A numerical study on the successive formation of Meddy-like lenses

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[1] We present a process study on the sustaining mechanism of lens formation using a series of numerical experiments of a density current over a sloping bottom. With a cape along the coastline, water parcels in the bottom density current are shed into the offshore region, leading to periodic formation of anticyclonic lenses as part of baroclinic dipolar vortices. The cyclonic partner is more prominent at the surface, and the coupled vortices are carried by the mean current established in the offshore region. Parameter dependence of dipole generation is examined, which suggests that the background current is necessary for the detached eddies to be coherent in the downstream direction and for shedding events to repeat [Nof and Pichevin, 1996]. It is also shown that the density mixing of the bottom current provides criteria for cyclogenesis at the sea surface. A detailed analysis is given by a five-layer model forced by a water mass source/sink, which reproduces the baroclinic dipolar vortices similar to those observed in the preceding z-coordinate model. We find that the dipole generation is due to the finite amplitude divergence/convergence of the baroclinic current passing the cape [Stern and Chassignet, 2000]. The overall analyses suggest three necessary conditions for successive eddy formation: (1) a localized variation in the coastline causing the finite amplitude disturbance, (2) mixing of Mediterranean Water with the surrounding fluids leading to anticyclonic rotation of Meddies as well as cyclogenesis at the surface, and (3) background currents that advect the detached vortices out of the source region. INDEX TERMS: 4255 Oceanography: General: Numerical modeling; 4219 Oceanography: General: Continental shelf processes; 4520 Oceanography: Physical: Eddies and mesoscale processes; 4528 Oceanography: Physical: Fronts and jets; KEYWORDS: eddy shedding, dense water plume, numerical simulation


1. Introduction

[2] The ubiquity of the submesoscale coherent vortices is now recognized in the world’s oceans [McWilliams, 1985; Shapiro and Meschanov, 1991; Maximenko and Yamagata, 1995; Chiswell and Sutton, 1998; Lukas and Santiago-Mandujano, 2001]. Many of these lens-like structures have been identified by their anomalous water mass characteristics or by anticyclonic rotation present in subsurface float trajectories. In particular, one of the best-known examples of such anticyclonic lenses are Meddies (Mediterranean salt lenses) which have been extensively surveyed in the eastern North Atlantic. Meddies are shed from a continental boundary current, called the Mediterranean Undercurrent, which originates in the dense water overflow from the Strait of Gibraltar. As the discharged Mediterranean Water flows westward along the Iberian continental slope [Rhein and Hinrichsen, 1993; Baringer and Price, 1997a, 1997b], it gradually cascades downslope and equilibrates at depths of 500–1500 m near Portugal [Zenk and Armi, 1990]. Further downstream, it leaves the continental slope in regions of steeply sloping topography [Bower et al., 1995; Cherubin et al., 2000]. Some Meddies show two vertically aligned maxima in salinity and temperature which resemble the double core feature observed in the Mediterranean Undercurrent near Cape St. Vincent, the southwestern corner of the Iberian Peninsula [Ambar and Howe, 1979a, 1979b; Zenk et al., 1991; Daniaux et al., 1994]. Bower et al. [1997] estimate that about ten Meddies form per year near Cape St. Vincent, and seven near the Tejo Plateau (the Estremadura Promontory) for a total of 17 per year.

[3] Recently, Serra et al. [2002] analyzed subsurface float data in the vicinity of Cape St. Vincent, finding that floats can become trapped either in anticyclonic lenses (Meddies) or in cyclonic vortices. This indicates that the anticyclones are accompanied by cyclones; this supports an early report by Prater and Sanford [1994]. Carton et al. [2002] presented the three-dimensional structure of two Meddies in the Gulf of Cadiz that were coupled with tall nearby cyclones. These observations may be compared with the
laboratory experiments of Sadoux et al. [2000], who simulated an intermediate current passing a corner along a sidewall. The most striking feature in the experiment was the shedding of dipolar vortices when the upstream current was unstable. A similar process was observed in the numerical experiment of Jungclaus [1999], in which a dense outflow over a sloping bottom was examined. The predominant baroclinicity of the density current led to the formation of a dipole that detached from the slope. The cyclonic partner eventually dissipated, leaving behind only a (Meddy-like) anticyclone. The above results suggest that an intrinsic generation mechanism exists for dipolar vortices associated with anticyclonic lenses [cf. Spall, 1995].

[5] There are many theories for the formation of Meddy-like lenses; see Serra et al. [2002] for extensive references. Here we abstract those relevant to the present study. McWilliams [1985] suggested that mixing in the Mediterranean Undercurrent is necessary for generating low potential vorticity water inside the anticyclones. According to D’Asaro [1988a, 1988b], the eddy shedding from a boundary current passing a corner can be ascribed to bottom frictional torques. Stern and Chassignet [2000] show that the inclusion of a finite amplitude velocity convergence in the boundary current leads to a complete separation of eddies. They further suggested that local topographic variations may create a blocking wave which induces separation, consistent with the measurements of Meddies shed from limited regions of steep coastal topography such as Cape St. Vincent and the Estremadura Promontory. In addition to the processes related to Mediterranean Water itself, some effects from the ambient waters have been suggested for the translation of Meddies. Hogg and Stommel [1990] considered how a Meddy can be advected by mean currents in the upper thermocline. Nof and Pichevin [1996] demonstrated that isolated vortices are successively formed in the presence of the β effect, and that newly formed vortices can exit the water source region in a short period of time [see also Cenedese and Whitehead, 2000]. Aiki and Yamagata [2000] showed that the fast drift of an anticyclonic lens realized in the 2.5-layer model is due to an additional pressure gradient force associated with the upper-layer vortex. Morel and McWilliams [1997] assumed a surface cyclone right above an anticyclonic lens to explain the southward drift of Meddies in the Canary Basin. Studying the origin of the Azores current, Özgökmen et al. [2001] demonstrated that a diapycnal volume flux due to bottom water mixing can induce cyclones in the upper layers.

[5] We present a process study on the sustaining mechanism of lens formation based on a series of numerical experiments. In section 2, the problem of water intrusion is formulated as simply as possible by introducing an idealized coastal ocean. The effect of a localized variation in the topography is analyzed in section 3. Successive formation of anticyclonic lenses as a part of baroclinic dipolar vortices is shown with full three-dimensional structure. The origin of dipolar vortices is examined in section 4. We demonstrate that eddy shedding is influenced by the mixing of the bottom water with the surrounding fluids. Parameter dependence of dipole generation is discussed in section 5. In section 6, we introduce a five-layer hydrostatic model forced by a source/sink of water mass to give a detailed look at the foregoing eddy shedding phenomenon. We conclude with a summary in section 7.

2. Model Description

[6] Numerical experiments are made using the nonhydrostatic Boussinesq equations. The computational domain is an idealized coastal ocean approximating the Iberian continental slope. The model is forced by dense water released from an opening in the lateral model boundary, while surface winds and surface heat-salinity fluxes are neglected. Two unique aspects of the calculation are the role of the nonhydrostatic pressure that hydraulically controls the dense water discharge, and the subgrid-scale parameterization of mixing between the dense water and the ambient fluid.

2.1. Formulation

[7] A rectangular basin with a uniformly sloping bottom is introduced as the model domain (Figure 1), which is an idealization of the Iberian continental slope. The model is forced by dense water released from an opening in the lateral model boundary, while surface winds and surface heat-salinity fluxes are neglected. Two unique aspects of the calculation are the role of the nonhydrostatic pressure that hydraulically controls the dense water discharge, and the subgrid-scale parameterization of mixing between the dense water and the ambient fluid.

Figure 1. Schematic view of the model configuration.
which deepens linearly away from the northern coastal wall located at $y_0 = -4$ km. The coastal ocean is connected to an imaginary marginal sea by a 40 km wide by 600 m deep breach, which is located in the northern wall at $(x, y) = (200$ km, 0 km). This represents the exit (not the sill) of the Strait of Gibraltar at around 6º30’W where the bottom is about 600 m deep. From the breach, dense water is arranged to overflow onto the continental slope, which generates currents inside the basin. The basin is also connected to another imaginary ocean at the southern end of the domain $(y = -400$ km), however, its influence is negligible because the flow in the far region is weak throughout the integration. Details of the open boundaries are described in section 2.2.

Jungclaus [1999] used a radiation condition at offshore boundaries and found rather complex movement of the detached eddies in the downstream region. Therefore we adopted a cyclic condition at the zonal boundaries, terminating simulation before deep water signals wrapped around the domain. A zonal channel is used instead of a meridional channel to consider possible influences of the planetary $\beta$ effect (this is evaluated in section 6.3.2).

The effect of irregular coastal topography is investigated by adding a cape at $(x, y) = (0$ km, $y_0)$. The cape, represented by an isosceles triangle, was 40 km wide in the zonal direction and 40 km long in the meridional direction; the cape’s sidewall height increased from 600 m at the northern coastline $(y = y_0)$ to 1000 m at the cape head $(y = y_0 - 40$ km). The bottom topography was thus made as simple as possible to find out minimum model components for the subsurface eddy shedding, but we note that our cape geometry is characterized by blocked PV (f/h) contours that are not realistic. Observations show that the Mediterranean Undercurrent experiences a strong cascade at the Portimao canyon before passing Cape St. Vincent [Serra et al., 2002]. Such local effects of leading the dense water to a deeper level is partly realized by the sidewall of the cape in the present model. This corresponds to the idealistic topography used in the numerical experiment of Carton et al. [2002], in which the southwestern corner of the Iberian peninsula is represented by a cape extending out to a flat bottom in the model Gulf of Cadiz (see their Figure 16).

The problem to solve is the time evolution of the dense water intruding into the surrounding environment. This suggests a two-fluid system in which the density $\rho$ of a water parcel is decomposed into two parts:

$$\rho(x,y,z,t) = \rho_{\text{Ad}}(x,y,z,t) + \rho_{\text{Med}}(x,y,z,t),$$

where $\rho_{\text{Ad}}$ is the density of the ambient water in the eastern North Atlantic and $\rho_{\text{Med}}$ is the density anomaly associated with the Mediterranean Water (MW) at the breach. The time evolution of $\rho_{\text{Ad}}$ and $\rho_{\text{Med}}$ are calculated separately:

$$\frac{\partial}{\partial t} \rho_{\text{Ad}} + \nabla \cdot (\mathbf{v}_{\text{Ad}}) = F_v(\rho_{\text{Ad}})$$

and

$$\frac{\partial}{\partial t} \rho_{\text{Med}} + \nabla \cdot (\mathbf{v}_{\text{Med}}) = F_v(\rho_{\text{Med}}),$$

where $F_v$ is the net effect of diffusion. This formulation is suitable for our purpose of investigating water and dynamical characteristics of the dense water plume. The integration period is 120 days (terminated before MW signals go around cyclic boundaries).

