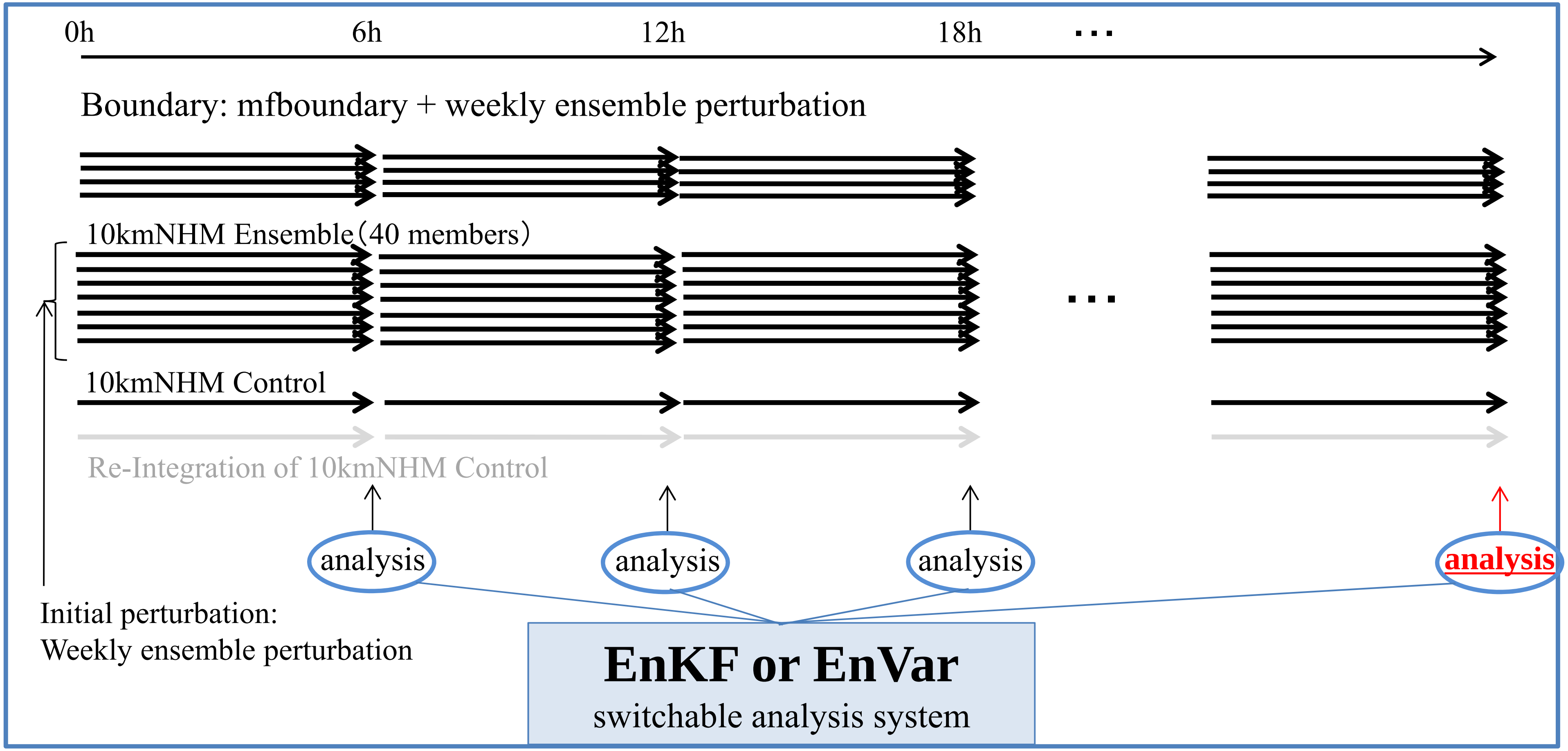
The 4th Research Meeting of Ultra-high Precision Mesoscale Weather Prediction
January 23, 2014 (Fri)   Kobe

Development of the NHM-EnVar system

Seiji ORIGUCHI, Kazumasa AONASHI, Takuya KAWABATA, Masaru KUNII
（MRI / JMA）

Analysis and Forecast system of EnKF or EnVar

NHM-EnVar system is developed based on NHM-LETKF system (Fujita ver2011.11)



Assimilating observation data
- Conventional observation data（Synop, Upper, Aircraft etc）
- Satellite observation data
- Precipitable water vapor（GPS, TRMM/TMI, SSM/I, Aqua/AMSR-E)）
- Doppler radial velocity
- Brightness temperature

<u>6hr window</u>
analysis slot (slot=7)

Ensemble Variational Assimilation（EnVar）

Zupanski 2005, Zupanski et al. 2008, Liu et al. 2008

○Cost function

$$J(\overline{\mathbf{x}}^{a}) = \frac{1}{2}[\overline{\mathbf{x}}^{a} - \overline{\mathbf{x}}^{f}]^{T}\mathbf{P}_{\mathbf{f}}^{-1}[\overline{\mathbf{x}}^{a} - \overline{\mathbf{x}}^{f}] + \frac{1}{2}[H(\overline{\mathbf{x}}^{a}) - \mathbf{y}]^{T}\mathbf{R}^{-1}[H(\overline{\mathbf{x}}^{a}) - \mathbf{y}]$$

$$\overline{\mathbf{x}}^{a} - \overline{\mathbf{x}}^{f} = \mathbf{P}_{\mathbf{f}}^{1/2} \cdot \overline{\mathbf{w}}^{a}$$

$$\mathbf{P}_{\mathbf{f}}^{1/2} = \left[\mathbf{p}_{1}^{f}, \mathbf{p}_{2}^{f}, \cdots, \mathbf{p}_{N}^{f}\right] = \frac{1}{\sqrt{N-1}}\left[\mathbf{x}_{1}^{f} - \overline{\mathbf{x}}^{f}, \mathbf{x}_{2}^{f} - \overline{\mathbf{x}}^{f}, \cdots, \mathbf{x}_{N}^{f} - \overline{\mathbf{x}}^{f}\right]$$

