Inversion of spatio-temporal variation in interplate slip rate from repeating earthquakes

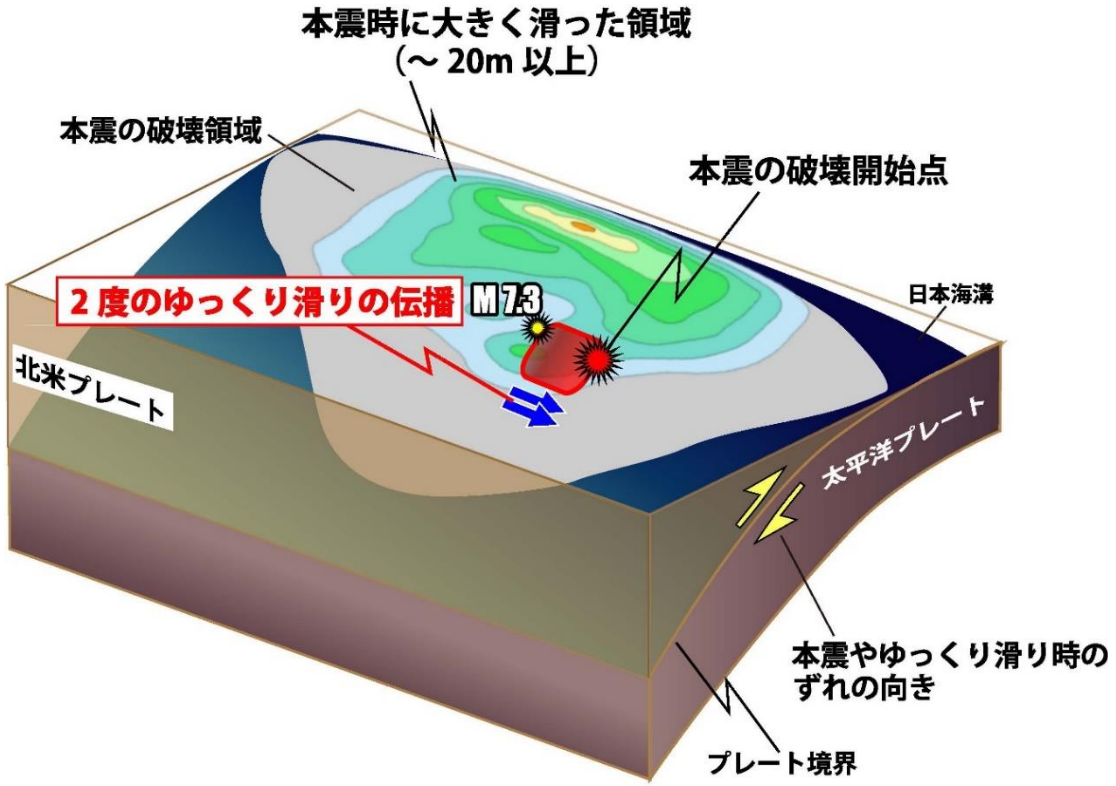
繰り返し地震によるプレート間滑り速度の時空間変動の逆推定

Shunichi Nomura
The Institute of Statistical Mathematics

10th Data Assimilation Workshop
2020/1/30 @JAMSTEC

Monitoring plate motion for earthquake prediction

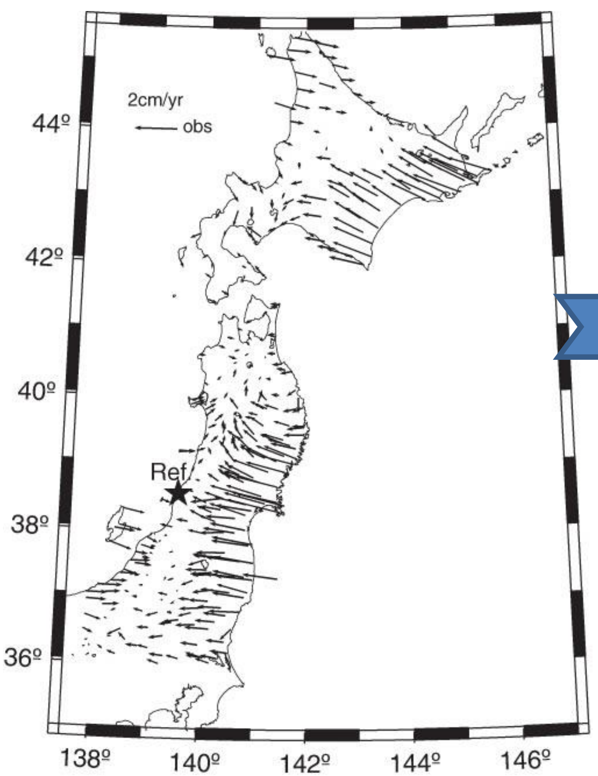
- Acceleration of plate motion called a slow slip event was found from 1 month before the 2011 Tohoku-oki earthquake (M9)
- Monitoring plate motion to find initiation of nucleation process is important to predict huge earthquakes



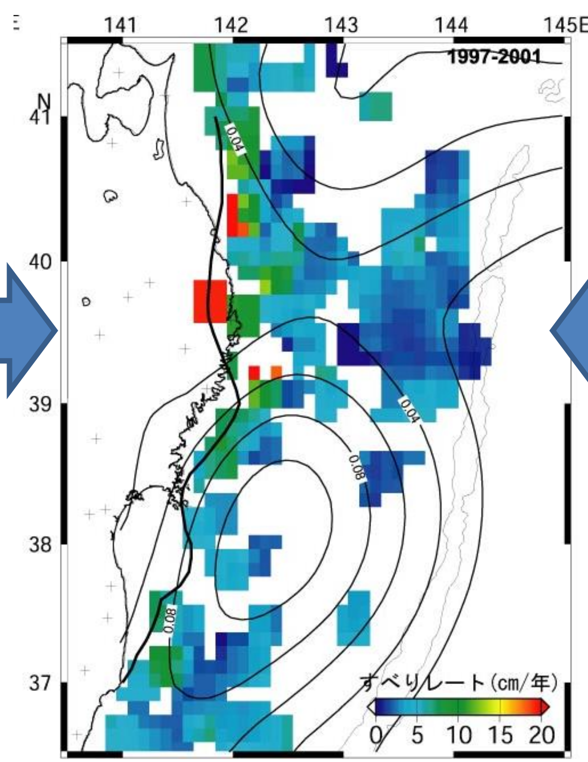
Slow slip events before Tohoku-oki earthquake
(Headquarter of Earthquake Research Promotion)

Inversion of interplate quasi-static slip rate

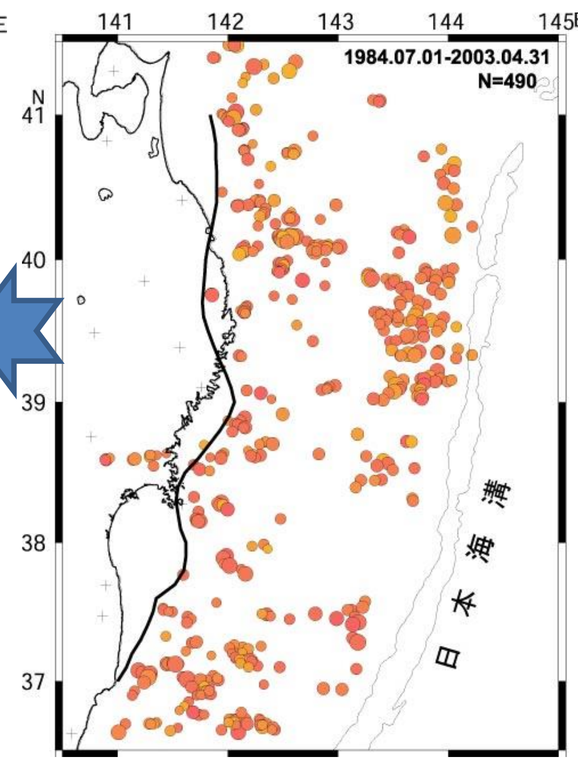
- GPS observation is conventionally used for inversion of interplate slip rate.
- Repeating earthquakes on interplate subduction zone can be used as complementary information to GPS data.



GPS displacement during 1997-2001
（Uchida et al.）



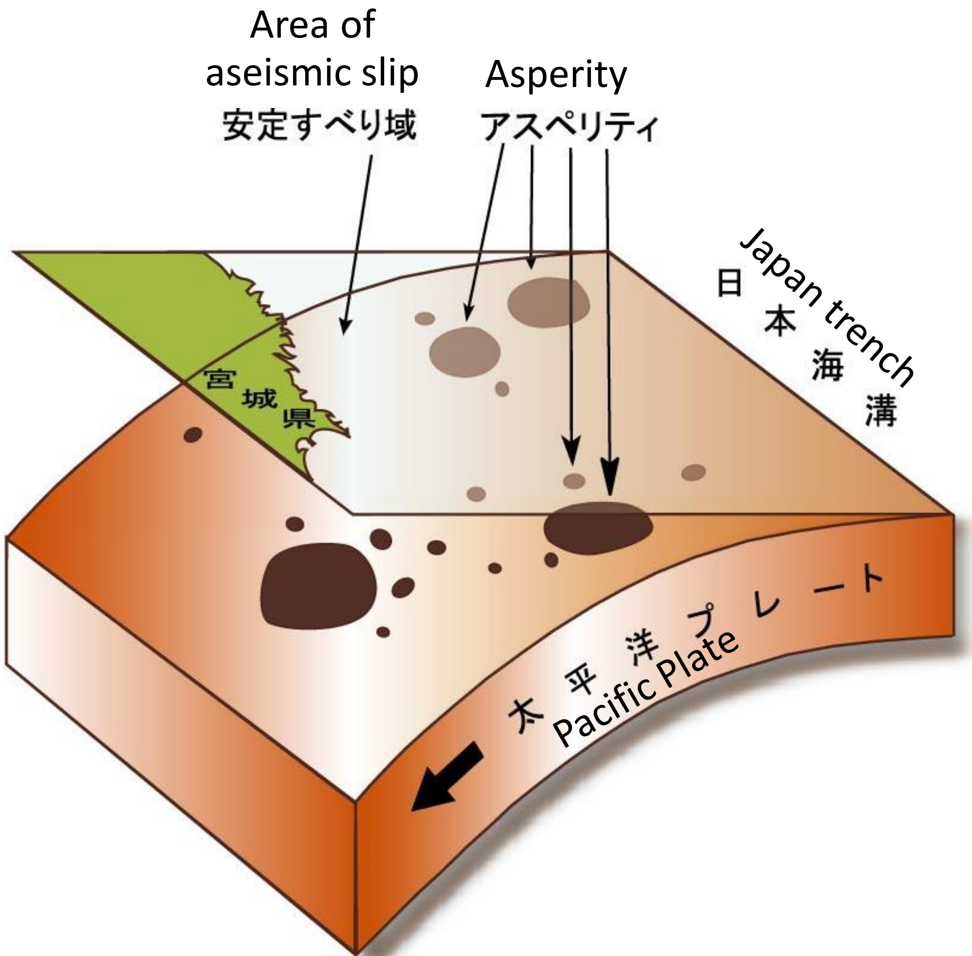
Average slip rate during 1997-2001
（Uchida et al.）



Repeating earthquakes
（ Uchida et al. ）

Stochastic model for repeating earthquakes

- On subducting interplate zones, there are many asperities that induce a sequence of characteristic earthquakes. On the asperity, elastic strain builds up by nearby static slip until it reaches to the failure level.
- While large asperities repeat their slip in tens or hundreds years, small ones slip by several years. So they can be used to monitor the change in quasi-static slip rate around the asperities. (c.f. Nadeau and McEvilly 1999)
- In this study, we propose a new stochastic model for repeating earthquake sequences to estimate spatio-temporal variation of quasi-static slip rate on the subduction zone of Pacific Plate in Eastern Japan.