The velocity $\mathbf{v} = (u, v, w)$ in equations (3) and (4) satisfies the nonhydrostatic incompressible Boussinesq equations

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \mathbf{f} \times \mathbf{v} = -\nabla p - \rho g \mathbf{z} + \mathbf{F}_v(\mathbf{v})$$

and

$$\nabla \cdot \mathbf{v} = 0,$$

where $\mathbf{f} = (0, f \tan \theta - \beta y \tan \theta, f + \beta y)$ is the Coriolis vector at $\theta = 37^\circ$N (parallel to the polar axis). Here, $p$ is the pressure written as a sum of three terms [Marshall et al., 1997]:

$$p(x,y,z,t) = p_\text{at}(x,y,t) + p_\text{ad}(x,y,z,t) + p_\text{med}(x,y,z,t).$$

The first term is the sea surface pressure, $p_\text{at} = g \rho_0 h$, with the reference density $\rho_0$, the gravity acceleration $g$, and the (free) sea surface height $h$. The second term is the hydrostatic pressure, $p_{\text{hy}} = \int_0^z g \rho g dz$, with the last term being the nonhydrostatic pressure. The $\mathbf{F}_v$ on the right hand side of equation (5) represents the net effect of viscosity. The Navier-Stokes equations, equations (3)–(6), are numerically solved using a finite difference method based on Marshall et al. [1997]. The flux-corrected transport (FCT) scheme [Boris and Book, 1973; Zalesak, 1979] is used for the advection of the tracers, $\rho_{\text{Ad}}$ and $\rho_{\text{Med}}$.

The evolution of the free surface elevation $\eta(x,y,t)$ is predicted using the semi-implicit scheme of Dokumovic and Smith [1994]. The discretization is made in the Cartesian coordinates with a uniform grid spacing of $\Delta x = \Delta y = 4.0$ km and $\Delta z = 40$ m.

Initially, the ambient fluid is at rest inside the domain, and the sea surface is level (i.e., $v = 0, \eta = 0$ cm). The interior fluid is stratified with an exponential profile (Figure 2):

$$\rho_{\text{at}}(z) = \rho_0 + \Delta \rho \left(1 - e^{-z/h}\right), \quad \rho_{\text{med}}(z) = 0.0 \text{ kg m}^{-3}.$$

The parameters are chosen from the basic state in the eastern North Atlantic; the reference density $\rho_0 = 1026.0$ kg m$^{-3}$, the density difference between the surface and bottom $\Delta \rho = 2.5$ kg m$^{-3}$, and the vertical scale of the thermocline $h = 1$ km. The Brunt-Väisälä frequency becomes 55.8$^\circ$ at the surface and 33.8$^\circ$ at $z = -1000$ m. As for the vertical modes at an area where the bottom is 1000 m deep (flat-bottom modes), the radii of deformation are 1131 km for the barotropic mode, 25.1 km for the first baroclinic mode and 33.8 km for the second baroclinic mode.

We introduce two quantities for convenience. Ertel’s potential vorticity is written as

$$q = \frac{f + \nabla \times \mathbf{v} : \nabla \rho}{f \Delta \rho / \| \mathbf{v} \|},$$

where $\| \mathbf{v} \|$ is the norm of the velocity vector. The Rossby number is defined as

$$R_o = \frac{\| \mathbf{v} \|}{f \Delta \rho / \| \mathbf{v} \|},$$

where $f$ is the Coriolis parameter, $\Delta \rho$ is the density difference between the ambient and the dense water, and $\| \mathbf{v} \|$ is the norm of the velocity vector. The Rossby number determines the time scale of Rossby waves. The time scale of Rossby waves is given by

$$\tau = \frac{f \Delta \rho}{\| \mathbf{v} \|}.$$
which is made nondimensional to achieve \( q = 1 \) at \( z = -1000 \) m (see Figure 2a). The vertical component of the relative vorticity is defined as \( \zeta = v_x - u_y \).

### 2.2. Interocean Water Exchange

[13] North of the breach \( y > 0 \) km, the MW initially occupies the bottom depths below \( z = -600 + l_b \) as illustrated in Figure 3. We release the bottom water at \( t = 0 \) and derive its subsequent evolution. The stratification is same as in the interior above this depth. The MW condition at the breach is thus

\[
\rho_{\text{Med}} = \begin{cases} 
\delta \rho, & -600 \leq z \leq -600 + l_b, \\
0.0 \text{ kg m}^{-3}, & -600 + l_b < z \leq 0 \text{ m},
\end{cases}
\]

where the density anomaly \( \delta \rho = 0.6 \) kg m\(^{-3}\) and the bottom layer thickness \( l_b = 120 \) m (Figure 2b). These values are based on the observations of the Mediterranean Undercurrent near the exit of the Strait of Gibraltar at 6\(^\text{30}'\)W [Johnson et al., 1994a, 1994b; Baringer and Price, 1997a, 1997b]. The virtual model Mediterranean Sea behind the breach is at rest, so that \( \rho_{\text{Atl}} \) and \( \rho_{\text{Med}} \) outside the analysis domain are fixed at their initial values during the integration. The sea surface height is additionally fixed at \( \eta = 0 \) cm for \( y > 0 \) km (i.e., in the Mediterranean Sea) during the integration and the nonhydrostatic pressure gradient \( \nabla P_{\text{NL}} \) is assumed to vanish at \( y = 0 \) km. These conditions for the density field yield a hydrostatic pressure which is always higher to the north of the breach. The normal velocity at the boundary is then computed by equation (5) in the same manner as in the interior. As a result, the MW at the bottom of the breach is discharged over the continental slope by the combined hydrostatic and nonhydrostatic pressure gradient force. Although the density of the incoming MW is specified, the corresponding transport of the overflow is initially unknown and determined during the calculation. The compensating surface transport out through the breach is dictated by the assumption of constant sea level in the Mediterranean Sea (see Figure 3). In other words, by using these boundary conditions for the pressure field, we idealize the hydraulically controlled water exchange at the Strait of Gibraltar. This is why nonhydrostatic pressure contributions are allowed in the present model. We note however that their contributions are significant only in the vicinity of the breach; elsewhere the flow field is basically hydrostatic.

[14] The southern open boundary, which is located at \( y = -400 \) km, is treated in a similar manner except their is no breach on this side. Thus \( \rho_{\text{Atl}} \) and \( \eta \) for \( y < -400 \) km are exactly the same as the initial state in the interior, while \( P_{\text{SH}} \) is assumed to vanish in order to reduce gravity waves. It turns out that this open boundary condition at \( y = -400 \) km acts like a quiescent sponge layer. The free surface, \( \eta_{\text{Atl}} = 0 \) km and \( \eta_{\text{Med}} = 400 \) km are kept equal (= 0 cm) in most of our experiments. Possible impacts of a difference in sea surface height between the northern and southern model boundaries will be investigated in section 5.1.

[15] It is convenient for our discussion to introduce the normalized concentration of the MW, defined by

\[
m(x, y, z, t) = \frac{\rho_{\text{Med}}(x, y, z, t)}{\delta \rho}.
\]

Thus \( m \) equals to unity at the bottom layer of the breach. The spatial evolution of the MW will be diagnosed by the vertical integral of the anomalous MW density,

\[
\int_{\eta_{\text{Med}}}^{0} \rho_{\text{Med}} dz/(\delta \rho) = \int_{\eta_{\text{Med}}}^{0} m dz/l_b
\]

(normalization is made with respect to the original value specified at the breach). This is suitable for lenses with three-dimensional structure, because the above integral captures the vertical elongation of the MW even if \( m \) is reduced during its dispersion.

### 2.3. Theoretical Background

[16] Prior to showing the numerical results, the dynamics related to the slope current are discussed briefly. The dense bottom water propagates westward parallel to the slope...
isobaths after geostrophic adjustment. If the overlying layers are motionless, the translation speed becomes [Nof, 1983]

\[ c_b = \frac{g'}{f} s = 0.65 \text{ms}^{-1}, \]  

(11)

where the reduced gravity acceleration \( g' = g \xi \rho / \rho_0 \). In the linear limit, this corresponds to the wave speed of the bottom-intensified long topographic wave discussed by Rhines [1970]. Despite its simplicity, equation (11) is applicable to any isolated features on a sloping bottom, including eddies and plumes.

[17] Barotropic topographic planetary waves are also excited in the present experiment. These waves, responsible for the establishment of surface currents, can be described using the linearized potential vorticity equation for the barotropic mode:

\[ \frac{\partial}{\partial t} \left( \nabla^2 \eta - \frac{f^2}{gH} \eta \right) - F \frac{H_e}{H} \eta_b = 0, \]

(12)

where the total depth \( H(y) = -\eta_\text{bottom} \). For the situation of strait coastline without cape, the boundary conditions are \( \eta_{y=0} = \eta_{y=-400 \text{km}} = 0 \) cm because of the large horizontal extent of the barotropic Rossby radius). Solving the eigenvalue problem (12) gives the phase speed of the meridional normal modes. The three fastest waves are 6.05 m s\(^{-1}\), 1.57 m s\(^{-1}\), and 0.69 m s\(^{-1}\) with increasing meridional wave number; the primary wave travels the analysis basin in two days. In diagnosing model results, we should keep in mind that these barotropic signals wrap around the cyclic boundaries and may influence the eddy study region.

2.4. Subgrid-Scale Parameterization

[18] The net effect of viscosity \( F_v \) in equation (5) is explicitly written as follows:

\[ F_v(u) = [v_h 2(u_x + v_x)]_x + [v_h (u_x + v_x)]_y - ru, \]

\[ F_v(v) = [v_h (v_x + u_x)]_y + [v_h (2v_x) + [v_h (v_x + v_y)]_y - rv, \]

\[ F_v(w) = [v_h (w_x + u_x)]_z + [v_h (v_x + v_y)]_y + [v_h (w_x + v_y)]_z, \]

(13)

where \( v_h \) and \( v_r \) are eddy viscosity parameters in the horizontal and vertical directions, respectively. At \( z = z_\text{bottom} \), the bottom friction of \( f = 1.25 \times 10^{-6} \text{s}^{-1} \) is applied [cf. Gawarkiewicz and Chapman, 1995; Tanaka and Akitomo, 2001], while the coastal walls are made slippery by setting the relative vorticity \( \zeta = 0 \) on the coastline. The net effect of diffusivity \( F_n \) in equations (3) and (4) is written as follows:

\[ F_n(p_i) = [\kappa_b (p_i)]_x + [\kappa_b (p_i)]_y + [\kappa_b (p_i)]_z, \quad (i = \text{Med}, \text{Atl}), \]

(14)

where \( \kappa_b \) and \( \kappa_n \) are eddy diffusivity parameters in the horizontal and vertical directions, respectively.