Square root of forecast error with ensemble perturbation

$$\overline{\mathbf{w}}^{a} = \left[\overline{w}_{1}^{a}, \overline{w}_{2}^{a}, \cdots, \overline{w}_{N}^{a}\right]^{T}$$

Increment in ensemble space（Control variable）

○Cost function in ensemble space

$$J(\overline{\mathbf{w}}^{a}) = \frac{1}{2}(\overline{\mathbf{w}}^{a})^{T}\overline{\mathbf{w}}^{a} + \frac{1}{2}[H(\overline{\mathbf{x}}^{f} + \mathbf{P}_{f}^{1/2} \cdot \overline{\mathbf{w}}^{a}) - \mathbf{y}]^{T}\mathbf{R}^{-1}[H(\overline{\mathbf{x}}^{f} + \mathbf{P}_{f}^{1/2} \cdot \overline{\mathbf{w}}^{a}) - \mathbf{y}]$$

○Gradient of cost function in ensemble space

Calculation of gradient vector using ensemble perturbation　TL＆AD don't need

$$\frac{\partial J}{\partial \overline{\mathbf{w}}^{a}} = \overline{\mathbf{w}}^{a} + \mathbf{R}^{-1}\left[\frac{\partial H}{\partial \overline{\mathbf{w}}^{a}}\right]^{T}[H(\overline{\mathbf{x}}^{f} + \mathbf{P}_{f}^{1/2}\cdot\overline{\mathbf{w}}^{a}) - \mathbf{y}]$$

$$\frac{\partial H}{\partial \overline{\mathbf{w}}^{a}} = \frac{\partial H}{\partial \overline{\mathbf{x}}^{a}}\cdot\frac{\partial \overline{\mathbf{x}}^{a}}{\partial \overline{\mathbf{w}}^{a}} = \frac{\partial H}{\partial \overline{\mathbf{x}}^{a}}\cdot\mathbf{P}_{f}^{1/2} = \frac{\partial H}{\partial \overline{\mathbf{x}}^{a}}\cdot[\mathbf{p}_{1}^{f},\mathbf{p}_{2}^{f},\cdots,\mathbf{p}_{N}^{f}]$$

$$\frac{\partial H}{\partial \overline{\mathbf{x}}^{a}}\cdot\mathbf{p}_{i}^{f} \approx H(\overline{\mathbf{x}}^{a} + \mathbf{p}_{i}^{f}) - H(\overline{\mathbf{x}}^{a}) \qquad \mathbf{p}_{i}^{f} = \frac{1}{\sqrt{N-1}}\left[\mathbf{x}_{i}^{f} - \overline{\mathbf{x}}^{f}\right]$$

Zupanski et al. 2008

○Hessian matrix

$$J''(\tilde{\mathbf{w}}^{a}) = I + \mathbf{R}^{-1}\left[\left.\frac{\partial H}{\partial \overline{\mathbf{w}}^{a}}\right|_{\tilde{\mathbf{w}}^{a}}\right]^{T}\left[\left.\frac{\partial H}{\partial \overline{\mathbf{w}}^{a}}\right|_{\tilde{\mathbf{w}}^{a}}\right]$$

$$\mathbf{P}_{a} = [J''(\tilde{\mathbf{w}}^{a})]^{-1}$$

○Ensemble update of analysis

$$\mathbf{w}_{i}^{a} = \overline{\mathbf{w}}^{a} + [\mathbf{P}_{a}^{1/2}(\tilde{\mathbf{w}}^{a})]_{i}$$

$$\mathbf{x}_{i}^{a} = \overline{\mathbf{x}}^{f} + \mathbf{P}_{f}^{1/2}\cdot\mathbf{w}_{i}^{a}$$

Algorithm of the NHM-3DEnVar

○Cost function in ensemble space

$$J(\overline{\mathbf{w}}^a) = \frac{1}{2}(\overline{\mathbf{w}}^a)^T \overline{\mathbf{w}}^a + \frac{1}{2}[H(\overline{\mathbf{x}}^f + \mathbf{P}_f^{1/2} \cdot \overline{\mathbf{w}}^a) - \mathbf{y}]^T \mathbf{R}^{-1} [H(\overline{\mathbf{x}}^f + \mathbf{P}_f^{1/2} \cdot \overline{\mathbf{w}}^a) - \mathbf{y}]$$