Asperity model (Tohoku Univ.)

Contents

- Non-stationary renewal process for repeating microearthquakes in Parkfield, California

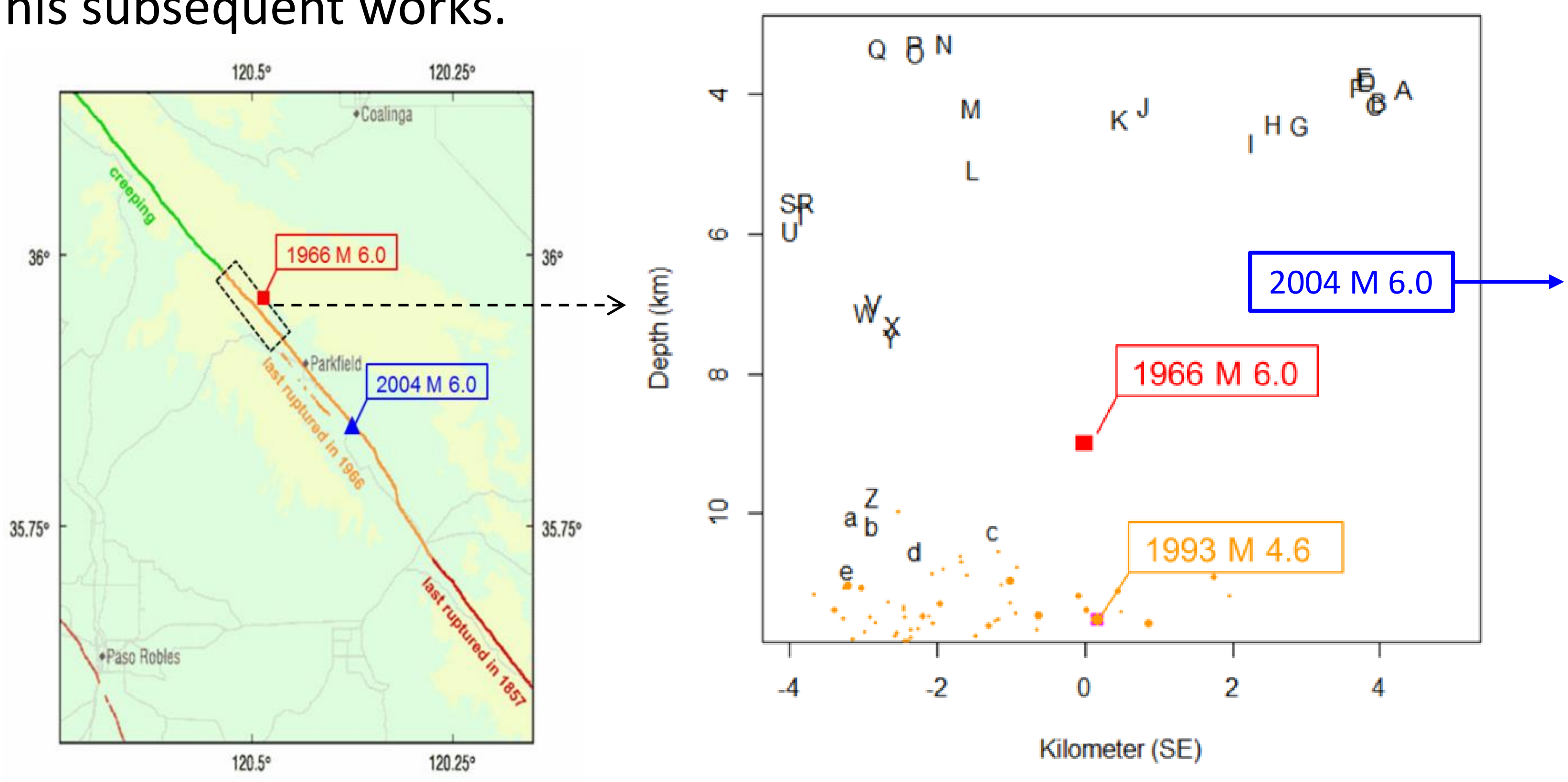
  Nomura, S., Ogata, Y. and Nadeau, R.M. (2014).
  Space-time model for repeating earthquakes and analysis of recurrence intervals on the San Andreas Fault near Parkfield, California, *Journal of Geophysical Research*, *119*, pp.7092-7122, doi:10.1002/2013JB010558.

- Inversion of spatio-temporal slip rate with locally discontinuous changes in Northeast Japan

  Nomura, S., Ogata, Y., Uchida, N. and Matsu'ura, M. (2017).
  Spatiotemporal variations of interplate slip rates in northeast Japan inverted from recurrence intervals of repeating earthquakes, *Geophysical Journal International*. **208**, pp.468-481, doi: 10.1093/gji/ggw395.

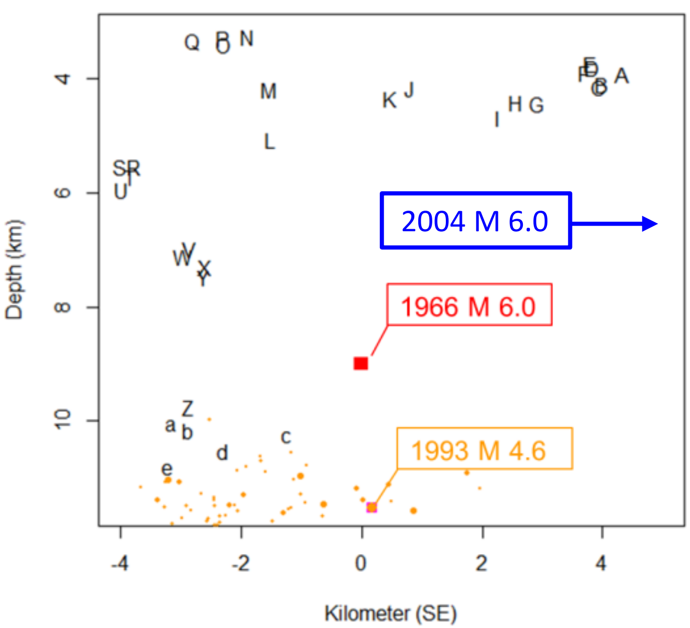
Repeating microearthquakes in Parkfield, California

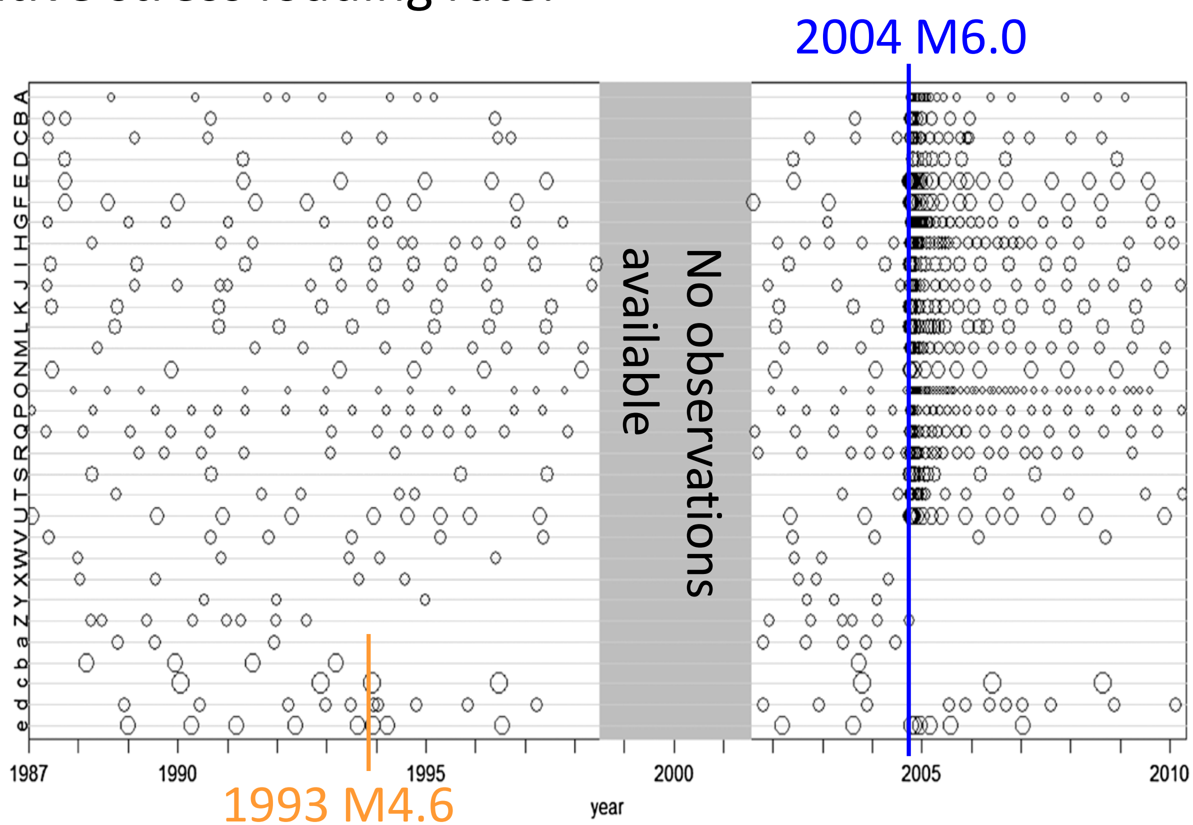
- High Resolution Seismic Network recorded microearthquakes of magnitudes below M2 around the Parkfield area of San Andreas Fault in California from 1987 to 2011.
- 31 repeating microearthquake sequences shown in indices from “A” to “e” are identified based on similarity of seismic waves by Nadeau et al., (1995) and his subsequent works.