[19] Although the grid spacing is fine enough to resolve Meddies, subgrid-scale parameterization is needed for the frontal instability of the density current on the bottom slope; its lateral scale is approximately given by the internal deformation radius of the bottom water: \( \sqrt{\frac{gH_s}{f}} = 7.9 \text{ km} \) [cf. Kikuchi et al., 1999; Gawarkiewicz, 2000]. For simplification, we assume that the overall effect of these small eddies is represented by the horizontal viscosity itself, while the vertical viscosity and diffusivity are fixed (\( \nu_v = \kappa_v = 0.1 \text{ cm}^2 \text{s}^{-1} \)). Here, we derive the horizontal eddy viscosity using a local balance in the turbulent kinetic energy (TKE). The present approach is almost the same as that of Smagorinsky [1963], except that our formula is designed for the dense water discharged over the sloping bottom. We assume that the mechanical production of TKE is mostly due to vertical distortion in the velocity field, and is in equilibrium with the dissipation of TKE:

\[ \nu_h \left[(u_x + w_x)^2 + (v_x + w_y)^2 + w_z^2 \right] \sim \epsilon, \]

(15)

where \( \epsilon \) is the dissipation rate. The form of \( \epsilon \) is determined from dimensional analysis:

\[ \epsilon \sim u_h^3 v_n^{-1} l^{-4}, \]

(16)

where \( l \) is a length scale. The simplest relationship is obtained for \( n = 2 \), yielding the horizontal eddy viscosity related to the vertical distortion

\[ \nu_h = C(\Delta x \Delta y)^{1/2} \Delta z \left[(u_x + w_x)^2 + (v_x + w_y)^2 + w_z^2 \right]^{1/2}. \]

(17)

Hereafter we adopt \( C = 0.8 \) in all experiments. The Prandtl number \( Pr = u_h / \kappa_h \) is unity unless otherwise noted. The smallest allowable value for both \( u_h \) and \( \kappa_h \) was 1 m s\(^{-1}\). [20] Note that the horizontal viscosity in equation (17) is derived considering the anisotropic nature of the small eddies. In other words, TKE is produced by the eddies in the vertical plane, whereas it is dissipated by the eddies in the horizontal plane. This contrasts with the Smagorinsky model used in OGCMS, in which horizontal viscosity is related to the horizontal distortion

\[ \nu_h = C(\Delta x \Delta y)^{1/2} \Delta z u_x \left[(u_x + w_x)^2 + (v_x + w_y)^2 + w_z^2 \right]^{1/2}. \]

(18)

Equation (18) is based on two-dimensional fluid dynamics, and is applicable when the horizontal scale is extremely large compared to the vertical scale. In the present calculation, however, isopycnals can slope steeply within the density current. Since \( u_x \) and \( v_x \) in equation (17) are approximately in the thermal wind balance, the eddy viscosity becomes large in the regions where there are steep isopycnal slopes. Physically, equation (17) represents the collapse of the thermal wind by small geostrophic eddies. Diapycnal mixing is thus represented by \( \kappa_h \) instead of \( \kappa_v \) in this model (i.e., overall effects of eddy diffusivity are included in the horizontal diffusivity parameter \( \kappa_h \) itself). With this simplification, we can better understand what is happening at the subgrid-scale by focusing only on the horizontal diffusivity (viscosity) parameter. Our choice of equation (17) is a first step toward parameterizing the mixing of dense overflow water with overlying fluids [cf. Price and Baringer, 1994].
Since many studies use diapycnal diffusivity estimated in the vertical direction rather than in the horizontal direction, we note here that the vertical evaluation is given by $k_h$ times the square of the isopycnal slope [Joyce, 1977], for example, it is 100 cm² s⁻¹ for $k_h = 100$ m² s⁻¹ and $s = 0.01$. Values of $k_h$ are presented in Figure 17a, which are of order 100 m² s⁻¹ for the density current and of order 10 m² s⁻¹ for the detached eddies. These values are acceptable both in the horizontal and vertical directions [cf. Daniault et al., 1994], and the Veronis effect, which is severe in low-resolution z-coordinate models, does not matter in the present study.

3. Simulated Flows

In this section, we present the results of two experiments. The first calculation (hereafter called as FLAT) is performed without the cape in Figure 1, and the second one (hereafter CAPE) uses the same initial and boundary conditions with the cape. In the FLAT experiment, the instability of the density current along the flat coastline is found insufficient to produce separated eddies, whereas the introduction of the cape leads to shedding of baroclinic dipolar vortices containing anticyclonic lenses underneath.

3.1. Water Intrusion in the Absence of the Cape (FLAT)

Driven by the pressure gradient force, the MW imposed at the bottom layer of the breach begins to intrude into the analysis basin. The MW immediately veers counterclockwise over the continental slope as a result of geostrophic adjustment, and forms a bottom-intensified boundary current flowing westward along the slope isobaths [cf. Jiang and Garwood, 1998; Etling et al., 2000]. The westward progression is clearly seen in Figure 4a with a slight offshore displacement of the current path. Although downstream disturbances appear in the slope current by $t = 40$ days (Figure 4b), the dense water continues propagating westward as seen at $t = 60$ days (Figure 4c). The translation speed of the western edge of the plume is about 1/5 of the analytical estimate of equation (11) for a 1.5-layer model. This suggests that the plume interacts with the surrounding environment.

A vertical cross section at $x = -400$ km is shown in Figure 5. The MW is trapped on the slope over a width of about 60 km (Figure 5a). Most of the water parcels are still heavy compared to the ambient water at the same depth, although the offshore edge of the plume approached its neutral density level. The neutral depth of the MW (around $z = 900$ m) is shallower than that expected if there were no mixing (around $z = 1200$ m, see Figure 2). This is due to the significant decrease in the concentration of the MW (Figure 5a); the maximum value at this section is 0.44. The density current experiences strong diffusion during its westward progression. As seen in Figure 5b, the density front around $z = -800$ m is associated with baroclinic currents in thermal wind balance. The bottom current is flowing westward with a (negative) maximum of $u = -20$ cm s⁻¹, while the surface current is flowing eastward.
with a maximum of $u = 15$ cm s$^{-1}$, yielding a Richardson number of about 30. This implies stability in terms of Kelvin-Helmholtz overturning. However frontal or baroclinic instability is possible because the timescale for the maximum growth rate, $0.3/\text{Ri}$, is about 10 days based on Eady’s theory [Eady, 1949]. Nevertheless, the density current does not produce pinched off eddies, but exhibits only disturbances embedded in the current. The FLAT experiment appears to lack a mechanism for ejecting eddies out of the boundary current. Moreover, the bottom and the upper currents are comparable in magnitude, and the lateral shift in the two maxima exhibits barotropic, cyclonic shear. The FLAT case is unable to simulate the observed structure of the anticyclonic lenses off the Iberian Peninsula.

This result contrasts with the numerical experiment of Jungclaus [1999], in which detachment is observed every 100 days for a density current on a flat bottom slope. He concluded that the ambient stratification prevented the downslope movement of the developing disturbances of the bottom boundary current, and so the dense plume penetrated offshore. However, his result shows rather complicated movement of the eddies in the offshore region; it is difficult to distinguish whether the eddies (or lenses) are fully separated from the coastal boundary current. We wonder if the numerical experiment of Jungclaus [1999] is significantly influenced by his downstream boundary condition.

### 3.2. Water Intrusion in the Presence of the Cape (CAPE)

As in the FLAT, the discharge of the MW on the slope produces a density current flowing westward in parallel to the slope isobaths until it reaches the cape (Figure 6). In contrast to FLAT, the southward deflection of the current by the cape generates a plume of MW at $t = 20$ days (Figure 6a). Instead of returning to $y = y_0$, the plume begins to move southwestward and lose contact with the bottom. At $t = 40$ days (Figure 6b), the southwest shift of the plume sheds a lens approximately 80 km in diameter. The lens initially moves southward and reaches the offshore region around $y = -200$ km at $t = 60$ days (Figure 6c). By this time, a new plume emerges between this first lens and the cape as a result of the continuous supply of MW from the breach. When the first lens drifts westward, the new plume develops a second lens. At $t = 80$ days (Figure 6d), the second lens moves westward leaving behind a new plume, while the first lens remains near $x = -200$ km where it was at $t = 60$ days. The primarily westward translation of the second lens contrasts with the movement of the first one that remains at nearly the same position even at $t = 100$ days. Consequently, there are three shed lenses in the basin at $t = 100$ days (Figure 6e). A vertical section crossing the westernmost lens (Figure 7) shows that it is clearly disconnected from the bottom, and floating at its neutrally buoyant level of $z = -900$ m with a thickness of 400 m. The concentration of the MW at the center of the lens is reduced to 0.43, which is consistent with its neutral depth. Among

![Figure 6](image-url)
the three lenses, the floating depths and thicknesses are almost the same (not shown). Interestingly, compared to the core of the MW exhibited in the FLAT (Figure 5), the MW lenses obtained in CAPE have very similar vertical extent and MW concentration, despite their drastically different spatial excursions.

[27] Figure 8 shows a three-dimensional distribution of the MW. There are actually two lenses in the offshore region. The third feature is sickle shaped at \( t = 100 \) days. Note that the three features are associated with negative relative vorticity, indicating anticycloic rotation. However, upstream of the cape where the majority of the MW is in contact with the bottom slope, we see disturbances in the relative vorticity of both signs. It follows that the anticycloic rotation of the lenses is induced just after the MW parcels are shed offshore.

[28] The model lenses are amazingly similar to features observed in the North Atlantic; the diameter, thickness and depth of the simulated lenses are in good agreement with the salinity anomalies associated with Meddies [cf. Richardson and Tychensky, 1998]. Moreover, the formation interval of about 30 days obtained in this CAPE experiment is very close to the recent estimate that about ten Meddies are formed in a year in the vicinity of Cape St. Vincent and seven near the Estremadura Promontory [Bower et al., 1997].

[29] The concurrent evolution of the vorticity field at intermediate depth is shown in Figure 9. At \( t = 20 \) days (Figure 9a), we see a localized signal of negative vorticity that corresponds to passage of the first plume past the cape (Figure 6a). At \( t = 40 \) days (Figure 9b), in agreement with the westward displacement and separation of the MW lens, the signal evolves into a dipole. At this time, the axis of the dipolar vortex, with an anticycloic (cycloic) part on its right (left), is directed southwestward. Note that the core of the MW is trapped only in the anticycloic part of the dipole. At \( t = 60 \) days (Figure 9c), the first dipole is swept southwestward with its axis rotating counterclockwise, while a second dipole emerges behind. At \( t = 80 \) days (Figure 9d), the first dipole remains near \((x, y) = (-200 \text{ km}, -220 \text{ km})\), but its axis is directed to the east. The aforementioned southward movement of the first lens (Figures 6c–6e) can thus be attributed to the rotation of the axis of the first dipole. North of the first dipole, the second dipole appears after detachment but its cyclonic part is relatively narrow. At this same time, a third dipole is generated off the cape. It is again noted that the MW is contained in the anticyclonic part. At a later time of \( t = 100 \) days (Figure 9e), when the second lens moves westward passing the first one, the two dipolar vortices merge their cyclonic parts that eventually dissipate, leaving behind two anticycloic monopoles. The relative vorticity of each monopole is about \(-0.2f\) near the center, with the swirling speed being as large as 20 cm s\(^{-1}\) at the periphery. Meanwhile, there is a relatively wide band of the westward current (about 4 cm s\(^{-1}\)) in the offshore region between \( y = -200 \) km and \( y = -20 \) km. This partly explains the westward translation of the dipoles, although the drifting speed varies between 4 and 8 cm s\(^{-1}\). This range of the translation speed is slow compared to the rotation of the core, so that parcels of MW are trapped in the anticyclones.

