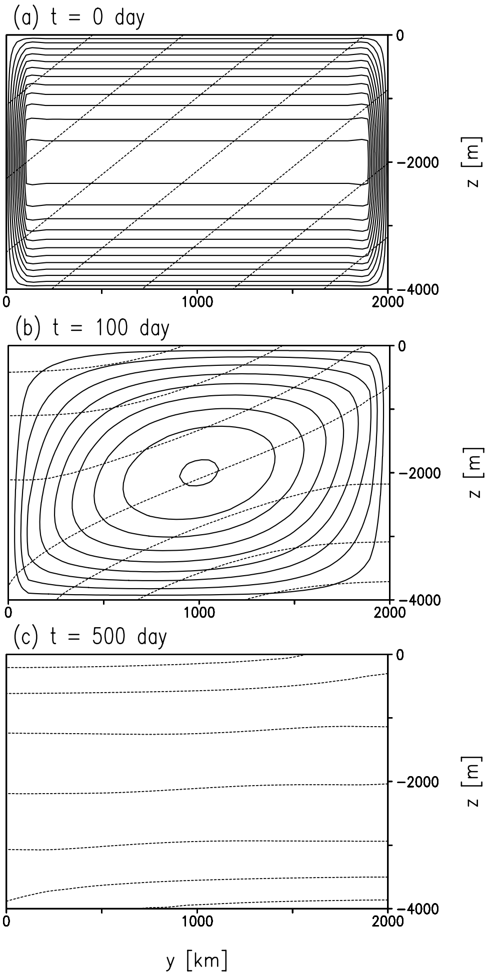

Figure 1. Experiment without momentum equations. The solid line is the overturning streamfunction $\Psi^{y+} = \int_{-h}^{z} v^{+} dz$ (contour interval of 5 m^2/s) and the dotted line is the density surface (contour interval of 0.5 kg/m^3).

[13] To examine flows near a sloping bottom, an experiment was conducted with a bottom ridge (Figure 2). Two cells are found in the quasi-Stokes stream function at the initial instant; one in each basin (Figure 2a). The maximum stream function magnitude of 60 m^2/s is same as that in the flat bottom case (Figure 1a). The parameterized velocity descends (ascends) the southern (northern) shelf of the ridge. Two-layer water exchange is found above the sill, which introduces the dense (light) water parcels in the northern (southern) basin to the south (north). In Figure 2b (100 days), the overturning center has shifted to the southern basin where dense water parcels cascade down the sloping bottom from the sill. The compensating returning flow is seen in the upper levels of the southern basin and continuing into the northern basin. The bottom dense water continues to flow downslope on the southern shelf after 500 days (Figure 2c) while the overturning has nearly ceased in the northern basin.

3.2. TRM Momentum Equation

[14] In the TRM framework, the horizontal momentum equations are written in terms of the total advective velocity (also called TRM velocity) $\hat{\mathbf{U}} \equiv \bar{\mathbf{U}} + \mathbf{U}^{+}$ with the additional terms (F^x, F^y) representing layer thickness form drag (i.e., eddy form drag or wave stress):

$$
\begin{aligned}
\hat{u}_t + \hat{\mathbf{U}} \cdot \nabla \hat{u} - f\hat{v} &= -P_x + F^x + M^x, \\
\hat{v}_t + \hat{\mathbf{U}} \cdot \nabla \hat{v} + f\hat{u} &= -P_y + F^y + M^y.
\end{aligned}
\tag{10}
$$

The quantity P is the hydrostatic pressure $\int_z^{\eta} g\tilde{\gamma}/\gamma_0 dz$ and M^x and M^y are small viscous terms. *McDougall and McIntosh* [2001] suggested (F^x, F^y) = ($-fv^+$, fu^+) in their equation (65). In this case, the Eulerian mean velocity is in near geostrophic balance so that the residual Coriolis term balances the form drag term, away from the equator at least [cf. *Greatbatch*, 1998]. Since the vertical integral of $\mathbf{V}^+$ vanishes, (F^x, F^y) causes no net force (a necessary condition for a form drag) in each vertical column. Combining geostrophic balance in the mean current, equation (4), constant C', and the identity $\hat{\mathbf{V}} = \overline{\mathbf{V}} + \mathbf{V}^+$, we can obtain a parameterization for the form drag in which only the total advective velocity appears:

$$
\begin{pmatrix} F_z^x \\ F_z^y \end{pmatrix} = \frac{-C'|f|}{1 + C'^2} \begin{pmatrix} \hat{u}_z \\ \hat{v}_z \end{pmatrix} + \frac{C'^2 f}{1 + C'^2} \begin{pmatrix} -\hat{v}_z \\ \hat{u}_z \end{pmatrix}. \tag{11}
$$