Repeating microearthquakes in Parkfield, California

- The 31 repeating microearthquake sequences have some spatio-temporal trends in the relative length of their recurrence intervals, which imply changes in stress loading rate caused by large seismic or aseismic events.
- We introduce a non-stationary renewal process and estimate spatio-temporal variation of relative stress loading rate.





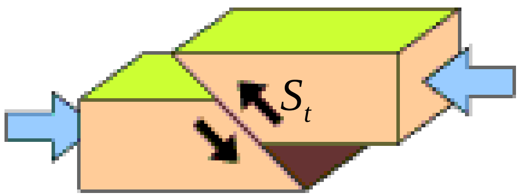
Renewal Process with BPT distribution

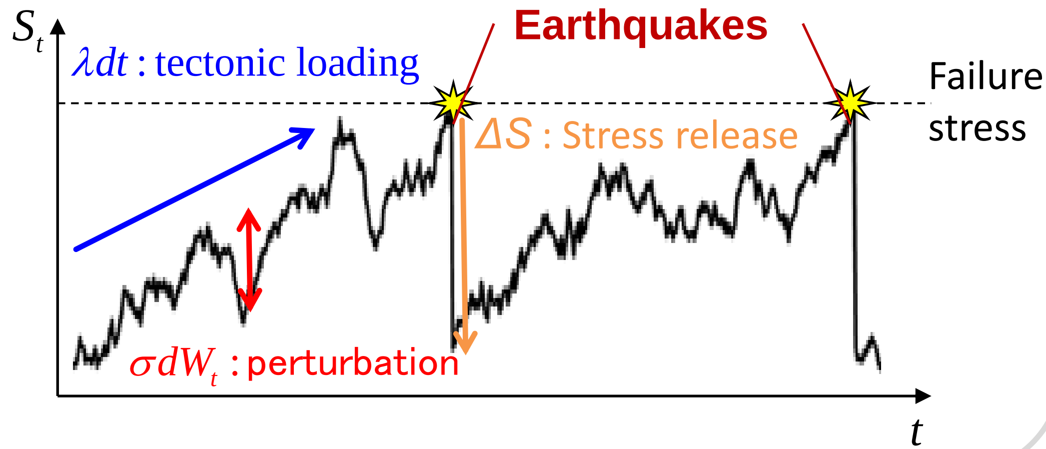
- Brownian Relaxation Oscillator（Matthews et al., 2002）

$$dS_t = \lambda dt + \sigma dW_t$$

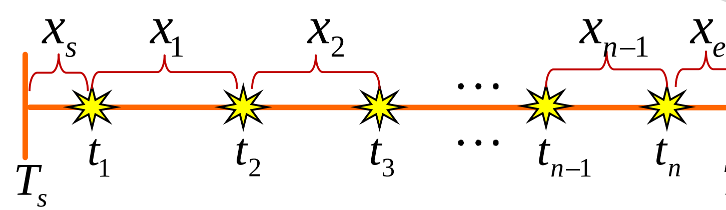
S_t :Coulomb failure stress

W_t :Standard Brownian motion



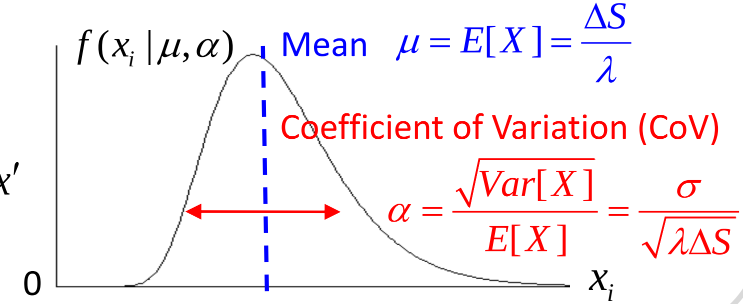


- Brownian Passage Time（BPT）renewal process



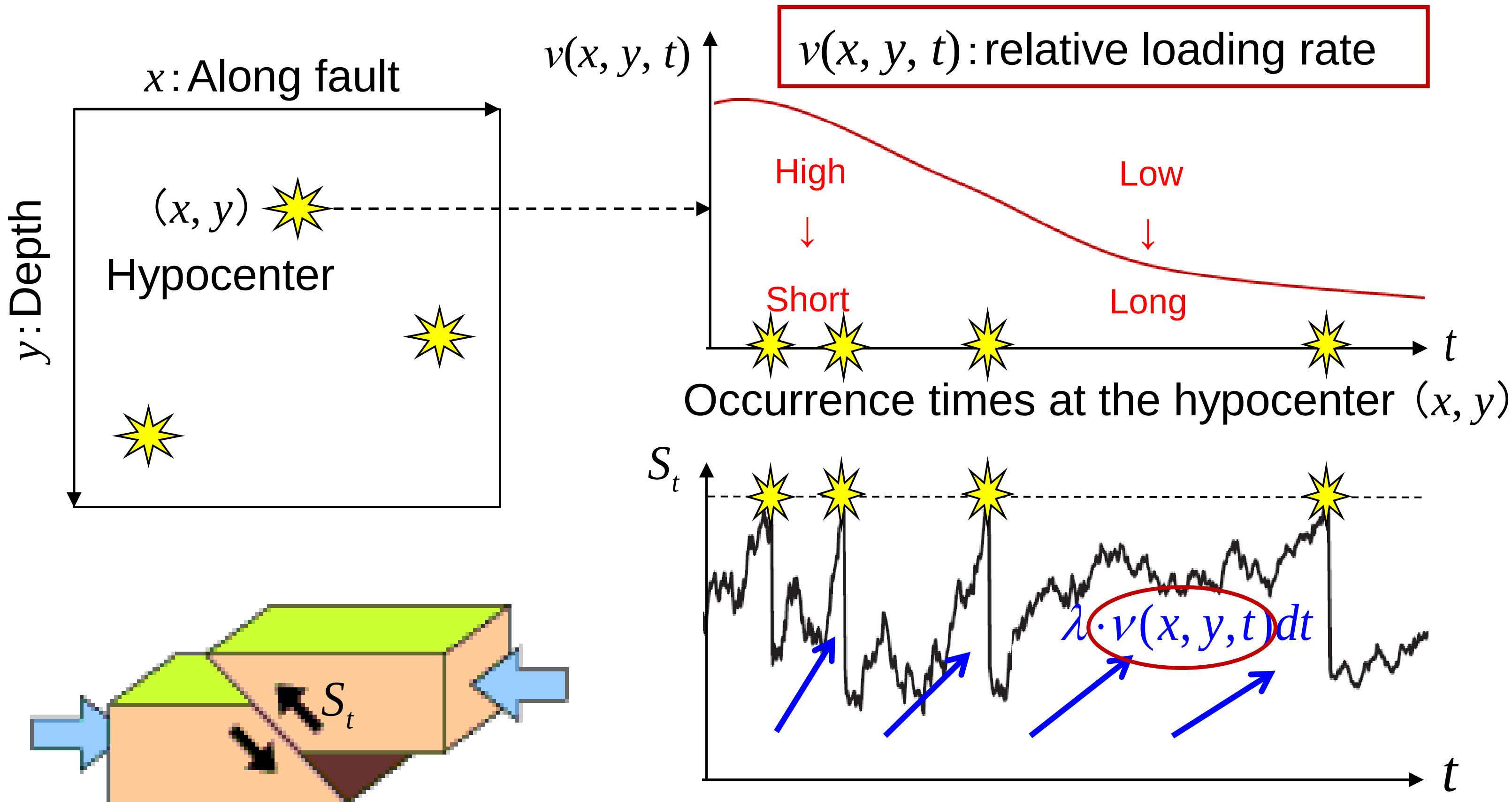
$$x_i \overset{i.i.d.}{\sim} \underbrace{f(x_i \mid \mu, \alpha)}_{\text{Probability density}} = \sqrt{\frac{\mu}{2\pi\alpha^2 x_i^3}} \exp\left\{ -\frac{(x_i - \mu)^2}{2\mu\alpha^2 x_i} \right\}$$

$$\underbrace{L(\mu, \alpha \mid t_1, \cdots, t_n)}_{\text{Likelihood}} = \frac{1}{\mu} \prod_{i=1}^{n-1} f(x_i \mid \mu, \alpha) \int_{x_s}^{\infty} f(x \mid \mu, \alpha) dx \int_{x_e}^{\infty} f(x' \mid \mu, \alpha) dx'$$



Non-stationary stress loading process

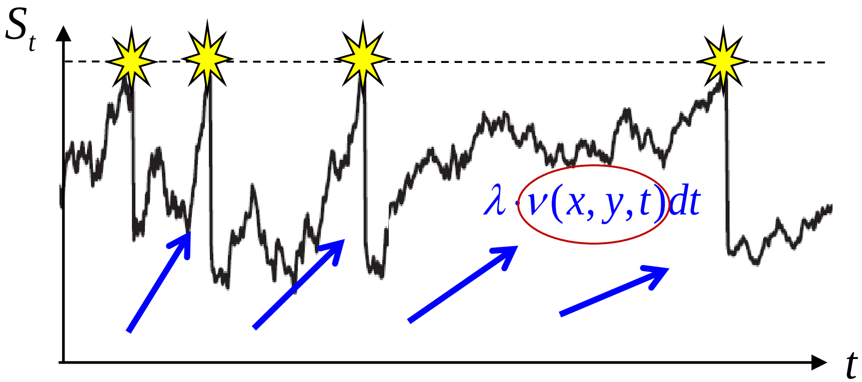
- Extend the stress loading process by incorporating relative loading rate $v(x, y, t)$ varying by location along the fault x, depth y and time t.
- Relative loading rate affects both stress loading and perturbation terms, and thus distribution of recurrence intervals.