[30] If the mutual induction is responsible for the translation of the dipolar vortices, translation speed of a dipole is given by 0.29 times the maximum axial velocity of the vortex, based on the analytical theory in Appendix A. This formula gives \( c = 8.0 \text{ cm s}^{-1} \) for the first dipole in Figure 9b, which actually moves southwestward with a speed of 6.3 cm s\(^{-1}\). The second dipole in Figure 9c moves westward at 8.3 cm s\(^{-1}\), and the formula gives \( c = 7.62 \text{ cm s}^{-1} \). Mutual induction is certainly needed to move newly formed dipoles offshore but it is not sufficient to account for the total westward translation of the dipoles. Moreover, the direction of translation is not exactly aligned with the axis of the dipole. Thus we have to take into account the surrounding flow field as suggested by Hogg and Stommel [1990]. Details of the translation mechanism will be discussed in section 6.3.2.

[31] In the reminder of this section, we will explore the three-dimensional structure of the dipolar vortices. Figure 10 shows a vertical section of relative vorticity through two dipolar vortices. The regions of negative vorticity are vertically bounded between \( z = -1500 \) m and \( z = -500 \) m, indicating that the anticyclones are concentrated at middepth. In contrast, the cyclones are surface intensified. Vortex pairs are thus vertically tilted although magnitude of the positive and negative vortices are approximately the same: \(|\zeta| \sim 0.2f\). The baroclinic coupling is clearly shown in Figure 11. There are two tall cyclones in the downstream region. Although the merged cyclone looks dissipated at \( z = -900 \) m (Figure 9e), this is actually not true. The cyclone is concentrated near the surface and is coupled with the two anticyclones underneath. On the other hand, the cyclone closer to the cape is surrounded by a filament of anticyclonic vorticity, corresponding to the sickle-like structure of the MW seen in Figures 6e and 8.

[32] The sea surface height (SSH) in Figure 12 also confirms the development of surface cyclones. At \( t = 20 \) days (Figure 12a), the SSH shows a zonally uniform crest along \( y = -60 \) km. This is established by topographic Rossby waves, whose propagation speed is as fast as \( c = -6.05 \text{ m s}^{-1} \) from equation (12). These waves are presumably associated with the interaction of the slope current and
the cape (this will be verified in section 6.3.1). An eastward jet is established along the northern coastline associated with the narrow front. There is a wide westward current in the offshore region where the SSH gradient reverses; this supports the existence of the middepth background current (Figure 9e). East of the cape, there is a small region of low SSH so that the MW plume underneath (Figure 6a) is in contact with the surface cyclone as early as this stage. At \( t = 40 \) days (Figure 12b), the nose of the low SSH separates into a cyclone of 100 km in diameter, which coincides with the formation of a middepth dipolar vortex (Figure 9b). The detached surface cyclone is surrounded by the mean current and thus travels westward. Another small cyclone is formed at \( t = 60 \) days (Figure 12c) when the second dipole emerges (Figure 9c). At \( t = 80 \) days (Figure 12d), the second cyclone merges into the first so that the new cyclone is coupled with the two lenses deeper down, the first lens to the south and the second one to the northwest (Figure 6d). At the same time, a third cyclone is about to separate from the cape. The detachment completes at \( t = 100 \) days (Figure 12e), thus the two cyclones are present at an offshore distance of \( y = -200 \) km (corresponding to the ones in Figure 10 and Figure 11). We

Figure 8. Isosurface of the MW concentration, \( m = 0.2 \), at \( t = 100 \) days for the CAPE experiment. The color on the isosurface indicates relative vorticity \( \zeta \) with a unit of \( f \); negative is red and positive is blue. The inset is an enlarged view of the eastern two cores of MW.
note that the three lenses exhibited in Figure 6 are all located beneath the sea surface front surrounding the cyclones. As the cyclones are shed intermittently, the background SSH exhibits constant growth of the crest height. At $t = 100$ days, the westward mean current attains a speed of about 4 cm s$^{-1}$ such that the background movement is synchronized between the surface and intermediate layers, while the eastward jet along the coastline is as fast as 30 cm s$^{-1}$.

[33] The overall results of the CAPE experiment are summarized schematically in Figure 13. At middepth, the oncoming slope water from the east is diverted at the cape to form anticyclones. At the surface, the boundary jet flowing eastward is established as a result of the combination of the surface water drainage into the breach and the topographic Rossby waves. Hence the region of the slope current, east of the cape, is predominantly baroclinic. Surface cyclones develop and grow east of the cape, and upon reaching a certain size, are successfully swept offshore. Then the surface vortices are carried by the weak westward current, while coupled with anticyclones deeper down. Because the cyclonic vortex is taller than the anticyclonic vortex, the pair revolves counterclockwise while drifting in the downstream region.

[34] Some “dynamical” characteristics (i.e., dipolar vortices, counter coastal current) of the above system are relevant to the observations in the eastern North Atlantic, although actual geophysical comparison is limited because of the idealistic model configuration. Käse and Zenk [1996] suggested that the fast translation of a Meddy is the result of a dipole structure. Richardson et al. [2000] presented trajectories of some Meddies surrounding a large cyclonic vortex. They also suggested that floats launched only in the warm salty cores are biased in favor of anticyclones; this is confirmed by our numerical results (e.g., Figure 8). Our eastward jet along the coastline may be relevant to the observation of a

Figure 9. Time evolution of relative vorticity $\zeta$ at $z = -900$ m for the CAPE experiment: $t = (a)$ 20 days, (b) 40 days, (c) 60 days, (d) 80 days, and (e) 100 days. The unit is $f$. Figure 9e is shown with velocity vectors at the same depth.
4. Formation Mechanism of the Dipolar Vortex

The preceding numerical results have demonstrated that the anticyclonic lenses are formed as part of baroclinic dipolar vortices. In this section, we discuss the origin of the anticyclonic and cyclonic vortex of the dipole. It will be shown that density mixing within the slope current is crucial in producing the vortices.

4.1. Formation of the Subsurface Anticyclones

The isolated MW features in the offshore region suggest that there are intrinsic differences in other physical quantities between the MW and the ambient water. Indeed, Figure 14 captures three regions of closed contours in the potential vorticity (PV) field at $z = -900$ m. These correspond to the three anticyclonic lenses (Figure 6). The minimum PV value at the center of each lens is less than

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**Figure 11.** Relative vorticity isosurfaces, $\zeta = -0.1f$ in red and $\zeta = +0.1f$ in blue, at $t = 100$ days for the CAPE experiment. The inset is an enlarged view of the eastern two anticyclones and the cyclone past the cape.

southward Portuguese current along the Iberian coast by Arhan and Verdière [1994], in which a southward transport of 2 Sv is suggested in the eastern boundary layer [cf. Fiuza et al., 1980; Mazé et al., 1997].
0.25; it is compared to the background value of 1.1 of the ambient water (Figure 2a). The low PV corresponds to the vertical elongation of the water column (Figure 7) since the Rossby number (<0.2) is relatively small. Were such low-PV patches to shrink vertically owing to horizontal divergence after shedding, we expect generation of negative relative vorticity on the basis of potential vorticity conservation [cf. McWilliams, 1985]. Once a lens becomes detached from the cape, vorticity will be conserved during its subsequent excursion.

[37] The vertical profile of PV is shown in Figure 15 to check the criterion of baroclinic and barotropic instability. In the source region (Figure 15a), we can see low PV values, $q < 0.25$, confined to the bottom layer, whereas there is a PV maximum, $q > 4$, just above the density current corresponding to the thin layer thickness in between the bottom water and the upper fluids (compare Figure 2b). It follows that there is change of sign in the vertical gradient of $q$. This satisfies the necessary condition of baroclinic instability [cf. Charney and Stern, 1962]. In contrast, there is no predisposition for barotropic instability in the horizontal PV gradient. The patch of high PV in the upper layer continues to the downstream region until the body of the MW hits the cape (Figure 15b). Thus the density current is baroclinically unstable in the whole region, $0 \text{ km} < x < 200 \text{ km}$, where the MW is on the bottom slope. Downstream of the cape (Figure 15c), there is no significant patch of high PV visible in the upper layers, but baroclinic instability is still indicated by the sign change in the vertical PV gradient between the upper and lower flanks of the intermediate current [cf. Morel and McWilliams, 2001].

[38] To describe the downstream evolution of PV inside the MW core, we introduce a space- and time-averaged quantity:

$$F(q,x) = \frac{\int_{0}^{120 \text{ days}} \int \int q(x,y,z,t) H(m) \, dydz \, dt}{\int_{0}^{120 \text{ days}} \int H(m) \, dydz \, dt},$$

where the surface integral $\int \int dydz$ covers the whole range of a meridional section at $x$. The discriminant $H(m)$ will collect values only for the concentration $m$ greater than 0.2, such as

$$H(m) = \begin{cases} 1, & \text{for } m > 0.2, \\ 0, & \text{for } m < 0.2. \end{cases}$$

Figure 12. Time evolution of the sea surface height for the CAPE experiment: $t =$ (a) 20 days, (b) 40 days, (c) 60 days, (d) 80 days, and (e) 100 days. The contour interval is 1 cm and the shade is for values under 1 cm. Velocity vectors are included in Figure 12e.

Figure 13. Schematic of the current system established in the CAPE experiment.
When necessary, $F$ is generalized for the other variables by substituting for $q$ in equation (19). Figure 16a shows the downstream evolution of the PV inside the MW core from the source to the downstream region beyond the cape. Near the breach at $x = 200$ km, the potential vorticity achieves almost the same value ($q = 1.5$) as that specified at the open boundary (Figure 2b). As a parcel flows westward, the mean PV starts to decrease and becomes lower than that of the surrounding fluid at about $x = 100$ km. The PV reduction is achieved by the mixing with overlying fluids, which makes $-f_0$ in equation (8) close to zero, and the vertical shear near the bottom, with which $-u_f p_r$ becomes negative. This tendency continues until the parcel reaches the cape at $x = 0$ km, where $q = 0.5$. Once the parcel passes the cape and separates from the boundary, it keeps a constant value ($q = 0.6$); the slight increase of $q$ is caused when the parcels of the slope water form a lens. The MW is thus transformed into low-PV water in the region between the breach and the cape where the majority of the MW stays on the bottom slope.