○Gradient of cost function

$$\frac{\partial J}{\partial \overline{\mathbf{w}}^a} = \overline{\mathbf{w}}^a + \mathbf{R}^{-1} \left[\frac{\partial H}{\partial \overline{\mathbf{w}}^a} \right]^T [H(\overline{\mathbf{x}}^f + \mathbf{P}_f^{1/2} \cdot \overline{\mathbf{w}}^a) - \mathbf{y}]$$

$$\frac{\partial H}{\partial \overline{\mathbf{w}}^a} = \frac{\partial H}{\partial \overline{\mathbf{x}}^a} \cdot \frac{\partial \overline{\mathbf{x}}^a}{\partial \overline{\mathbf{w}}^a} = \frac{\partial H}{\partial \overline{\mathbf{x}}^a} \cdot \mathbf{P}_f^{1/2} = \frac{\partial H}{\partial \overline{\mathbf{x}}^a} \cdot [\mathbf{p}_1^f, \mathbf{p}_2^f, \cdots, \mathbf{p}_N^f]$$

$$\frac{\partial H}{\partial \overline{\mathbf{x}}^a} \cdot \mathbf{p}_i^f \approx H(\overline{\mathbf{x}}^a + \mathbf{p}_i^f) - H(\overline{\mathbf{x}}^a) \qquad \mathbf{p}_i^f = \frac{1}{\sqrt{N-1}} [\mathbf{x}_i^f - \overline{\mathbf{x}}^f]$$

$$\overline{\mathbf{w}}^a = [\overline{w}_1^a, \overline{w}_2^a, \cdots, \overline{w}_N^a]^T$$

Control variables (analysis slot)

$$J(\overline{\mathbf{w}}^a), \quad \frac{\partial J}{\partial \overline{\mathbf{w}}^a} \qquad \overline{\mathbf{w}}^a \Big|_{step=0} = 0$$

LBFGS

$$\overline{\mathbf{w}}^a \Big|_{step} = \tilde{\mathbf{w}}^a$$

○Optimal analysis of ensemble mean

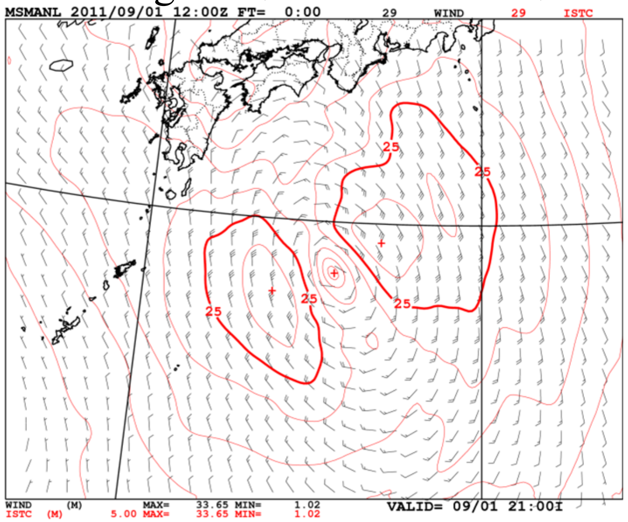
$$\tilde{\mathbf{x}}^a = \overline{\mathbf{x}}^f + \mathbf{P}_f^{1/2} \cdot \tilde{\mathbf{w}}^a$$

$$\mathbf{P}_f^{1/2} = [\mathbf{p}_1^f, \mathbf{p}_2^f, \cdots, \mathbf{p}_N^f] = \frac{1}{\sqrt{N-1}} [\mathbf{x}_1^f - \overline{\mathbf{x}}^f, \mathbf{x}_2^f - \overline{\mathbf{x}}^f, \cdots, \mathbf{x}_N^f - \overline{\mathbf{x}}^f]$$

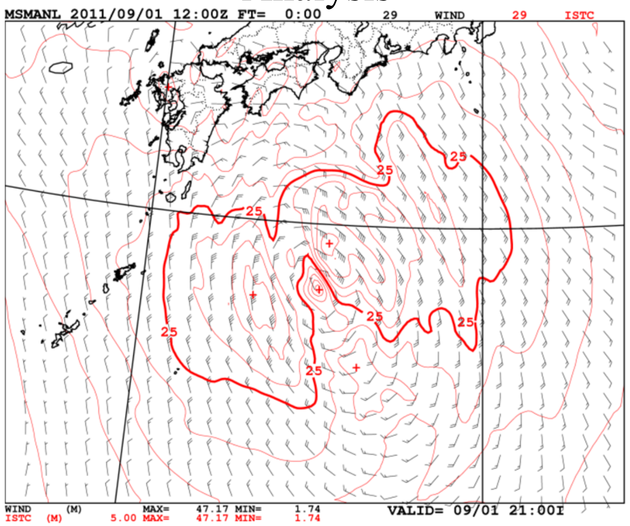
Assimilation experiment of 1-point observation Case1

Event; 2011 typhoon Talas Location; Northeast side of central position・Height of 5 km
Obs; Wind vector U=−35.54(m/s), V=45.68(m/s), obserr=0.5(m/s)