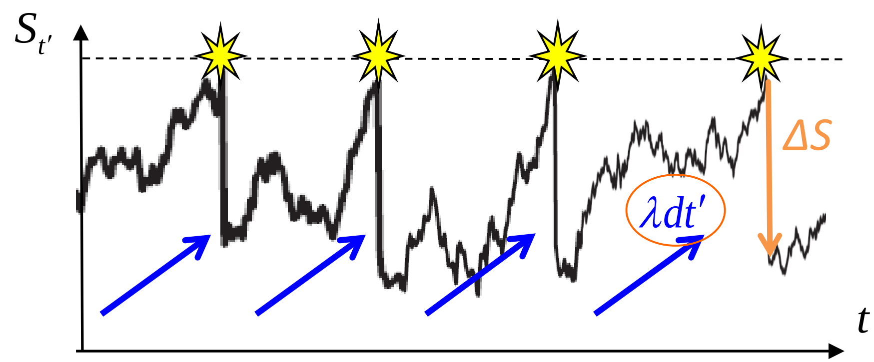
$$dS_t = \lambda \cdot v(x,y,t)dt + \sigma\sqrt{v(x,y,t)}dW_t$$

Time transformation to stationary process

Non-stationary process



Stationary process



$$dS_t = \lambda \cdot v(x,y,t)dt + \sigma\sqrt{v(x,y,t)}dW_t$$

$$dS_{t'} = \lambda dt' + \sigma dW_{t'}$$

Mean $\mu = \dfrac{\Delta S}{\lambda}$

CoV $\alpha = \dfrac{\sigma}{\sqrt{\lambda \Delta S}}$

Time transformation : $dt' = v(x,y,t)dt$

Likelihood function

$$L(\mu,\alpha,v \mid t_1,t_2,\cdots,t_n) = L(\mu,\alpha \mid t'_1,t'_2,\cdots,t'_n)\cdot\prod_{i=1}^{n} v(x,y,t_i)$$

$$= \underbrace{\mu^{-1}\int_{t'_1}^{\infty} f(x\mid\mu,\alpha)dx \prod_{i=1}^{n-1} f(t'_{i+1}-t'_i \mid \mu,\alpha)\int_{T'_C-t'_n}^{\infty} f(x'\mid\mu,\alpha)dx'}_{\text{Likelihood in the transformed time}} \cdot \underbrace{\prod_{i=1}^{n} v(x,y,t_i)}_{\text{Relative loading rate at occurrence times}}$$

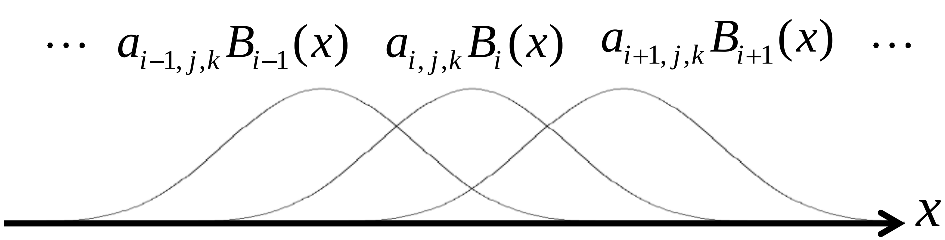
Likelihood in the transformed time

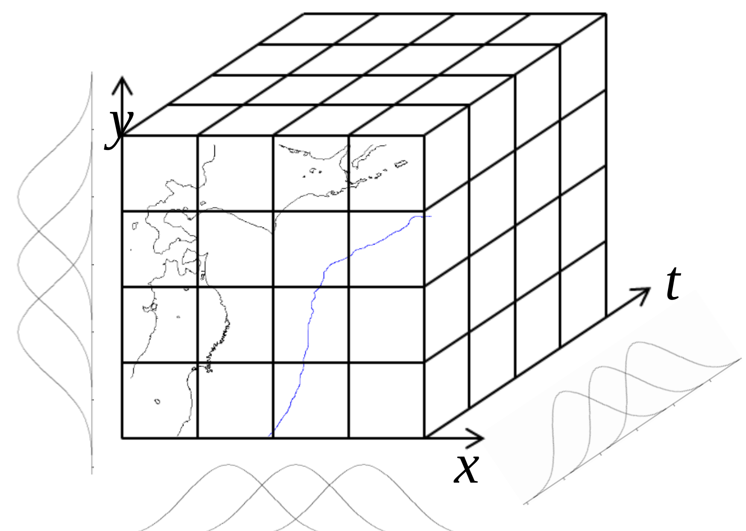
Relative loading rate
at occurrence times

B-spline representation of relative loading rate

- We represent the relative loading rate function $v(x, y, t)$ by using a cubic B-spline function, which is a linear combination of local B-spline bases.
- Then, the set of coefficients for B-spline bases is the parameter to be estimated.

$$v(x,y,t) = \exp\left\{ \sum_{i,j,k} \underbrace{a_{i,j,k}}_{\text{Parameter}} \underbrace{B_i(x) B_j(y) B_k(t)}_{\text{cubic B-spline basis}} \right\}$$





Parameter inference by penalized likelihood

- Consider M repeating earthquake sequences with distinct parameters $\boldsymbol{\mu} = (\mu_1,\ldots,\mu_M)$ and $\boldsymbol{\alpha} = (\alpha_1,\ldots,\alpha_M)$ for distribution of recurrence times.
- Then, log likelihood of the M sequences is defined by

$$\log L(\boldsymbol{\mu},\boldsymbol{\alpha},\nu) = \sum_{m=1}^{M} \log L(\mu_m,\alpha_m,\nu(x_m,y_m,\cdot) \mid t_{m1},t_{m2},\cdots,t_{mn_m}).$$

- Parameters are estimated by maximizing the penalized log likelihood with smoothness constrains:

$$\log Q(\boldsymbol{\mu},\boldsymbol{\alpha},\nu) = \log L(\boldsymbol{\mu},\boldsymbol{\alpha},\nu) - \underbrace{\Phi(\nu \mid w_1,\cdots,w_6)}_{\text{Penalty function}}$$

Penalty function = Gaussian prior for $a_{i,j,k}$

Quadratic form of $a_{i,j,k}$

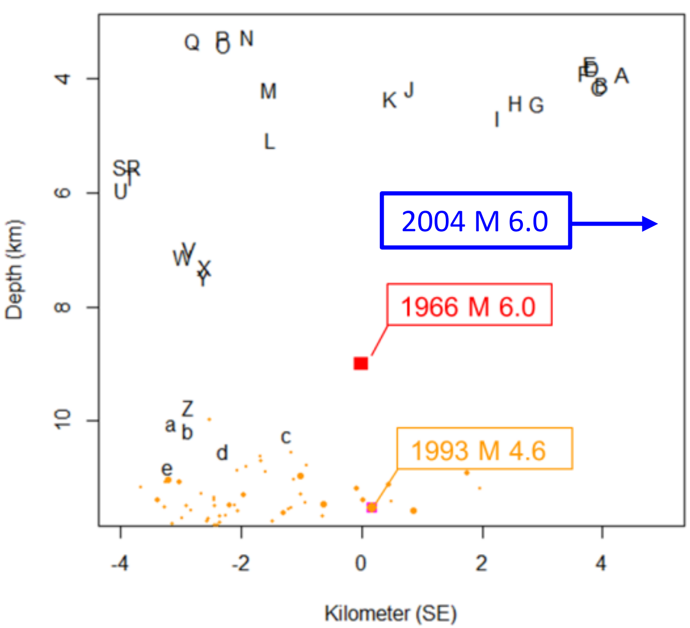
$$\Phi(\nu \mid \underbrace{w_1,\cdots,w_6}_{\text{Hyper-parameters}}) = \iiint \overbrace{\left\{ w_1\left(\frac{\partial \nu}{\partial x}\right)^2 + w_2\left(\frac{\partial \nu}{\partial y}\right)^2 + w_3\left(\frac{\partial \nu}{\partial t}\right)^2 + w_4{}^2\left(\frac{\partial^2 \nu}{\partial x^2}\right)^2 + w_5{}^2\left(\frac{\partial^2 \nu}{\partial y^2}\right)^2 + w_6{}^2\left(\frac{\partial^2 \nu}{\partial t^2}\right)^2 \right.}^{\text{Quadratic form of } a_{i,j,k}}$$
$$\left. + 2w_4w_5\left(\frac{\partial^2 \nu}{\partial x \partial y}\right)^2 + 2w_4w_6\left(\frac{\partial^2 \nu}{\partial x \partial t}\right)^2 + 2w_5w_6\left(\frac{\partial^2 \nu}{\partial y \partial t}\right)^2 \right\} dxdydt$$