In Figure 16b, we find that the relative vorticity within the MW core changes sign at the cape, from positive value to negative. In the upstream region near the source ($100$ km $< x < 200$ km), we find positive relative vorticity ($\zeta < 0.1 f$). This is presumably generated by the stretching of the water column when it descends the slope. As the parcel of MW moves westward, the relative vorticity vanishes, indicating considerable disturbances associated with the frontal or baroclinic instability. Further downstream where the parcel hits the sidewall of the cape, the horizontal convergence produces positive vorticity. However, west of the cape where the MW is ejected into the offshore region, the vorticity...
becomes negative and remains almost constant at \( \zeta = -0.1 f \). This is consistent with the eventual appearance of the anticyclone. It is noteworthy that only after the parcel of the MW is ejected offshore that negative vorticity appears. Therefore the anticyclone is first formed when the low-PV water is pumped into the offshore environment of higher PV.

[40] The density current descends between the breach and the cape from \( z = -600 \) m to \( z = -900 \) m beyond the cape where the lenses are formed. This downslope sinking, followed by frontal instability, causes mixing of the MW with the surrounding water; this is responsible for the aforementioned PV conversion. This process is parameterized by means of the eddy diffusivity in this model as expressed in equation (17) and is shown in Figure 17a. Relatively large values of \( \kappa_h \) (\( \sim 100 \) m\(^2\) s\(^{-1}\)) are achieved in the region of the slope current. The maximum near the breach (\( \kappa_h > 200 \) m\(^2\) s\(^{-1}\)) is associated with intrusive turbulence induced by the nonhydrostatic pressure. The MW concentration is as small as \( m = 0.5 \), half of the original value, near the breach (Figure 17b). On the other hand, downstream from the cape, the diffusivity is reduced to \( \kappa_h \sim 10 \) m\(^2\) s\(^{-1}\). It follows that the MW concentration as well as the PV is approximately conserved as the parcels travel beyond the cape. In summary, the low-PV water is formed in a limited area where an inherent diffusive mechanism operates; PV is conserved once the MW loses contact with the bottom.

[41] The model lenses manifest how the initial MW is transformed into low-PV water. In Figure 6, the vertical integral of the MW density anomaly is around 1.0 at the core of the lens. Interestingly, this is almost unchanged from the original value at the breach, despite the fact that the concentration at the lens center is greatly reduced (\( m < 0.3 \)) (Figures 7 and 17b). This indicates that an effect equivalent to vertical diffusion is induced by the combination of horizontal diffusion and horizontal convergence. The density current on the bottom slope appears to involve an intrinsic PV conversion mechanism, contrasting with open ocean convection in which low-PV water is produced by isotropic eddy activity within the mixed layer [Marshall and Schott, 1999].

[42] The mixing process within the slope current causes amplification of the plume transport (Figure 17c). The along-slope transport is 0.5 Sv near the source region at \( x = 200 \) km, which is in good agreement with observational estimates [Baringer and Price, 1997a]. Further downstream, as the mixing leads to the entrainment of ambient water, the transport reaches 1.0 Sv near the cape at \( x = 0 \) km. Indeed, this 100% amplification is consistent with the observation of the Mediterranean Undercurrent [cf. Rhein and Hinrichsen, 1993; Baringer and Price, 1997a]. West of the cape (\( x < 0 \) km), where MW parcels are ejected into the offshore, the transport remains at about 0.8 Sv. The finite amplitude undulation about \( x < -200 \) km (Figure 17c) is due to the southward migration of the first lens (Figures 6c–6e), and the noise is due to the limited number of lenses farther downstream.

### 4.2. Formation of the Surface Cyclones

[43] At the surface, the water parcels exiting the basin through the breach is likely to produce a narrow jet along the coastline (this will be further examined in section 6.3.1). Note that the jet flows eastward, opposite to the density current underneath, and experiences meanders in the wake (east) of the cape (Figure 12). Since we have adopted the free-slip lateral boundary, we cannot ascribe this meandering to the boundary friction at the cape. Thus the positive vorticity in the wake region must be produced by the stretching of surface water columns.

[44] Two mechanisms are suggested for the stretching in the vicinity of the cape. When the coastal jet passes the cape from the west to the east, the water columns along the coastline are deflected southward at the cape’s western sidewall. The increase in bottom depth leads to the stretching of water columns. This mechanism will be addressed in detail in section 6.2, because this kind of adiabatic process is better captured by the multilayer model introduced in section 6. The second mechanism stems from the diapycnal volume flux associated with the mixing and entrainment of the bottom water. In fact, the volume flux due to the entrainment is comparable with the downstream transport.
of the slope current (Figure 17c). Here, we examine this second mechanism which is essentially diabatic. Adopting a 1.5-layer model, Özgökmen et al. [2001] introduced a sink in the surface layer to represent the diapycnal volume flux entrained into the underlying layers. The connection between the cyclogenesis and the bottom entrainment is illustrated in Figure 18 for a continuously stratified fluid. Let us consider an isopycnal, \( r = r_1 \), between the surface and bottom fluids over the bottom slope. Near the bottom, diffusion of the cascading density current leads to a down-gradient transfer of density, thus the isopycnal is displaced upward in accordance with the increase of the in situ density. However, to achieve steady state, there must be an up-gradient density flux which cancels the down-gradient effect (i.e., isopycnals only move if there is a divergence in the buoyancy flux). Because of the stable stratification (\( \rho < 0 \)), this can be represented by vertical advection, as follows:

\[
wp_{\text{T}} = k_{\text{h}} \nabla^2 \rho, \quad \text{at } \rho = \rho_1, \tag{21}
\]

where it is noted that diapycnal mixing is represented by the horizontal diffusivity \( k_{\text{h}} \) in the present model (see section 2.4 for details). Equation (21) indicates that there should be a downward movement, \( w < 0 \), of surface water parcels crossing the isopycnal, \( \rho = \rho_1 \). Therefore surface water columns are always subjected to stretching, which accounts for the development of the positive vorticity near the sea surface:

\[
\frac{\partial \zeta}{\partial t} = - \left( f + \frac{c}{n} \right) \frac{w}{H}, \quad \text{at } \rho < \rho_1, \tag{22}
\]

The essence for this cyclogenesis is the diapycnal volume flux associated with the mixing-induced entrainment into the dense plume.

[45] In order to understand the effect of the entrainment on the vorticity production, the foregoing CAPE experiment was repeated for various Prandtl numbers, keeping the other parameters unchanged. With increasing Prandtl number, we expect that cyclogenesis will weaken owing to the reduction in entrainment. Indeed, for the extreme case of \( Pr = 18 \) (= 10^{1.25}), a narrow density current occurs beyond the cape with its nose located at around \( x = -500 \) km (Figure 19a). Although the cape diverts the density current offshore, the majority of the MW remains in contact with the bottom with its offshore edge descending below \( z = -1000 \) m. The sea surface height for this run in Figure 19b shows no detached cyclones apart from one weak feature at \( x = -470 \) km; this corresponds to the nose of the density current underneath. The background sea surface height exhibits the zonally extending ridge along \( y = -60 \) km with a maximum of \( h = 8 \) cm. Hence the surface current system is as in the CAPE experiment (for \( Pr = 1 \)) with a narrow eastward jet along the coastline and a wide offshore region with a weak westward current. The essential difference between the two experiments is the nature of the subsurface current. This result confirms that diabatic effects, together with baroclinic interaction, are necessary for cyclone formation.

[46] The present approach is not perfect in identifying the formation mechanism of the surface cyclones, however, because increasing the Prandtl number also changes the
characteristic of the MW. Less mixing of MW retains high plume density values leading to a deeper current path. Using Figure 16, we check the downstream evolution of the vorticity in the MW core for the case with $Pr = 18 (= 10^{1.25})$. PV is certainly higher than that of the ambient fluids in the region between the cape and the breach (the dotted line in Figure 16a). However, we find a drop in PV at $x = 0$ km where the dense water experiences finite amplitude convergence as it hits the cape. This may indicate underestimate of the vertical shear near the bottom in the calculation of PV (namely $u_d$, at $z = z_{bottom}$). However, it is out of the scope of the present study to go into detail of technical issues related to model resolution and boundary condition of the staggard grid. The relative vorticity suddenly becomes negative at $x = 0$ km (the dotted line in Figure 16b) as in the CAPE experiment. This contrasts with the foregoing explanation in section 4.1 based on the shrinking of the water columns, and remains to be examined. We will again touch this issue in section 6 using a simplified model of water intrusion.

[47] Figure 20 summarizes the series of experiments for $1 \leq Pr \leq 10^{1.25}$. The eddy diffusivity inside the body of the slope current is inversely proportional to the Prandtl number (Figure 20a), indicating that the corresponding eddy viscosity is of the same magnitude ($\nu_h \sim 100 \text{ m}^2 \text{ s}^{-1}$). As expected, the larger Prandtl number leads to less detachment of the MW past the cape (Figure 20b). The largest southward shift occurs when $Pr = 1$. For $1 \leq Pr \leq 10^{0.5}$, where the diffusivity is larger than $80 \text{ m}^2 \text{ s}^{-1}$, parcels of MW detach from the bottom slope to form anticyclonic lenses. In this range, the diapycnal volume flux is sufficient to produce surface cyclones. For $10^{1.25} \leq Pr \leq 10^{1.25}$, the majority of the MW remains on the bottom without shedding eddies. These experiments demonstrate that the strength of the mixing provides a criteria for generating dipolar vortices.

[48] One distinct difference between successive formation of isolated eddies and instability of a jet, such as the barotropic and baroclinic instability, is the reestablishment of the initial state [cf. Nof and Pichevin, 1996]. In the present experiment, once a surface cyclone is swept sufficiently far away, the flow condition near the cape returns to its original state. The recovery of the mean current is crucial for the separation events to be repeated. Since the mean current at the surface is produced by topographic Rossby waves in the present model (this will be verified in section 6.3.1), the boundary condition of the sea surface height at the breach ($y = 0$ km) and at the southern end ($y = -400$ km) should have certain impact on the movement of the dipolar vortices. We will examine these issues in the following section.

5. Remote Impacts of the Separation Event

[49] The foregoing analysis suggests that separation of dipolar eddies results from a collection of dynamically linked processes: topographic effects, low PV formation within the bottom current, offshore ejection of MW at the cape, cyclogenesis induced by entrainment, baroclinic coupling within the dipolar vortices, and mean current advection of the surface cyclones. It indicates that the overall behavior of the MW may be well affected by fluctuations of these processes. In this section, we investigate the robustness of the generation of dipolar vortices and explore possible pathways of the MW. This is also relevant to the seasonal variation of the background environment in the eastern North Atlantic [cf. Arhan and Verdière, 1994; Mazed et al., 1997]. Indeed, historical observations of spreading cores of Mediterranean Water in the Iberian Basin have been classified into several branches [Shapiro and Meschlanov, 1996].