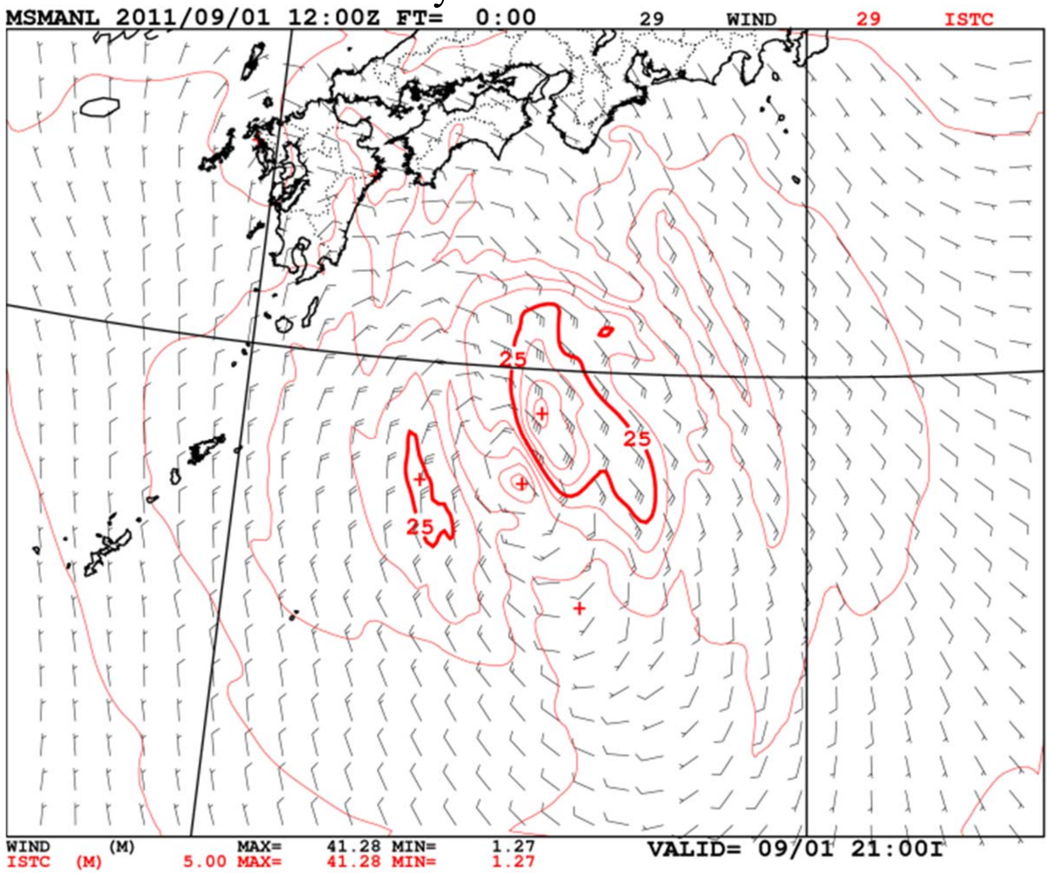
First guess（ensemble mean）



Analysis



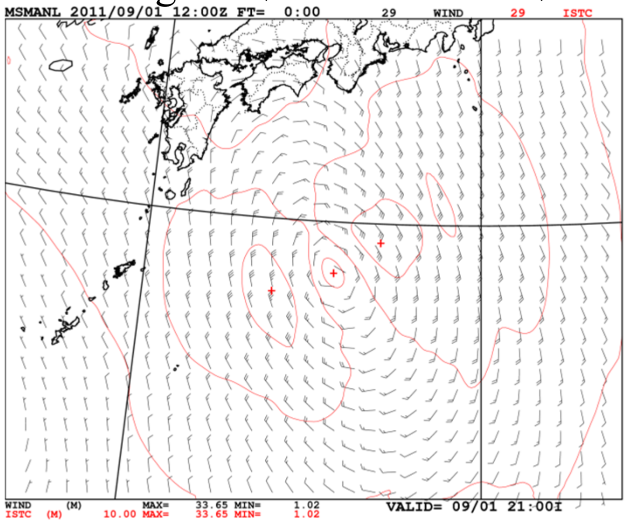
Analysis increment



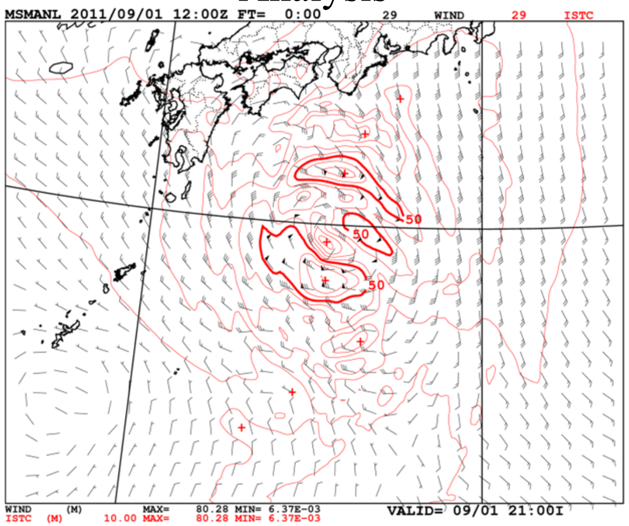
Assimilation experiment of 1-point observation Case2

Event; 2011 typhoon Talas Location; South side of central position・Height of 5 km
Obs; Wind vector U=100.0(m/s), V=0.0(m/s), obserr=0.5(m/s)