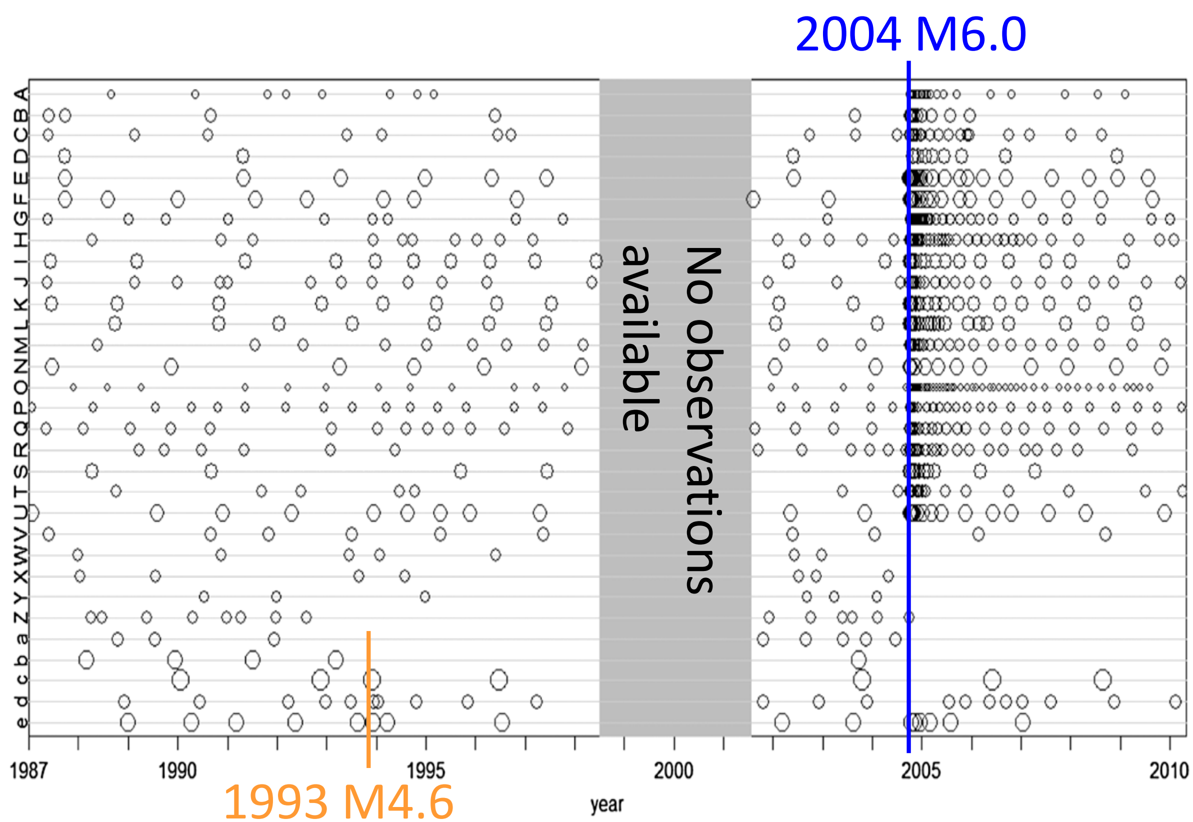
Hyper-parameters

Empirical Bayes estimation

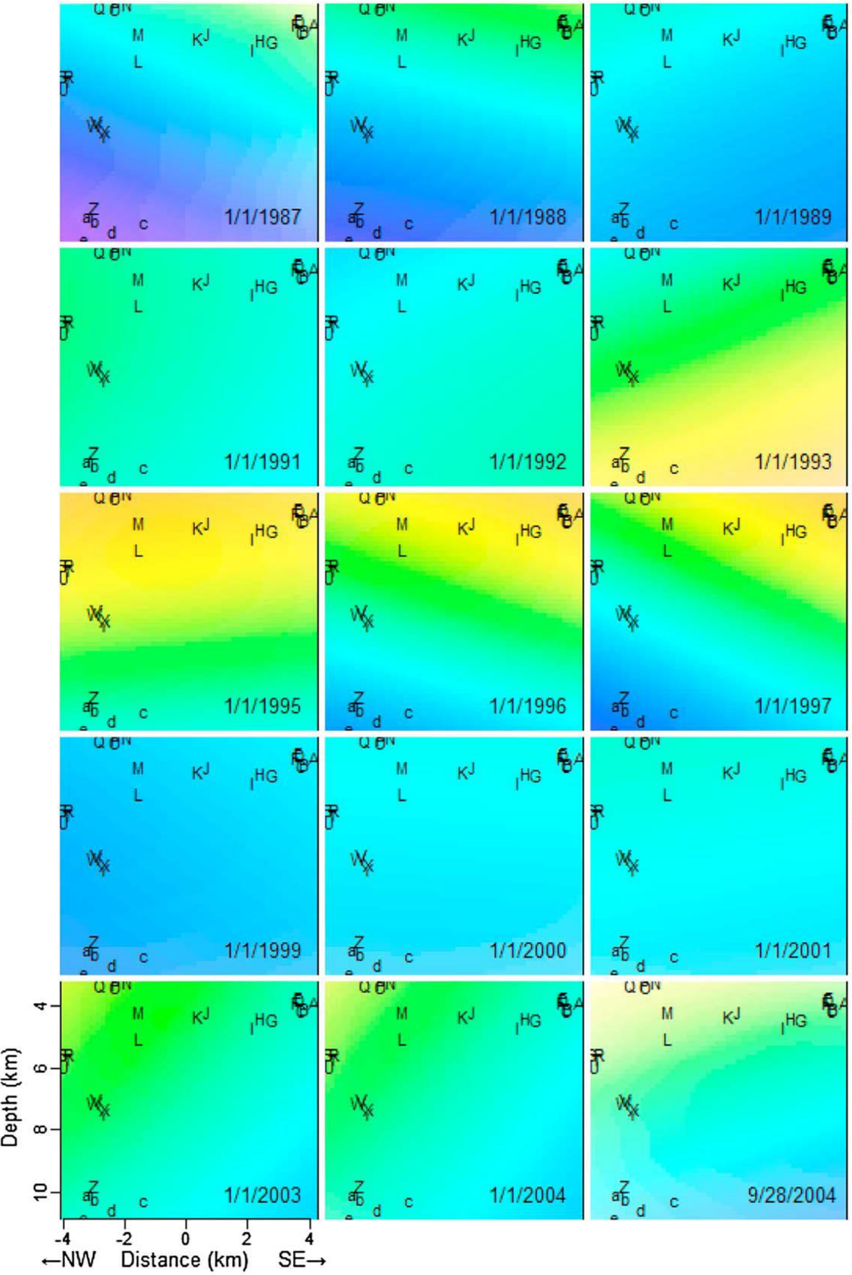
Repeating microearthquakes in Parkfield, California

- Using the proposed model, we estimated spatio-temporal loading rate from the repeating earthquake sequences in the Parkfield area.
- We analyzed two periods before and after the 2004 Parkfield earthquake of M6.0, separately.

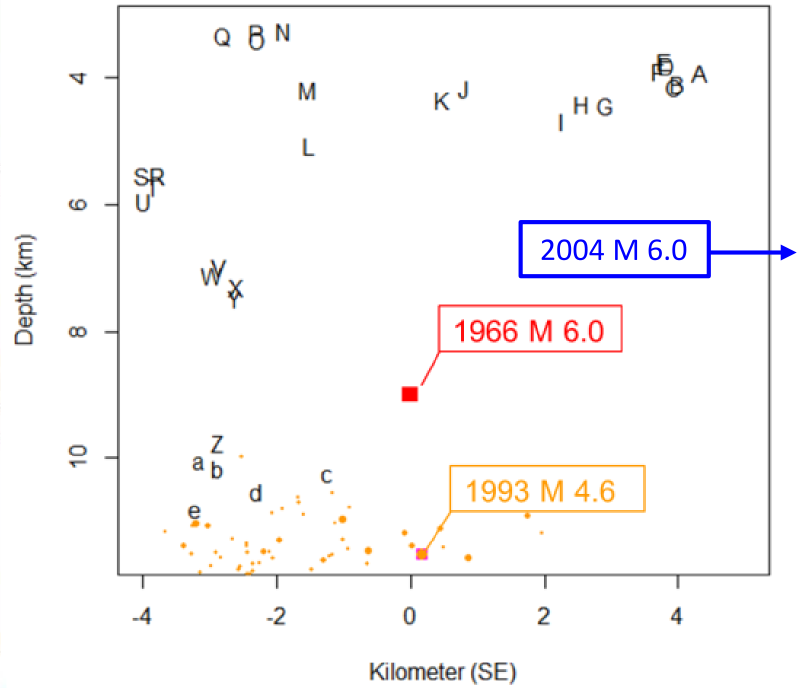


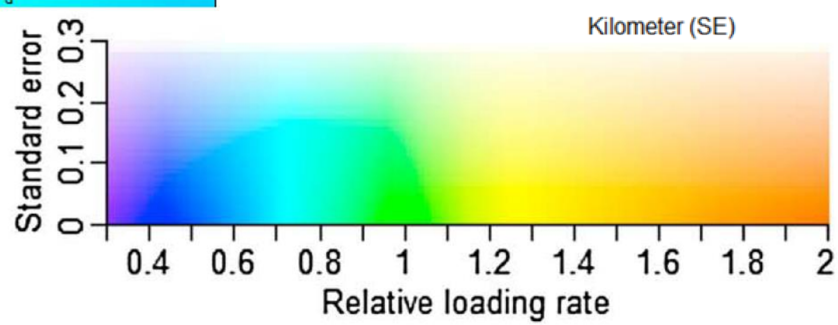


Loading rate before the 2004 Parkfield M6



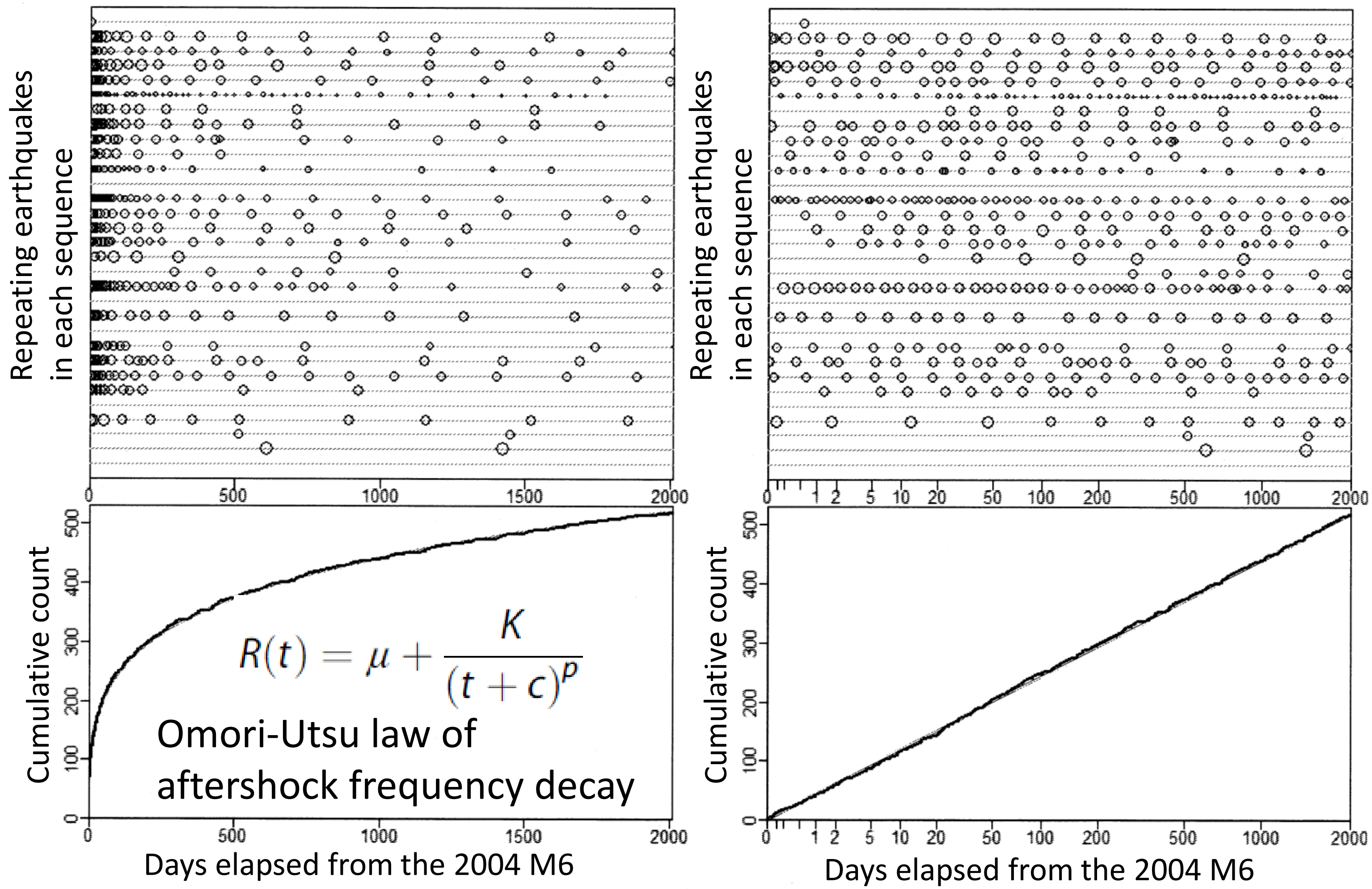
Acceleration of stress loading had begun at depth with the M4.6 event and propagated to the shallower zone and then moved toward the southeastern zone