5.1. Offshore Mean Current

[50] At the surface, cyclones are successively swept offshore and carried by the westward mean current. As demonstrated by Nof and Pichevin [1996] using a 1.5-layer model, the separation is attributed to the background condition rather than the localized instability near the coast. In order to examine the role of this mean current, the CAPE experiment was repeated by changing the sea surface height at the southern boundary; a lowered (lifted) $\eta_{y=-400\text{km}}$ increases the offshore zonal current to the west (east). The other parameters were kept the same.

[51] When there is an additional westward current, we see three lenses at $t = 100$ days (Figure 21a). The first lens is located at $x = 170$ km after passing the cyclic boundary, followed by the second and the third lenses. As in the CAPE, the lenses are found in the anticyclonic part of baroclinic dipolar vortices (not shown). Interestingly, the axes of the dipoles are each directed southwestward. The fast downstream advection causes larger intervals between the vortices and reduces their nonlinear interaction. Thus a periodic array of the lenses is realized when the vortices are rapidly swept away from the cape region. When there is an additional eastward current, the MW spreads rather irregularly (Figure 21b) (note that the offshore current is still flowing westward). The first lens is now located at $(x, y) = (0 \text{ km}, -200 \text{ km})$, hardly different from its detachment site. The subsequent ejection of MW produces a plume extending westward beyond the first lens, whose nose is located at $(x, y) = (-250 \text{ km}, -120 \text{ km})$. At this time, a large surface cyclone is present at $(x, y) = (-150 \text{ km}, -180 \text{ km})$ (not shown) and is surrounded by the patches of MW. The overall movement of MW in the offshore region is highly
influenced by this surface cyclone. In both experiments, surface cyclones appear and promote the ejection of MW, whether its structure is a lens or a plume.

5.2. Source Water Density

A southward veering of the slope current at the cape is needed for the MW to form anticyclonic lenses after being pumped into a high-PV environment. In order to investigate how differences of the slope current alter the spreading of the MW, we conducted experiments by changing $d\rho$, the MW density anomaly at the breach. The other parameters were kept the same as the CAPE run.

When the source water is relatively light, we see two small lenses in an array at $t = 100$ days (Figure 22a). Regardless of the weakening of the upstream density current, periodic formation of the dipolar vortices is observed. As in the run of the fast offshore advection (Figure 21a), the two dipolar vortices are detached with their axes directed southwestward. In this case, the small size of the eddies reduces the interaction between the dipoles. It appears that the two experiments shown in Figure 22a and Figure 21a have captured a similar regime of eddy separation. On the other hand, when the source water is relatively dense, a wide slope current is generated with no separated eddies (Figure 22b). The current axis is shifted southward, which is in agreement with the total descent of the body of MW. The slope current is now too wide to be blocked by the cape; thus ejection of MW lenses is impossible. This suggests that the pathway of the MW depicted in Figure 22b is similar to the FLAT experiment without the cape. In order to shed eddies, the slope current must reach its neutral depth before it is blocked by the cape. Therefore in the present topography, lenses cannot be produced at deeper layers below $z = -1000$ m, where the head of the cape intersects the bottom slope. This is confirmed by the fact that all lenses obtained in the present work extend vertically in the range between $z = -1000$ m and $z = -600$ m (not shown, but one in Figure 7). We suggest that the vertical range of the detached lenses is generally determined by the structure of the cape.

6. A Detailed Analysis Using a Layer Model

In this section, we introduce a five-layer hydrostatic model with a source/sink of water mass to investigate the essence of the eddy shedding simulated in the CAPE experiment. The introduction of a transposed point source of MW next to the cape [cf. Defant, 1955] proves to reproduce baroclinic dipolar vortices similar to those observed in section 3.2. A series of experiments is presented to clarify (1) how the dipolar vortices separate from the cape, (2) whether or not the surface cyclones are due to the diapycnal volume flux and (3) the source of the background current.

6.1. Five-Layer Source/Sink Model

We use a five-layer shallow water equation model to simplify the problem of water intrusion, with the computa-
mixing of the dense water in the upstream region, and focus our attention on the spreading of the injected water mass.

Our formulation is partly relevant to the numerical studies of Aiki and Yamagata [2000] in which a subduction process is idealized by a point source in the intermediate layer, and Özgökmen et al. [2001] in which a diapycnal volume flux due to bottom water mixing is represented by a regional sink in the surface layer. Introducing the injection (detrainment) of water mass $Q_{\text{inj}}$ ($Q_{\text{det}}$) in the continuity equation of the third (second) layer, we write the governing equations as follows:

$$\frac{D\mathbf{u}_i}{Dt} + (f + \beta y) \mathbf{u}_i \times \mathbf{u}_i = -\frac{g}{\rho_0} \nabla \eta - \nabla p_i - \frac{\varepsilon}{\delta^2} \frac{Q_{\text{det}}}{\delta^2} \mathbf{u}_2 - \frac{\varepsilon}{\delta^2} \frac{Q_{\text{inj}}}{\delta^2} \mathbf{u}_3,$$

(23)

$$\frac{\partial h_i}{\partial t} + \mathbf{u}_i \cdot \nabla (\mathbf{u}_i h_i) = \frac{\varepsilon}{\delta} Q_{\text{det}} + \frac{\varepsilon}{\delta} Q_{\text{inj}},$$

(24)

where $\mathbf{u}_i = (u_i, v_i)$, ($i = 1, 2, 3, 4, 5$), are the horizontal velocity with $h_i$ being the layer thickness ($\delta_j$ is Kronecker’s delta). The other notation is conventional: the Coriolis parameter $f + \beta y$, on a $\beta$ plane at $37^\circ$N, the gravity acceleration $g$, the sea surface elevation $\eta$, the hydrostatic pressure $p_i$, the Lagrangian derivative $D/Dt = \partial/\partial t + \mathbf{u}_i \cdot \nabla$ together with the horizontal gradient operator $\nabla = (\partial/\partial x, \partial/\partial y)$. The localized source in the third layer is located just west of the cape at $(x_c, y_c) = (-20 \text{ km}, 0 \text{ km} - 35 \text{ km})$ with a Gaussian form:

$$Q_{\text{inj}} = \frac{4I_{\text{inj}}}{\pi R_{\text{inj}}^2} \exp \left[ -\frac{4 \left( (x-x_c)^2 + (y-y_c)^2 \right)}{R_{\text{inj}}^2} \right],$$

(25)

where $I_{\text{inj}}$ is the injection rate. The injection radius is set to $R_{\text{inj}} = 12 \text{ km}$. Uniform detrainment in the second layer is arranged over a box region defined by $0 < x < 220 \text{ km}$ and $-40 < y - y_0 < 0 \text{ km}$, so that $Q_{\text{det}} = I_{\text{det}}/(210 \text{ km} \times 40 \text{ km})$; the detrainment rate divided by the area of the trapezoid. To avoid vanishing layer thickness during the calculation, the detrainment is conditionally stopped at areas where $h_2 < H_2/2$.

We have conducted a series of numerical experiments for a variety of $(I_{\text{inj}}, I_{\text{det}})$. In particular, the combination of $(I_{\text{inj}}, I_{\text{det}}) = (1.0 \text{ Sv}, -0.5 \text{ Sv})$ is suggested by the CAPE experiment in which the transport of the density current is about $0.5 \text{ Sv}$ $(= I_{\text{cor}})$ near the covering area and about $1.0 \text{ Sv}$ near the cape (see Figure 17c): $I_{\text{inj}} = I_{\text{cor}} + (-I_{\text{det}})$ with $-I_{\text{det}}$ being the effect of the entrainment. The numerical integration of equations (23) and (24) is carried out using a finite difference scheme based on Arakawa and Lamb [1981]. The flux-corrected transport (FCT) scheme [Boris and Book, 1973; Zalesak, 1979] is used for the layer thickness. The development of the free surface elevation $\eta(x, y, t)$ is given by the semi-implicit scheme. The discretization is made in Cartesian coordinates with a uniform grid spacing of $\Delta x = 4 \Delta y = 4.0 \text{ km}$. Although not explicitly expressed in equation (23), we adopt eddy viscosities in the horizontal and vertical directions. The horizontal viscosity is given by the conventional Smagorinsky model with a constant of $C = 0.1$ [Smagorinsky, 1963]. The stress at the interfaces of the adjacent layers is estimated
using a coefficient corresponding to $\nu_c = 0.1 \text{ cm}^2 \text{s}^{-1}$. As in section 2.4, a Rayleigh friction of $1.25 \times 10^{-6} \text{s}^{-1}$ is applied on the bottom, while the lateral coastal boundary is slippery. The model is integrated for 100 days from the initial condition of no motion (i.e., $h_i = H_i$ and $u_i = 0$), keeping a constant injection (detrainment) rate $I_{\text{inj}}$ ($I_{\text{det}}$). During the calculation, $\eta$ and $h_i$ are fixed at their initial values beyond the two open boundaries: at $y > 0$ km and $y < -400$ km (see section 2.2 for details).

6.2. Simulation Results

This subsection discusses results of two experiments for $(I_{\text{inj}}, I_{\text{det}}) = (1.0 \text{ Sv}, 0.5 \text{ Sv})$ and $(1.0 \text{ Sv}, 0.0 \text{ Sv})$. The first case corresponds to the density current established in the CAPE experiment and is successful in reproducing the baroclinic dipolar vortices observed previously. The second case purposefully investigates whether or not the surface cyclones are due to the diapycnal volume flux.

6.2.1. Flow With the Source and the Sink

When the injection and detrainment rates are fixed at $(I_{\text{inj}}, I_{\text{det}}) = (1.0 \text{ Sv}, 0.5 \text{ Sv})$, baroclinic dipolar vortices are shed repeatedly (Figure 24). The injected water mass is carried away by lenses of about 100 km in diameter and 400 m in thickness; they move westward with a constant speed of 4 cm s$^{-1}$. In contrast to the CAPE, there is no collision of vortices during their excursion so that the three lenses in Figure 24a are simply aligned in order of age. As shown in Figure 24b, the lenses are all anticyclones and each has a cyclonic counterpart; the relative vorticity is about $|\zeta| \sim 0.3f$ at the centers of each pole. As seen in Figure 24c, the surface current system is remarkably similar to that in CAPE: a narrow eastward jet along the northern coastline, a weak westward current in the offshore region, and an array of cyclones present in the offshore front. Noteworthy is that the amplitude of $\eta = 8$ cm for the zonal SSH crest is almost the same as that of the $z$-coordinate calculation (see Figure 12e). As in CAPE, the cyclones are generated east of the cape and repeatedly shed downstream. All these results validate our introduction of the source-sink model. It is also noted that the source of the water mass is located west of the cape. Thus the present result suggests that eddies can be shed from the cape even in the absence of the frontal and baroclinic instability of the density current in the upstream region (i.e., between the cape and the breach).