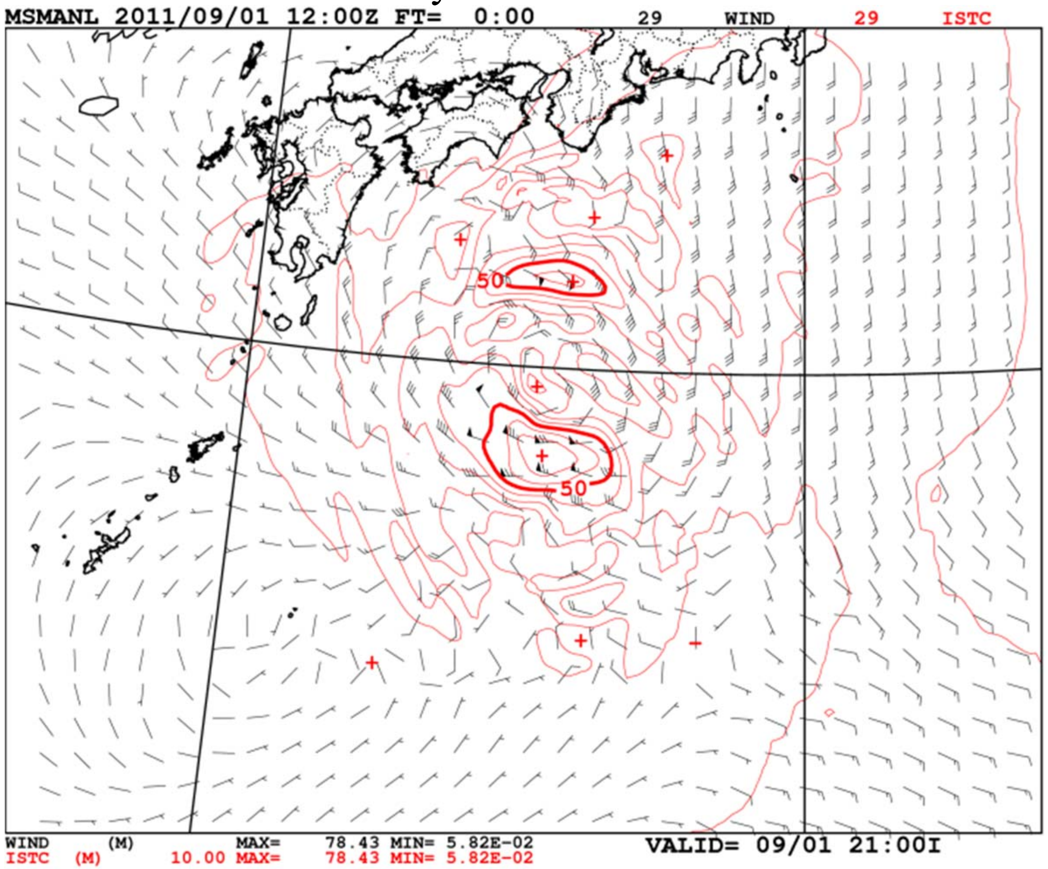
First guess（ensemble mean）



Analysis



Analysis increment



Temporal localization in observation term

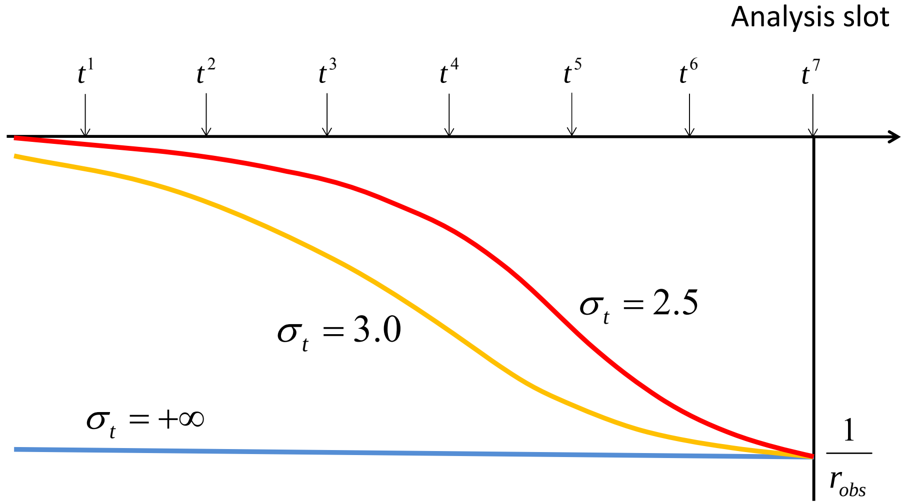
Temporal localization is implemented by multiplying the reciprocal of the localization function for observation error.

$$R_{obs,t} = diag(r^2_{obs,t^1}, \cdots, r^2_{obs,t^i} \cdots, r^2_{obs,t^n} \cdots)$$

$$r_{obs,t^i} \sim r_{obs} \cdot \exp\left(\frac{1}{2} \cdot \left(\frac{t^i_{obs} - t_{anl}}{\sigma_t}\right)^2\right)$$

σ_t : Localization scale

i : Slot number



- Load in localized function with a distance between observation and analysis slots
- No localization, if standard deviation is +∞.