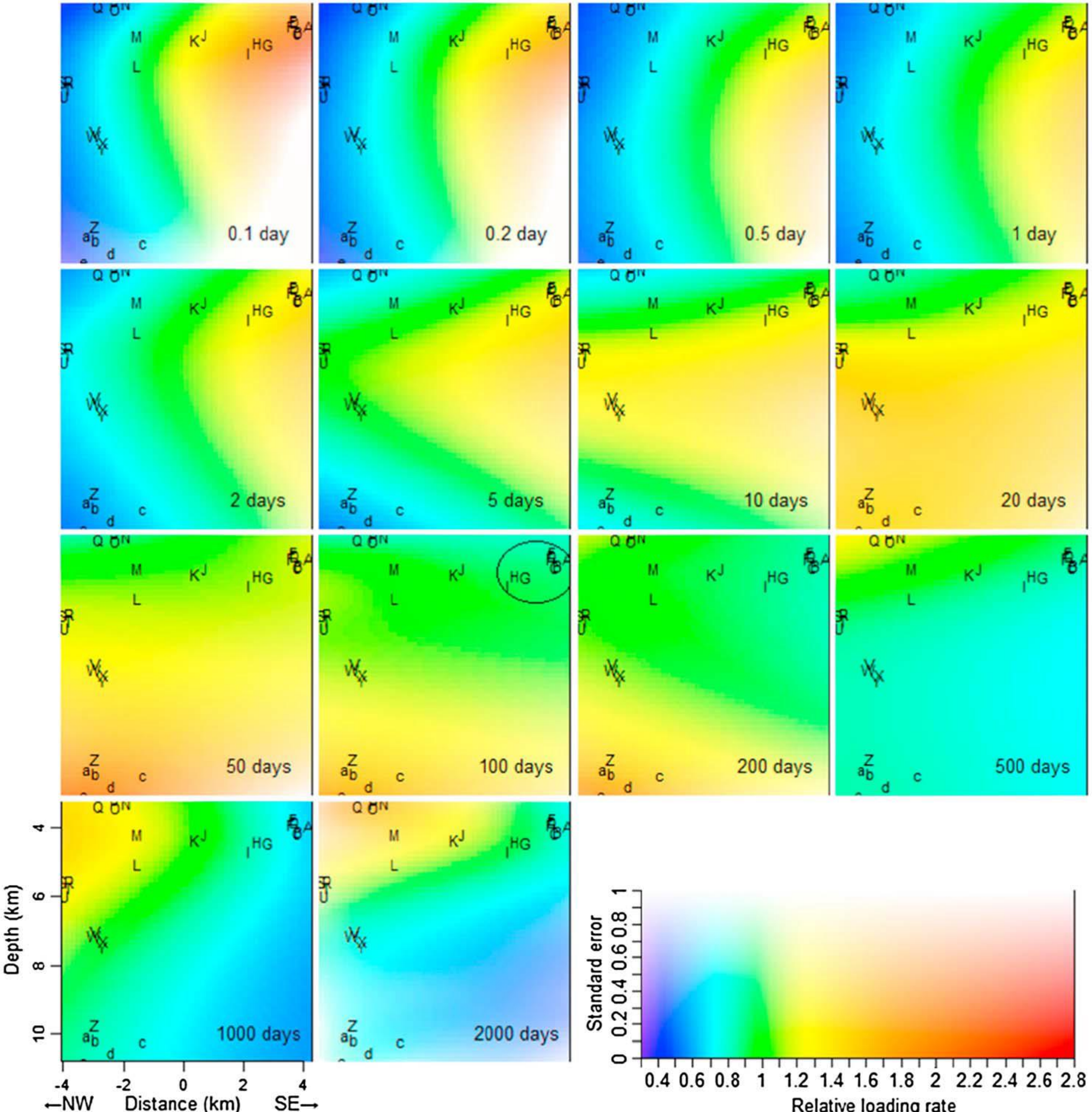


Repeating earthquakes after the 2004 Parkfield M6

- The 2004 Parkfield earthquake of M6.0 had triggered many repeating earthquakes as its aftershocks whose frequency had decayed by time.
- If we transform time along the aftershock decay law, we can obtain repeating earthquakes with quite periodic recurrence intervals.



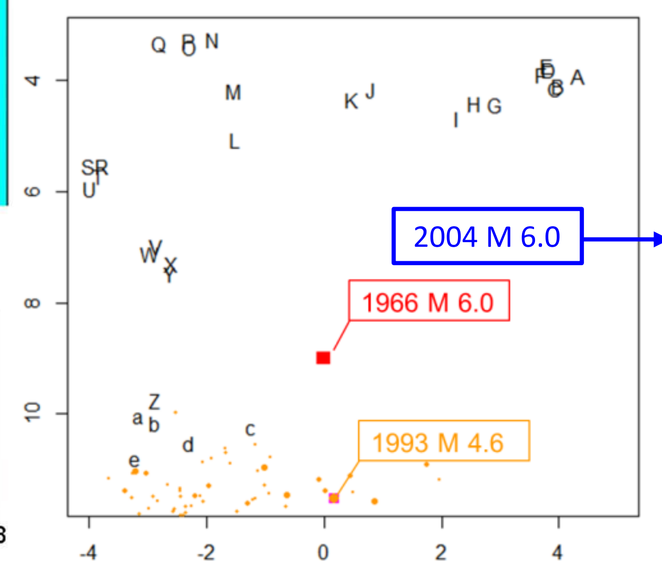
Loading rate after the 2004 Parkfield M6



We estimated the relative loading rate in the transformed time as shown in the previous slide.

There still remain regional differences in aftershock triggering and decay.

The repeating events at depth started after 20 days from the mainshock.



Contents

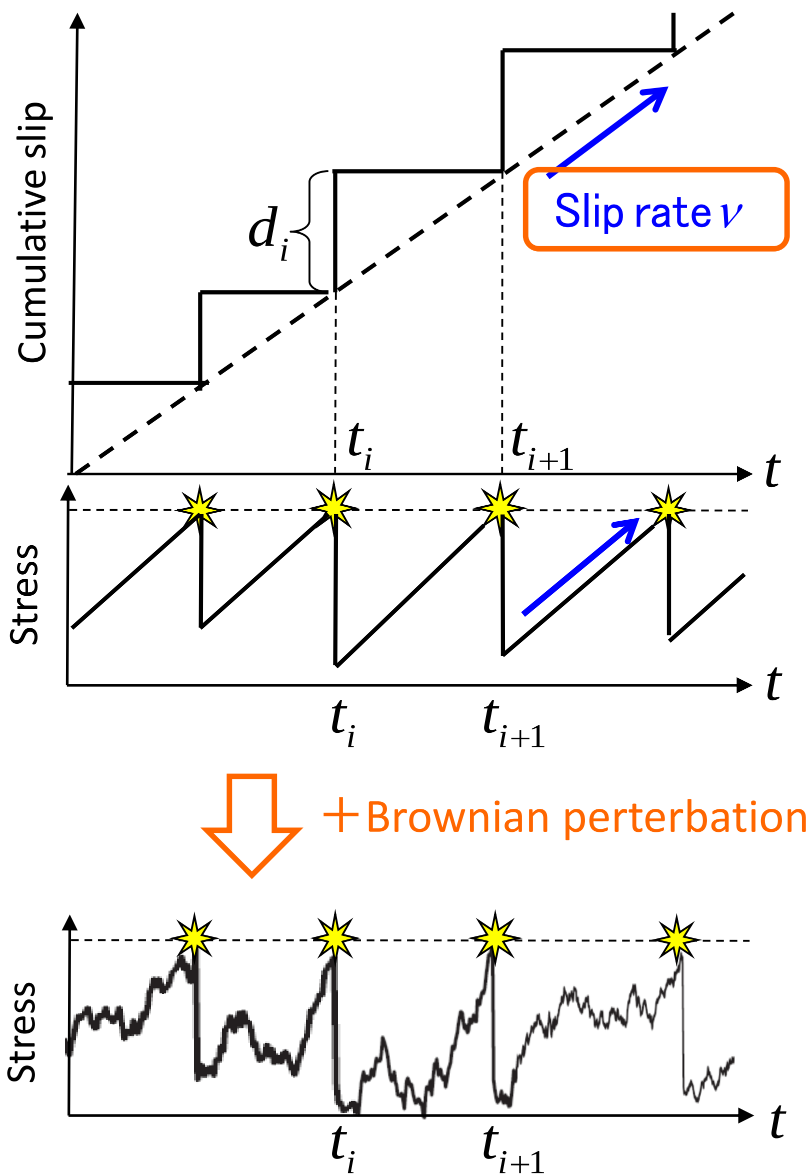
- Non-stationary renewal process for repeating microearthquakes in Parkfield, California

  Nomura, S., Ogata, Y. and Nadeau, R.M. (2014).
  Space-time model for repeating earthquakes and analysis of recurrence intervals on the San Andreas Fault near Parkfield, California, *Journal of Geophysical Research*, *119*, pp.7092-7122, doi:10.1002/2013JB010558.