In order to discuss the origin of the surface cyclones, the second-layer thickness is shown in Figure 24d. The layer thickness is relatively thin ($h_2 = H_2/2$) in the region between the cape and the breach due to the regional detrainment, such that this region is characterized by high potential vorticity (PV) values. The high-PV water parcel is evidently shed from the cape to form the core of the surface cyclones. Hence the surface cyclones carry the water parcels from the coastal boundary current, just like the anticyclonic lenses advect the injected water parcels in the intermediate layer.

6.2.2. Flow Without the Sink

When the first experiment is repeated without detrainment ($I_{\text{det}} = 0$ Sv), we see irregular spreading of the coastal boundary current, just like the anticyclonic lenses advect the injected water parcels in the intermediate layer.

**Figure 24.** Solution for $(I_{\text{inj}}, I_{\text{det}}) = (1.0 \text{ Sv}, 0.5 \text{ Sv})$ at $t = 100$ days. (a) Third-layer thickness $h_3$ (contour interval of 50 m and shade for values larger than 400 m). (b) Relative vorticity $\nabla \times u_3$ in the third layer (contour interval of $0.05f$). (c) Sea surface height (contour interval of 1 cm and shade for regions lower than 1 cm), with velocity vectors at the surface. (d) Second-layer thickness $h_2$ (contour interval of 20 m and shade for values less than 320 m).
water mass from the point source (Figure 25). The injected water parcels form into a series of lenses and plumes which move westward with a speed of 4 cm s\(^{-1}\). Although three blobs are not fully isolated at \(t = 100\) days, all of them are associated with anticyclonic rotation (Figures 25a and 25b). Patches of negative vorticity are coupled with nearby patches of positive vorticity. We observed this kind of irregular spreading in section 5 when the mean current in the offshore region was weak. At the sea surface, two cyclones are embedded in the offshore front (Figure 25c). The dynamical characteristics of these baroclinic eddies are qualitatively similar to those observed in previous runs of both models. It is quite interesting to note that just pumping subsurface water can reproduce the whole current system: anticyclonic lenses at the middepth, surface cyclones, a coastal jet at the sea surface and offshore front associated with the weak westward flow.

Figure 25d provides us with a key to understand how surface cyclones are formed. The detached cyclones are relatively weak compared to the ones in Figure 24, and are subjected to complicated interaction in the downstream region. Patches of the high-PV water exist in the vicinity of the cape in spite of the absence of the detrainment, and are shed as cyclones. The high-PV water originates in the eastward jet along the coastline where the second layer is in contact with the bottom (see Figure 23) and the layer thickness is as small as \(h_2 = 200\) m (\(H_2/2\)) along \(y = y_0\). When the coastal jet passes the cape from west to east, the water column along the coastline is deflected southward at the western sidewall of the cape. The deepening of the bottom leads to the stretching of the water column such that positive (cyclonic) relative vorticity is induced at the head of the cape. This suggests that the diapycnal volume flux is not a unique mechanism for surface cyclogenesis. The situation is similar to that discussed by Stern and Chassignet [2000], in which they demonstrate that finite amplitude convergence (or divergence) of the multilayer coastal current produces the pinch off eddies. The detrainment significantly strengthens the surface cyclones and promotes the subsurface lenses to leave the coast [cf. Morel and McWilliams, 2001]. Indeed, the detached lenses are located at an offshore position of about \(y = -150\) km in Figure 24a (with the sink) whereas it is about \(y = -100\) km in Figure 25a (without the sink). Having said that, we should note that even the second experiment without the sink includes the effects of some diapycnal volume flux, because the specified injection rate of 1.0 Sv regards the sum of the outflow rate \(I_{out}\) and the entrainment rate \(I_{det}\). Both surface current and bottom current are affected by the density mixing.

6.3. Background Current

The preceding analysis demonstrates that cyclones are shed from the coastal boundary current at the sea surface. Here we investigate the origin of the background current and its impact on the translation of eddies.

6.3.1. Topographic Waves

It is essential to know how the basin-wide anticyclone is established to form the eastward jet along the coastline along with the weak westward current in the offshore. In the \(X-T\) diagram of the sea surface height shown in Figure 26, we find a crest in SSH starts from the cape and propagates westward. The background current
is thus produced by the passage of long topographic waves which go around the cyclic boundary. In section 2.3, we estimated the phase speed of the barotropic topographic waves. These waves are sufficiently fast to spread information throughout the basin in a matter of a few days. To identify the source of the wave, another series of experiments is conducted for smaller injection rates ($0.01 \sim 1.0$ Sv) without the effect of the detrainment. In Figure 27, we find that the observed sea surface height is proportional to the injection rate for $I_{\text{inj}} = 0.01 \sim 1.0$ Sv. This suggests that the propagation speed of the raised sea surface is constant (i.e., independent of amplitude). Thus the linear wave theory applies to the response of the sea surface height [cf. Longuet-Higgins, 1965; Davey and Killworth, 1989]. We conclude that the background current is induced by the water pumping at middepth. This raises the interfaces of the upper layers and generates barotropic topographic waves. Also shown in Figure 27 is the dependence of the third-layer thickness that is approximated by $h_3^{\text{max}} - H_3 \propto I_{\text{inj}}^{3/4}$. This indicates that the injected parcels are more efficiently swept off with increasing injection rate. The increase in the propagation speed is reasonable because increasing the amplitude of the third layer leads to responses with lower baroclinic modes; this is clearly shown by Aiki and Yamagata [2000].

### 6.3.2. Translation of Eddies

We here discuss the translation speed of an anticyclonic lens in the presence of a background current. When a lens is embedded in the third layer, the center of the lens is given by $X = \int (h_3 - H_3) x dS / M$ with $M = \int (h_3 - H_3) dS$. According to the integral theorem (see Appendix B), the time evolution of $X$ is expressed as follows:

$$X_t = -\frac{\beta}{M} \int \psi dS - \frac{g}{\rho_0 FM} \int (h_3 - H_3) \eta_y dS - \frac{1}{M} \int (h_3 - H_3) p_y dS \quad (26)$$

$$Y_t = -\frac{\beta}{M} \int \phi dS + \frac{g}{\rho_0 FM} \int (h_3 - H_3) \eta_x dS + \frac{1}{M} \int (h_3 - H_3) p_x dS. \quad (27)$$

In each of the above equations, we call symbolically the first contribution with $\beta$ as “planetary divergence,” the second contribution with the surface height $\eta$ as “barotropic advection,” and the last contribution with the hydrostatic pressure $p$ as “baroclinic advection.” The term with the pressure gradient in each equation is unique to baroclinic vortices; if the third layer alone is active and the other layers are motionless, the integral vanishes because $\int (h_3 - H_3) \nabla (g' h_3) dS = \int \nabla (h_3 - H_3)^2 dS = 0$. In other words, a lens may be advected by influences from the other layers. Table 1 summarizes the translation speeds of an anticyclonic lens. Tracking the simulated lens gives a westward speed of 6.71 cm s$^{-1}$ and a southward speed of 4.39 cm s$^{-1}$. The theoretical speed is in good agreement with the observed speed; the theory explains more than 90% of the translation in the zonal and meridional directions. We find that a major contribution comes from the barotropic advection term so that the lens is carried by the barotropic current. The sea surface height exhibits a gradient region right above the lens; the lens basically follows the contours of this sea surface front. The second largest contribution is from the baroclinic advection term, which is negative. This implies that the hydrostatic pressure tends to reduce the drift of the lens. The smallest among the three contributions is the theoretical speed.

### Table 1. Comparison of the Translation Speeds of a Lens

<table>
<thead>
<tr>
<th>Direction</th>
<th>Westward, cm s$^{-1}$</th>
<th>Southward, cm s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed speed</td>
<td>6.71</td>
<td>4.39</td>
</tr>
<tr>
<td>Theoretical speed</td>
<td>6.48</td>
<td>3.92</td>
</tr>
<tr>
<td>Planetary divergence</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>Barotropic advection</td>
<td>6.88</td>
<td>4.07</td>
</tr>
<tr>
<td>Baroclinic advection</td>
<td>-0.56</td>
<td>-0.30</td>
</tr>
</tbody>
</table>

*See section 6.2.1. The observed speed is simply obtained by tracking the most western lens (in Figure 24) from $t = 80$ days to $t = 100$ days. The theoretical speed is estimated using equations (26) and (27) at $t = 90$ days. Values of the three terms in equations (26) and (27) are listed under the titles of “planetary divergence,” “barotropic advection,” and “baroclinic advection.”
planetary divergence, thus the self-rotation of the lens (i.e., nonlinear planetary wave propagation) is less important in the presence of the barotropic current. From these results, we conclude that the background current is necessary for the subsurface lenses to drift a significant distance from the coastal generation region, in which case isolated lenses are shed repeatedly [cf. Nof and Pichevin, 1996].

7. Summary

[65] We have presented a process study of the sustaining mechanism of lens formation based on a series of numerical experiments of a density current over a sloping bottom. The idealistic coastal ocean used in this paper turns out to be an excellent platform to understand the physical mechanisms involved. It puts into perspective several interacting components of the Mediterranean Outflow system, e.g., Mediterranean Undercurrent, Meddies, Portuguese current and Azores current. We have introduced some useful ideas to elucidate the shedding of Meddy-like eddies. For example, it is shown that vorticity as well as passive tracers are necessary for the investigation and visualization of Meddy-like lenses. This is associated with the way in which Meddies (Mediterranean Salt Lenses) are observed (or defined) in the North Atlantic; Meddies are anticyclonic lenses when they are seen from the anomalously (warm and salty) water characteristic but also they are (not always) part of baroclinic dipolar vortices based on the velocity or vorticity distribution [cf. Morel and McWilliams, 1997; Richardson et al., 2000]. The evolution of the dense water is described by the vertical integral of the anomalous density of Mediterranean Water (Figure 6). Interestingly, this integral is basically a conserved quantity during the excursion of the anticyclonic lenses despite of the strong mixing of Mediterranean Water with the ambient fluids. The established flow field is hydrostatic over the regions of the density current and the separated eddies except for the vicinity of the breach. The nonhydrostatic pressure is needed for the dense water to be discharged from the breach. This is unique to the open boundary condition of the present model which specifies only the pressure field (instead of the inflow/outflow rate across the breach). The open boundary condition is designed to produce reasonable currents in the overlying layers, and is contrasted with previous studies in which the movement of the ambient fluids is not carefully considered.