○Cost function in ensemble space

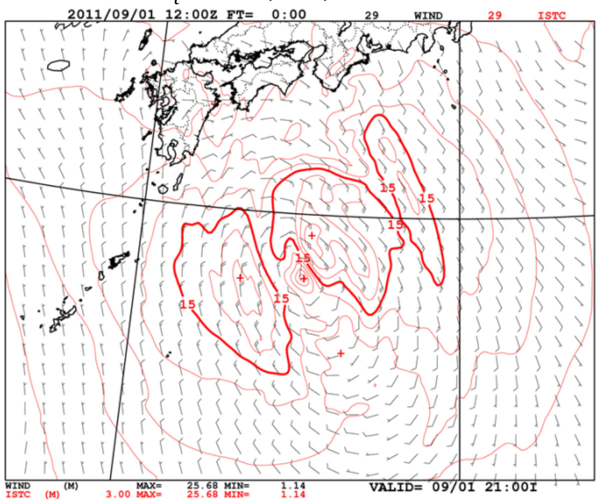
$$J(\overline{\mathbf{w}}^a) = \frac{1}{2}(\overline{\mathbf{w}}^a)^T \overline{\mathbf{w}}^a + \frac{1}{2}[H(\overline{\mathbf{x}}^f + \mathbf{P}_f^{1/2} \cdot \overline{\mathbf{w}}^a) - \mathbf{y}]^T R^{-1}_{obs,t} [H(\overline{\mathbf{x}}^f + \mathbf{P}_f^{1/2} \cdot \overline{\mathbf{w}}^a) - \mathbf{y}]$$

Suppress of sampling error

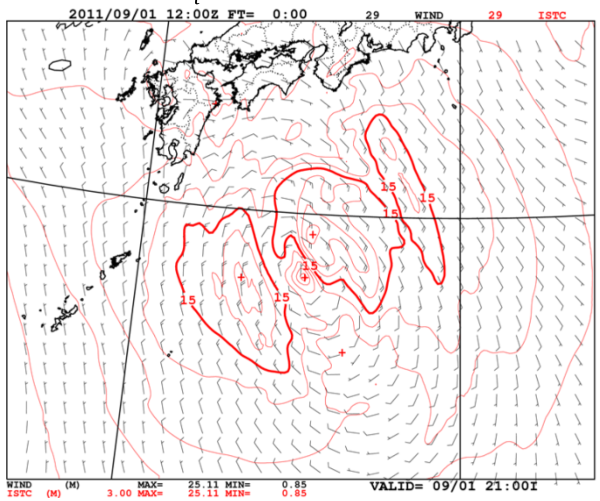
Assimilation experiments of 1-point observation for each slot

Event; 2011 typhoon Talas Location; Northeast side of central position・Height of 5 km
Obs; Wind vector U=−27.27(m/s), V=32.84(m/s) (Guess of analysis time＋10m/s)
Observation error is set so small.

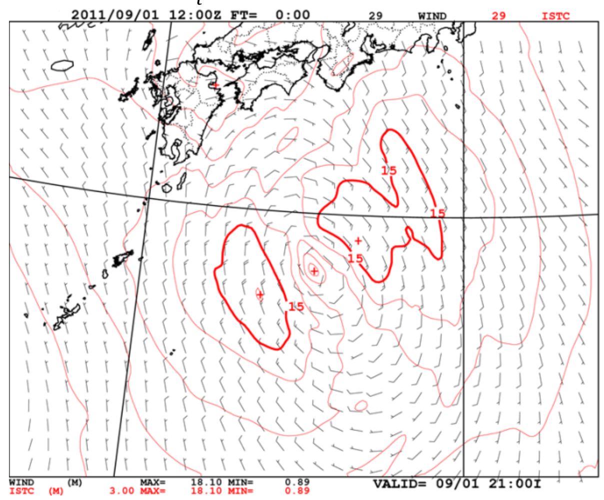
$\sigma_t = 2.5, 3.0, +\infty \quad slot = 7$



$\sigma_t = 3.0 \quad slot = 4$



$\sigma_t = 3.0 \quad slot = 1$

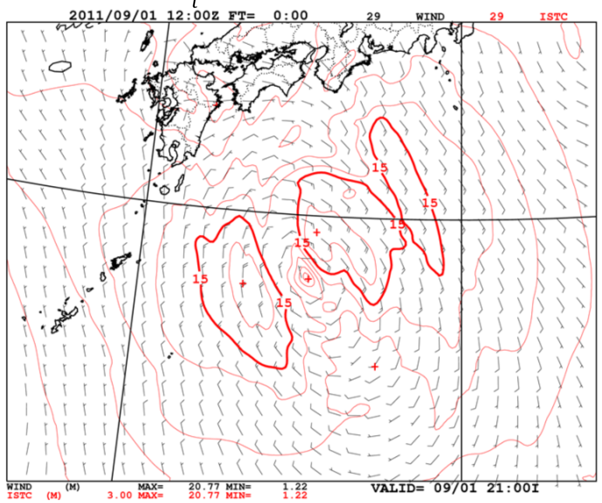


Analysis increment
(Height 5 km)

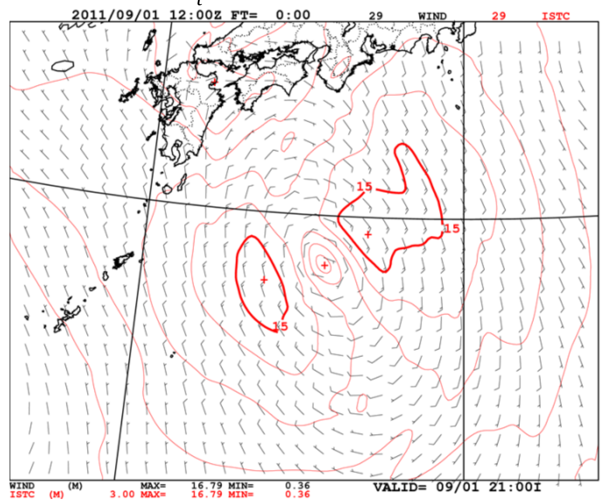
- Positive increment with a circulation enhancement
- Flow dependency (3DVar)
- Weight with a distance between observation and analysis slots (Effect of temporal localization)