- Inversion of spatio-temporal slip rate with locally discontinuous changes in Northeast Japan

  Nomura, S., Ogata, Y., Uchida, N. and Matsu'ura, M. (2017).
  Spatiotemporal variations of interplate slip rates in northeast Japan inverted from recurrence intervals of repeating earthquakes, *Geophysical Journal International*. **208**, pp.468-481, doi: 10.1093/gji/ggw395.

Introducing slip rate & amount into renewal model

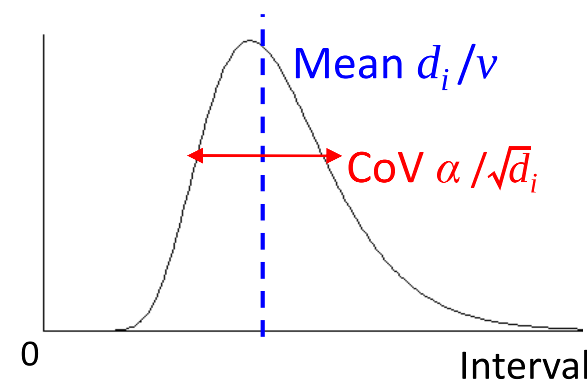


Slip amount：$\log d_i = -2.36 + 0.17 \log M_i$

（Nadeau and Johnson, 1998）

Interval：$t_{i+1} - t_i = d_i / v$

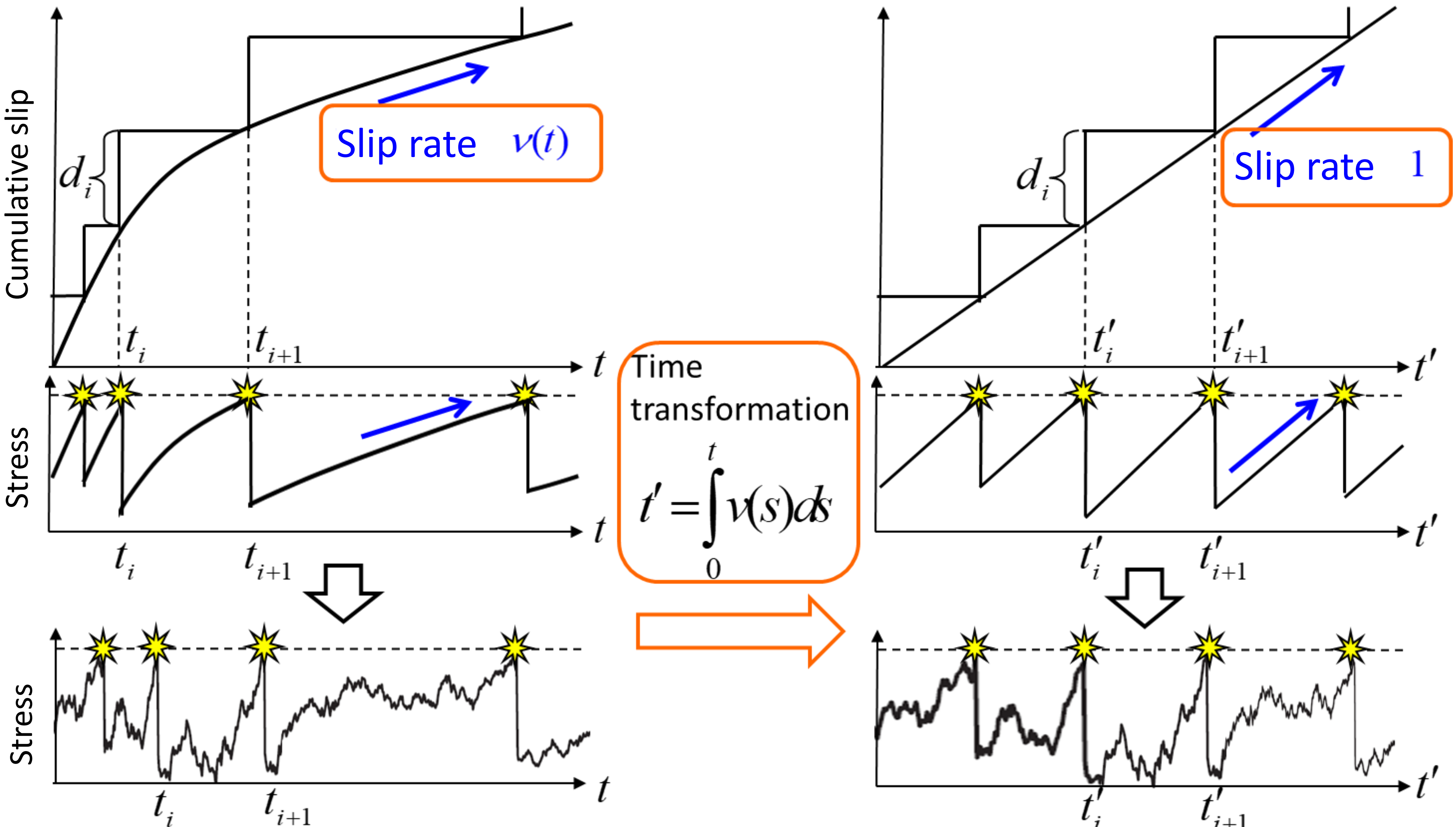
（Shimazaki and Nakata, 1980）



Interval：$t_{i+1} - t_i \sim \mathrm{BPT}(\underbrace{d_i / v}_{\text{Mean}}, \underbrace{\alpha / \sqrt{d_i}}_{\text{CoV}})$

（Ogata, 2002）

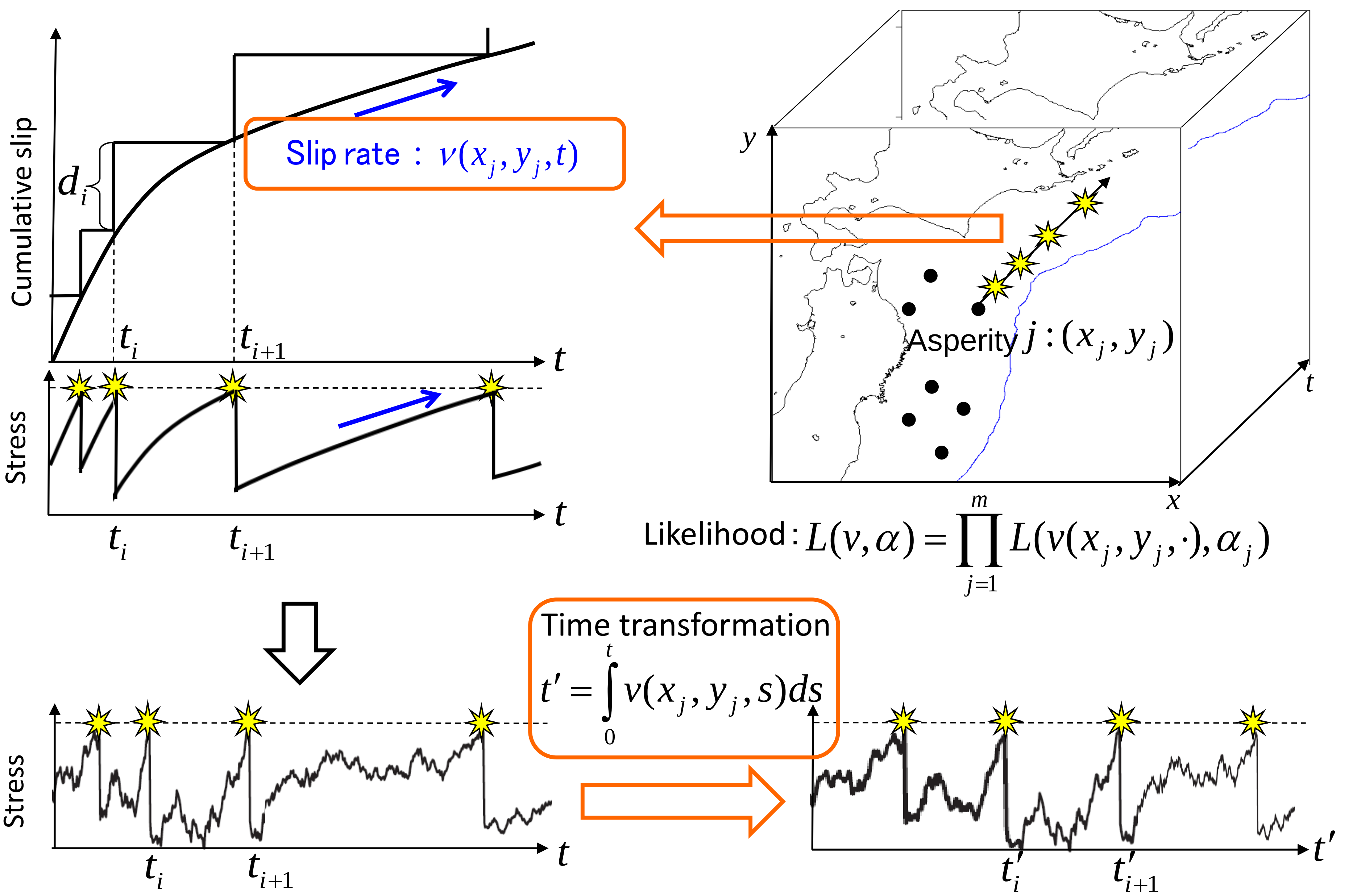
Time dependent slip rate model



Interval in transformed time $: t'_{i+1} - t'_i \sim \mathrm{BPT}(d_i, \alpha / \sqrt{d_i})$

Likelihood $: L(v, \alpha \mid t_1, \cdots, t_n, d_1, \cdots, d_n) = L(1, \alpha \mid t'_1, \cdots, t'_n, d_1, \cdots, d_n) \cdot \prod_{i=1}^{n} v(t_i)$