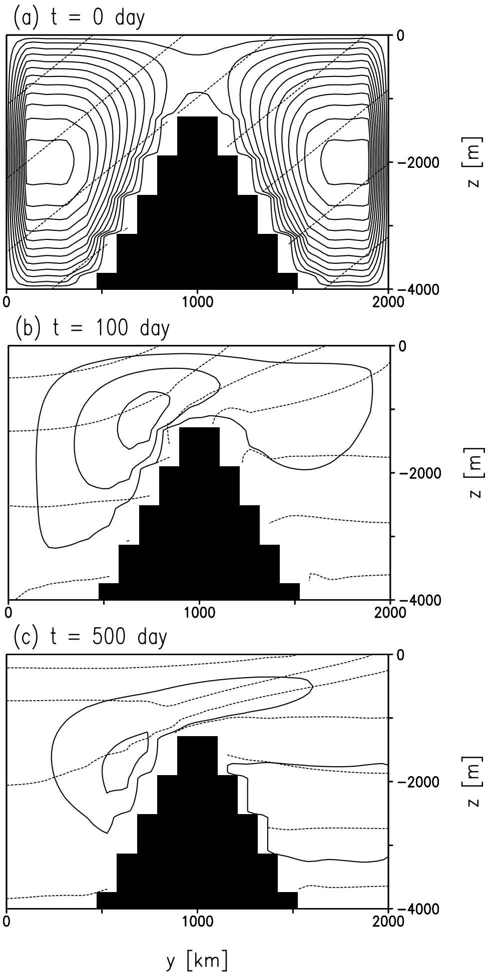

Figure 2. Same as Figure 1 except that a bottom ridge is included.

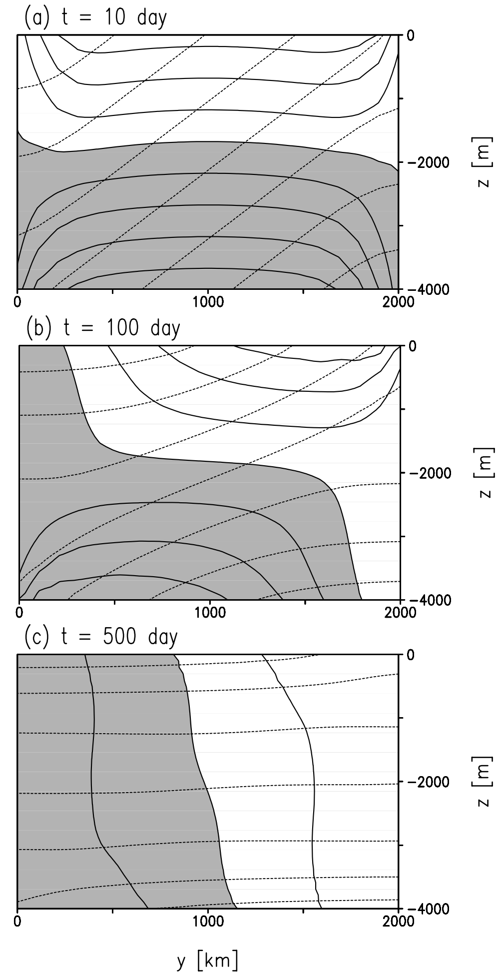

Figure 3. Experiment with TRM momentum equations. The solid line is the zonal velocity $\hat{u}$ (contour interval of 5 cm/s and shaded for negative values) and the dotted line is the density surface (contour interval of 0.5 kg/m^3).

The derivation is still valid if $C' = C'(x, y)$. The first term on the rhs behaves like a Rayleigh friction. If C' is somewhat less than one, the damping time scale $1/(C'|f|)$ is at least several inertial periods. However, the main current will not vanish in a few inertial periods because of the abundance of the potential energy reservoir. The second term of equation (11) appears to reduce the effect of rotation but will be small if C'^2 is small. In our numerical code, the vertical integral of equation (11) is put into the momentum equation (10) after subtracting the barotropic component. The TRM velocity $\hat{\mathbf{V}}$ is stepped forward as a prognostic variable.

[15] Figure 3 shows the numerical result of integrating equation (10) with prescription equation (11) from an initial condition of no motion ($\hat{\mathbf{U}}$ = 0). The integration is two-dimensional, and again, C' = 0.3. It takes several inertial periods before the TRM velocity achieves geostrophic balance (Figure 3a) with eastward (westward) velocities in the upper (lower) levels. If there was no assistance from the eddy form drag, the density slopes would not change further, remaining in thermal wind balance. In Figure 3b, however, overturning continues while the zonal current loses its vertical shear. Eastward (westward) velocity is found in the top-northern (bottom-southern) portion of the channel section. It appears that the relative vorticity of the main current is advected and overturned together with the density field. In Figure 3c (500 days), when the density surfaces are flattened into the horizontal plane, the TRM zonal velocity remains with an enhanced barotropic structure. It has a maximum of about 10 cm/s at both the southern and northern sidewalls.

4. Summary

[16] We have suggested several possibilities for parameterization of mesoscale eddy activity. These parameterizations guarantee that mean potential energy is removed by the eddy field and may apply to situations for which this is thought to be the correct physical principle. In the simplest case, we suggest that the additional advective velocity of mesoscale eddies is given by relating its vertical shear to the buoyancy torque of the main geostrophic current. To our knowledge, this is the first time that such a parameterization has been suggested. Interestingly, the parameterized velocity is naturally nonsingular at the bottom and the sea surface, contrasting with previous theories. However, in many respects our formulation in Section 2 is rudimentary and requires further study: this applies to basic physical justifications and to comparisons with available data from simulations and observation. Such studies are part of our ongoing work. We note that the eddy form drag in the TRM momentum equation may be useful in understanding the local and global energy budget. Meanwhile, three-dimensional simulations of the parameterization and eddy resolving three-dimensional numerical studies may shed light on the behavior of our parameterization in realistic climate modeling simulations.

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H. Aiki, FRCGC, JAMSTEC, 3173-25 Showa-machi, Kanazawaku, Yokohama-city, 236-0001, Japan. (aiki@jamstec.go.jp)

T. Jacobson and T. Yamagata, Department of Earth and Planetary Science, The University of Tokyo, Tokyo, 113-0033, Japan. (tivon@eps.s.u-tokyo.ac.jp)