3DEnVar-FGAT

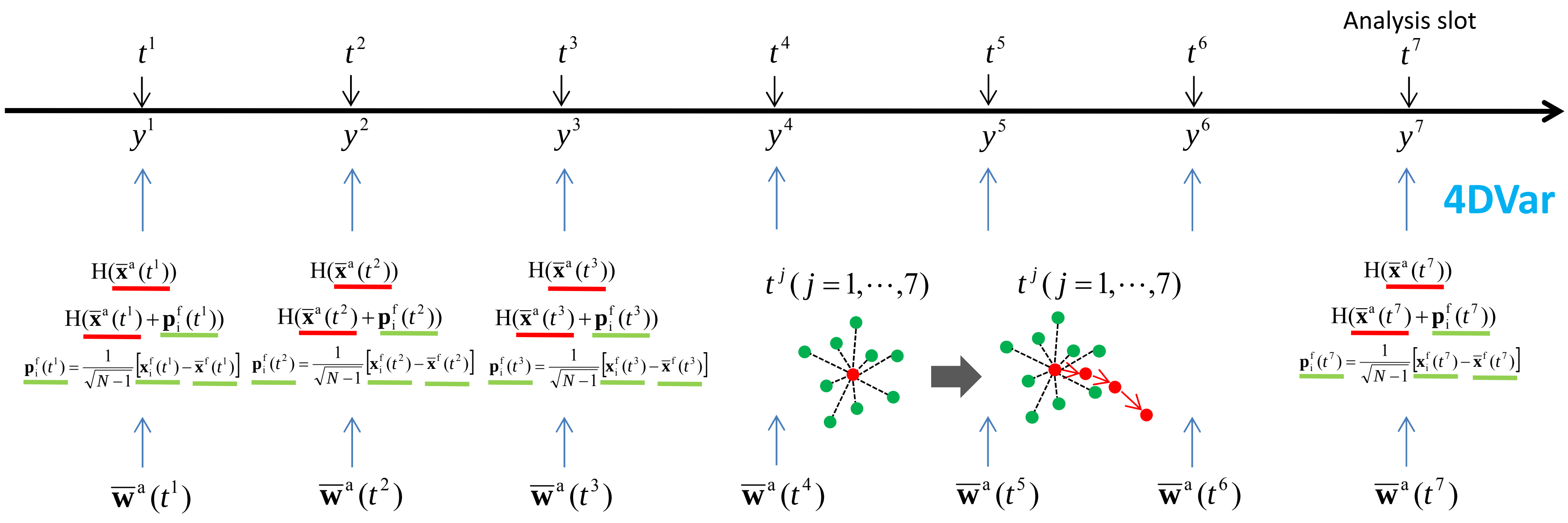
$\sigma_t = 2.5 \quad slot = 4$



$\sigma_t = 2.5 \quad slot = 1$

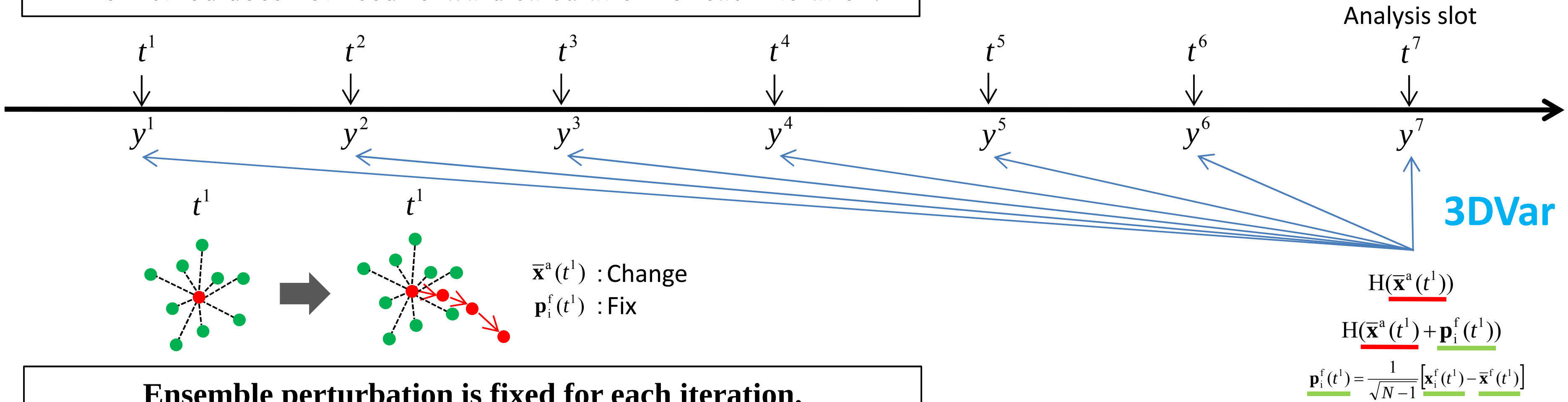


4D method of EnVar (Extension of control variables)



Ensemble perturbation is fixed for each iteration.
This method does not need forward calculation for each iteration.