[66] Numerical results in section 3 show that the spreading of dense water is highly dependent on a localized shape of the coast. The slope current along a straight coast sheds no vortices. In contrast, with a cape along the coastline, water parcels in the bottom density current are remarkably shed into the offshore region, leading to periodic formation of lens-like structures. The detached water parcels are trapped only in the anticyclonic part of baroclinic dipolar vortices; this is qualitatively consistent with the result of Sadoux et al. [2000]. Interestingly, the cyclonic partner is more prominent at the surface. The coupled vortices are carried by the mean current established in the offshore region. This supports the translation mechanism of Hogg and Stommel [1990], in which they discussed how mean currents in the upper thermocline could advect Meddies deeper down. The size and rotation of the anticyclonic lenses simulated in the model are in good agreement with Meddies in the Iberian Basin. Note-worthy is the formation interval of about 30 days, consistent with the measurements of Meddies shed from Cape St. Vincent or the Estremadura Promontory [Bower et al., 1997]. Comparing the present result and Jungclaus [1999], we find that the major difference comes from the boundary condition which determines the movement of the ambient fluid. A larger-scale current in the offshore region seems to be necessary to displace eddies away from the generating coastal boundary current.

[67] In section 4, we investigated how the anticyclonic and cyclonic parts of the dipole are excited. As the released dense water flows along the isobaths of the sloping bottom, it entrains the ambient fluid and produces low-potential-vorticity water near the bottom and high-potential-vorticity water in the upper layers. The vertical profile of the potential vorticity gradient satisfies the necessary condition for baroclinic instability. However, comparing the two experiments with and without the cape (CAPE and FLAT), it seems that baroclinic instability is not sufficient to produce separated eddies [cf. Stern and Chassignet, 2000]. This highlights the role of the cape in eddy separation. The anticyclonic rotation of the detached lenses is largely due to shrinking of water columns as they are ejected from the cape into the offshore stratification [cf. McWilliams, 1985]. As for the cyclogenesis at the surface, we examined how it depends on the diapycnal volume flux due to the bottom water mixing. Additional experiments with reduced mixing show no separated eddies from the density currents, thus mixing appears necessary for lens formation. Another important result given by the parameter study in section 5 is that the spreading of the MW is highly dependent on the differences of the mean current in the offshore region. Horizontal shear seems to be essential to pinch off surface cyclones, which occur in all cases and help the ejection of the MW.

[68] A detailed analysis was given in section 6 using a five-layer model forced by a source-sink distribution of water mass that clarified (1) the formation mechanism of the dipolar vortices, (2) the impact of the diapycnal volume flux on surface cyclones and (3) the origin of the background current and its role in eddy shedding. It was shown that baroclinic eddies are shed even without the effect of detrainment of upper-layer fluids. This suggests that the diapycnal volume flux is not a unique mechanism for surface cyclogenesis. Since the five-layer model ignores frontal and baroclinic instability of the density current in the region between the cape and the breach, the dipolar vortices are basically generated from the nonlinear response of the coastal boundary current to the cape; this corresponds to the pinching off mechanism of Stern and Chassignet [2000]. The detrainment provides an additional effect that strengthens the surface cyclones and promotes the subsurface lenses to leave the coast [cf. Morel and McWilliams, 2001], leading to successive formation of baroclinic dipolar vortices. The counter boundary current at the sea surface is established after the passage of barotropic topographic waves in our experiments. Since the coastal jet is found to shed cyclonic vortices as it passes the cape, the observation
of the coastal boundary current off the Portuguese coast should be interesting. This is probably relevant to the report by Arhan and Verdière [1994] of a southward Portuguese current along the Iberian coast (as part of the North Atlantic subtropical gyre); they suggest a southward transport of 2 Sv in the eastern boundary layer [cf. Fiuza et al., 1980; Mazé et al., 1997]. In the present experiments, the background currents are also important in the offshore region where, by examining the momentum balance of separated eddies, we find that eddies are advected by the barotropic component of those flows. For there to be a successive lenses, a large-scale current is necessary [cf. Nof and Pichevin, 1996].

The analyses of the z-coordinate and layer models suggest three necessary conditions for the successive formation of lenses: (1) a localized variation in the coastline which causing finite amplitude disturbances, (2) mixing of Mediterranean Water with the surrounding fluids leading to anticyclonic rotation of Meddies as well as cyclogenesis at the surface, and (3) background currents which advect the detached vortices out of the source region. Although this is the subject of future study, there might be a nondimensional parameter which can categorize the regimes of Meddy formation. This is along the line of Baey et al. [1995] who presented a diagram of the nonlinear intermediate current behavior in Rossby, Burger number parameter space. Taking into account the effects of density mixing and the cape, the new parameter of 

\[ \text{CAPE parameter} = \frac{\text{density difference} \times \text{speed of the offshore current}}{\text{viscosity} \times \text{size of the obstacle}} \]

where the distance between the two bifurcation points, “B” and “C,” defines the dipole diameter (= 2a). Kelvin’s circulation theorem is applied to a semicircle surrounding the positive vorticity region.

\[ \psi_{\text{int}} = \left( r - \frac{2J_1(3.83) \text{c}}{3.83} \right) \frac{c \sin \theta}{a^2}, \]

where \( J_0 \) and \( J_1 \) are the ordinary Bessel functions \( J_i(3.83) = 0 \) is the gravest zero point. From these solutions, the axial velocity is as in the following:

\[ u = \begin{cases} \frac{a^2}{x^2}, & |x| > a, \\ -c & |x| \leq a. \end{cases} \]

\[ u = -c \left( 1 - \frac{2J_1(3.83)c}{3.83} \right) \frac{\text{a}}{\text{b}(3.83)}, \]

Since the line integral from “A” to “D” vanishes at infinity, the circulation around the semicircle is determined. Thus the translation speed is related to the circulation (= \( \Gamma \)) and the radius of the dipolar vortex, as follows:

\[ c = 0.2926 \frac{\Gamma}{a}. \]

Note that this formula is applicable to any dipolar vortices with arbitrary distribution as long as the axial velocity is represented as in equation (A2). Moreover, a useful relationship is derived from equation (A2),

\[ c = 0.2871 \times \text{the maximum axial velocity}. \]

which directly estimates the mutual induction once the velocity maximum is obtained.

**Appendix A: Translation Speed of a Dipolar Vortex**

[70] Dipolar vortices observed in the present calculation can be compared with analytical solutions found in classical geophysical fluid dynamics, such as a pair of point vortices or a Modon [Lamb, 1932; Larichev and Reznik, 1976]. However, the former solution is known to overestimate the mutual induction of the dipole’s movement in a rotating system, and the latter is limited to the quasi-geostrophic framework [cf. Umatani and Yamagata, 1987]. Here, a universal formula for the mutual induction is derived based on Kelvin’s circulation theorem.

[71] We consider a dipole translating with a constant speed \( c(>0) \) in a two-dimensional flow field (Figure A1). This poses a nonlinear problem of \( J(\psi + cy, \nabla^2 \psi) \). The exterior solution is given by

\[ \psi_{\text{ext}} = \frac{a^2}{r} c \sin \theta, \]

where the origin of the cylindrical coordinate is fixed at the center of the dipole of radius \( a \). In order to evaluate the circulation around a semicircle surrounding the region of the positive vorticity, the axial velocity must be fixed between the bifurcation points (“B” and “C”). From Lamb [1932], the interior solution is given by

\[ \psi_{\text{int}} = \left( r - \frac{2J_1(3.83) \text{c}}{3.83} \right) \frac{c \sin \theta}{a^2}, \]

Figure A1. Schematic of a dipolar vortex under discussion. The solid (dotted) contours denote the positive (negative) vorticity region such that this dipole drifts in the direction of the arrow from “D” to “A” with a speed \( c \). The distance between the two bifurcation points, “B” and “C,” defines the dipole diameter (= 2a). Kelvin’s circulation theorem is applied to a semicircle surrounding the positive vorticity region.

**Appendix B: Translation Speed of a Lens**

[72] We consider the translation speed of an anticyclonic lens in the presence of a background current. In a planet-
ary geostrophic limit, the momentum and continuity equations in the third layer, equations (23) and (24), are expressed as

\[ \mathbf{u} = u_x, \quad p = (g/\rho_0)h + p_2 \quad \text{and} \quad h = h_3. \]

\[ M = \int (h - H) dS \] being the volume anomaly of the third layer \((H = H_3)\), the center position of a lens is defined by \( \mathbf{X} = \int (h - H) dS/M \), where the integral area covers all anomalous fields of \( h \). The movement of the lens is determined from time evolution of the layer thickness; this is given by \( \Delta X/dt = \int hu dS \) in a general form [see Killworth, 1983; Cushman-Roisin et al., 1990]. However, in the presence of a background current, we cannot assume static flows \((u = 0)\) in the far field. With this constraint, the movement is better described by an additional partial integral such that \( \int (h - H) dS = \int (h_x/c^2/2, h_y/c^2/2) dS \). For example, the zonal component becomes

\[ M \frac{dX}{dt} = \frac{1}{2} \int (hu_x + u_y) \mathbf{x}^2 dS + \frac{1}{2} \int (h - H) u_x \mathbf{x}^2 dS. \]  

(B2)

Substituting \( u_x + u_y = -\frac{\beta}{2f} u + u = -\frac{\beta u}{2f} u - p_2 f \) yields

\[ -\frac{\beta}{2f} \int (Hv_y + (h - H)u_y) \mathbf{x}^2 dS \quad \text{and} \quad \frac{\beta}{2f} \int (Hv_x + u_x) \mathbf{y}^2 dS \quad \text{and} \quad \frac{\beta}{2f} \int (hu_y) \mathbf{y}^2 dS \quad \text{and} \quad \frac{1}{f} \int (h - H)p_x dS. \]

The first integral on the right hand side again contains the divergence so it is negligible relative to the second integral. The thickness flux in the second is written as

\[ hu = HU_{\text{back}} + x \times \nabla \psi_{\text{lens}} - \nabla \phi_{\text{lens}}. \]

where \( U_{\text{back}} \) is the background velocity and \( \psi_{\text{lens}}(\phi_{\text{lens}}) \) is the vector (scalar) potential of the anticyclonic lens vanishing in the far field. We assume \( U_{\text{back}} \) to be a constant over the integral area because the background quantities are less dependent on \( x \) and \( y \). As a result, the zonal speed becomes

\[ X_t = -\frac{\beta}{FM} \int \psi_{\text{lens}} dS - \frac{1}{\rho_F FM} \int (h - H)p_x dS. \]  

(B3)

The meridional speed, derived in a similar manner, is given by

\[ Y_t = -\frac{\beta}{FM} \int \phi_{\text{lens}} dS + \frac{1}{\rho_F FM} \int (h - H)p_y dS. \]  

(B4)

In spite of the inclusion of the background current, equations (B3) and (B4) are almost the same as equations (4.5)–(4.6) by Aiki and Yamagata [2000] [see also Cushman-Roisin et al., 1990]; one exception is the addition of \( \phi_{\text{lens}} \) in the meridional speed; this is examined in section 6.3.2. The first terms in equations (B3) and (B4) are due to the planetary geostrophic divergence inside the vortex. The second terms in equations (B3) and (B4) are unique for baroclinic vortices, and are associated with the sloping of the overlying (or underlying) layers. Since the first terms in equations (B3) and (B4) only take into account the circulation inside the lens, effects of the background current appear in the second terms with the pressure gradients.

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