$\overline{\mathbf{x}}^{a}(t^j),(j=1,\cdots,7)$: Change
$\mathbf{p}_i^f(t^j),(j=1,\cdots,7)$: Fix



Ensemble perturbation is fixed for each iteration.
All slots are considered as with analysis slot.

Algorithm of the NHM-4DEnVar

○Cost function in ensemble space

$$J(\hat{\overline{\mathbf{w}}}^a) = \frac{1}{2}(\hat{\overline{\mathbf{w}}}^a)^T \hat{\overline{\mathbf{w}}}^a + \frac{1}{2}[H(\overline{\mathbf{x}}^f + \hat{\mathbf{P}}_f^{1/2} \cdot \hat{\overline{\mathbf{w}}}^a) - \mathbf{y}]^T \mathbf{R}^{-1} [H(\overline{\mathbf{x}}^f + \hat{\mathbf{P}}_f^{1/2} \cdot \hat{\overline{\mathbf{w}}}^a) - \mathbf{y}]$$

○Gradient of cost function

$$\frac{\partial J}{\partial \hat{\overline{\mathbf{w}}}^a} = \hat{\overline{\mathbf{w}}}^a + \mathbf{R}^{-1} \left[\frac{\partial H}{\partial \hat{\overline{\mathbf{w}}}^a}\right]^T [H(\overline{\mathbf{x}}^f + \hat{\mathbf{P}}_f^{1/2} \cdot \hat{\overline{\mathbf{w}}}^a) - \mathbf{y}]$$

$$\frac{\partial H}{\partial \hat{\overline{\mathbf{w}}}^a} = \frac{\partial H}{\partial \overline{\mathbf{x}}^a} \cdot \frac{\partial \overline{\mathbf{x}}^a}{\partial \hat{\overline{\mathbf{w}}}^a} = \frac{\partial H}{\partial \overline{\mathbf{x}}^a} \cdot \hat{\mathbf{P}}_f^{1/2}$$

$$\frac{\partial H}{\partial \overline{\mathbf{x}}^a(t^k)} \cdot \mathbf{p}_i^f(t^k) \approx H(\overline{\mathbf{x}}^a(t^k) + \mathbf{p}_i^f(t^k)) - H(\overline{\mathbf{x}}^a(t^k)) \qquad \mathbf{p}_i^f(t^k) = \frac{1}{\sqrt{N-1}}\left[\mathbf{x}_i^f(t^k) - \overline{\mathbf{x}}^f(t^k)\right]$$

n x n matrix

$$\hat{\mathbf{P}}_f^{1/2} = \begin{bmatrix} \mathbf{P}_f^{1/2}(t^1) & \mathbf{P}_f^{1/2}(t^2) & \cdots & \mathbf{P}_f^{1/2}(t^n) \\ \mathbf{P}_f^{1/2}(t^1) & \mathbf{P}_f^{1/2}(t^2) & & \mathbf{P}_f^{1/2}(t^n) \\ \vdots & & \ddots & \vdots \\ \mathbf{P}_f^{1/2}(t^1) & \mathbf{P}_f^{1/2}(t^2) & \cdots & \mathbf{P}_f^{1/2}(t^n) \end{bmatrix}$$

Correlation between slots in 4D system is considered by $P_f^{1/2}$ with no-diagonal components

$$J(\hat{\overline{\mathbf{w}}}^a),\ \frac{\partial J}{\partial \hat{\overline{\mathbf{w}}}^a} \qquad \hat{\overline{\mathbf{w}}}^a\Big|_{step=0} = 0$$

LBFGS

$$\hat{\overline{\mathbf{w}}}^a\Big|_{step} = \tilde{\overline{\mathbf{w}}}^a$$

$$\hat{\overline{\mathbf{w}}}^a = \left[\overline{\mathbf{w}}^a(t^1), \overline{\mathbf{w}}^a(t^2), \cdots, \overline{\mathbf{w}}^a(t^n)\right]^T$$

Extension of control variables (all slots)

4D system has been implemented and is being tested

○Optimal analysis of ensemble mean

$$\tilde{\overline{\mathbf{x}}}^a = \overline{\mathbf{x}}^f + \hat{\mathbf{P}}_f^{1/2} \cdot \tilde{\overline{\mathbf{w}}}^a$$

$$\mathbf{P}_f^{1/2} = \left[\mathbf{p}_1^f, \mathbf{p}_2^f, \cdots, \mathbf{p}_N^f\right] = \frac{1}{\sqrt{N-1}}\left[\mathbf{x}_1^f - \overline{\mathbf{x}}^f, \mathbf{x}_2^f - \overline{\mathbf{x}}^f, \cdots, \mathbf{x}_N^f - \overline{\mathbf{x}}^f\right]$$

Summary

In the case of typhoon Talas in 2011, we carried out assimilation experiments of 1-point observation, and confirmed the following things;

- Positive increment was appeared along typhoon's circulation, and flow-dependent pattern was shown with a circulation enhancement.（3DEnVar）

- Experiment results were weighted with a distance between observation and analysis slots. Correlation between observation and analysis slots were not considered. (3DEnVar-FGAT)

4D system

- 4D system is currently being tested for the case of typhoon Talas.