Extension to space-time model



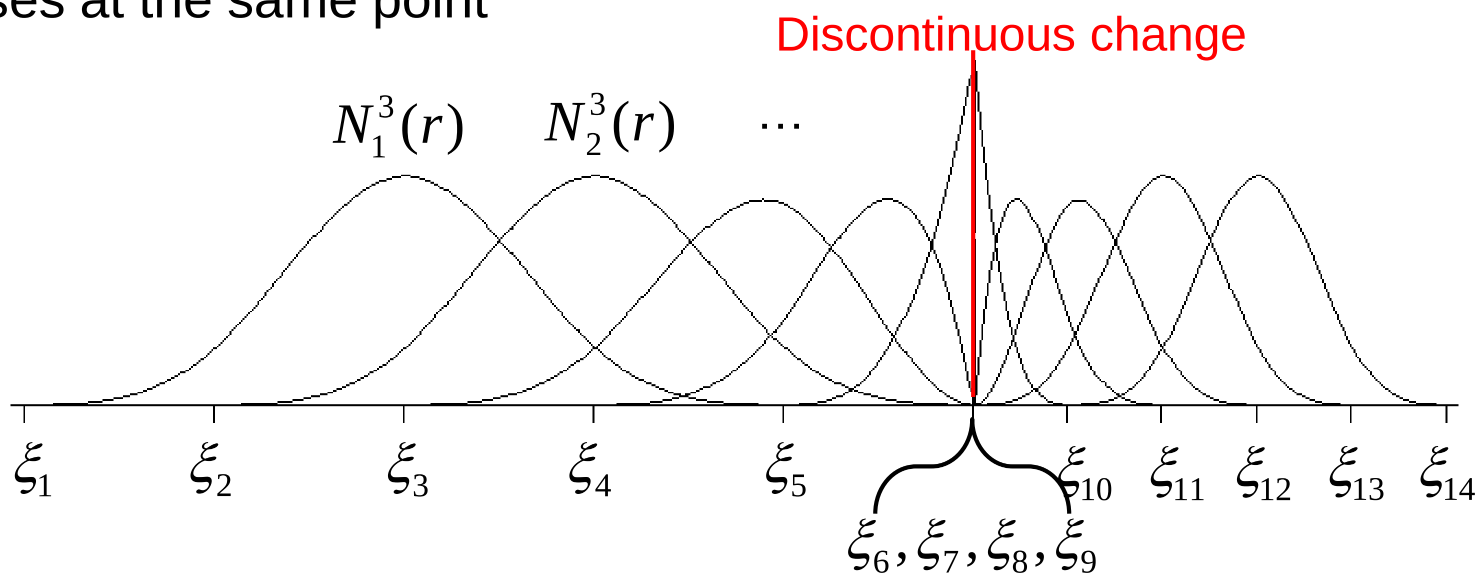
B-spline bases with discontinuous points

- B-spline basis $N_j^3(r)$ is constructed by de Boor's recursion formula:

$$N_j^1(r) = \begin{cases} 1 & (\xi_j \le r < \xi_{j+1}) \\ 0 & \text{otherwise} \end{cases}$$

$$N_j^k(r) = \frac{\xi_{j+k} - r}{\xi_{j+k} - \xi_{j+1}} N_{j+1}^{k-1}(r) + \frac{r - \xi_j}{\xi_{j+k-1} - \xi_j} N_j^{k-1}(r)$$

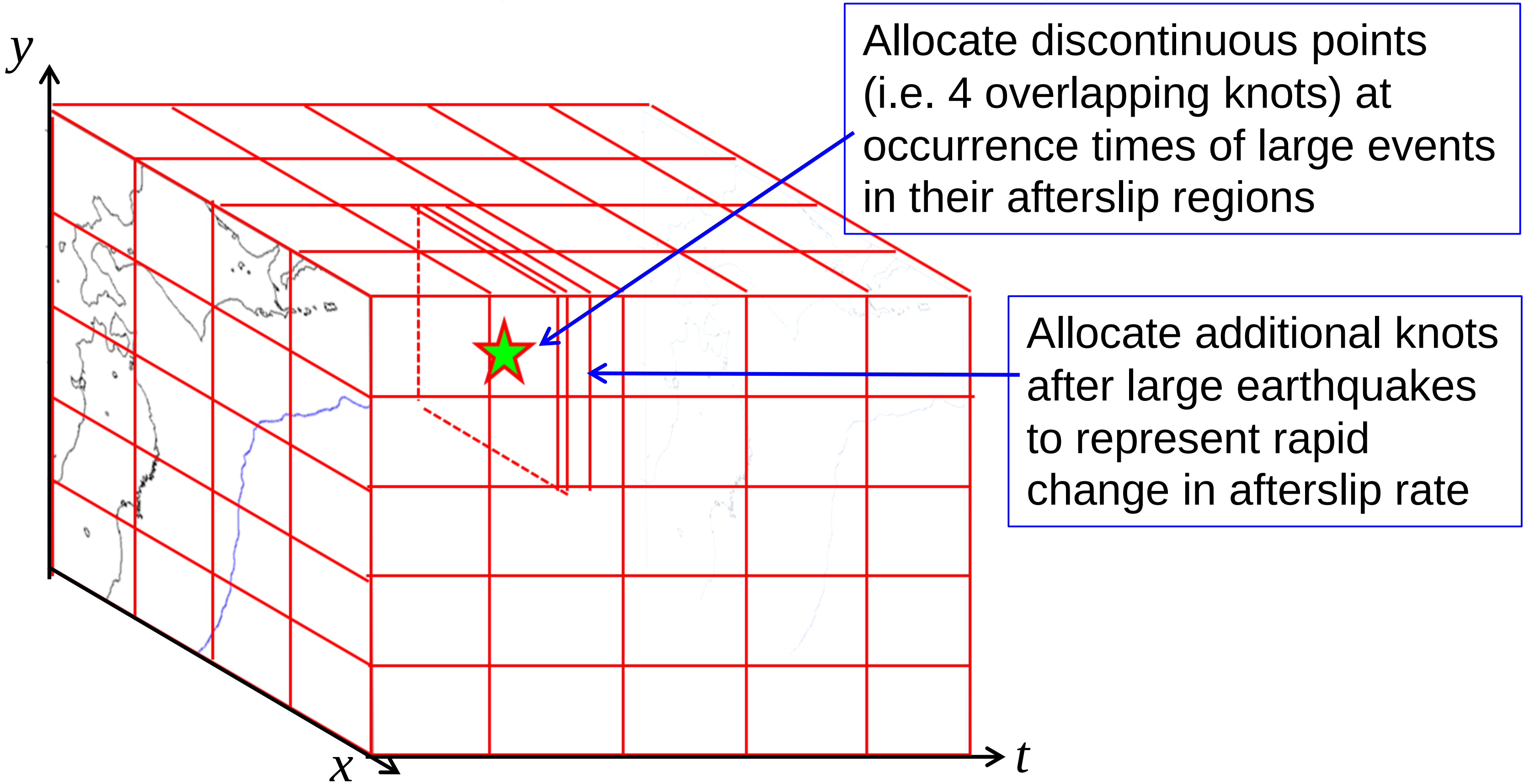
- A discontinuous change point can be set by allocating 4 knots of B-spline bases at the same point



B-spline representation of slip rate change

- We represent slip rate function $v(x, y, t)$ by using a cubic B-spline function, which is a linear combination of local B-spline bases.
- Then, the set of coefficients for B-spline bases is the parameter to be estimated.

$$v(x,y,t) = \exp\left\{ \sum_{i,j,k} a_{i,j,k} N_i^3(x) N_j^3(y) N_k^3(t) \right\}$$



Dataset and Settings

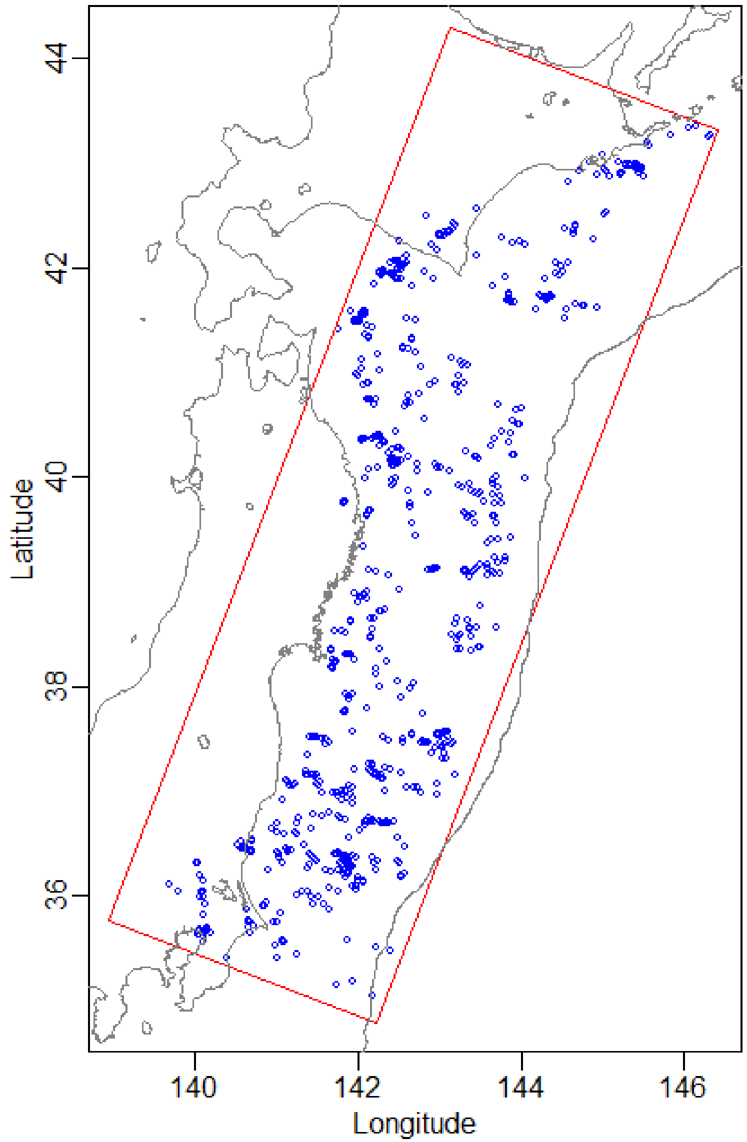
Dataset

- 778 earthquake sequences with 2901 events repeating on subduction zone from off North-Kanto to off Hokkaido
- Magnitude range: M2.8～6.05
- Observation period：1993.7.15 - 2011.3.10

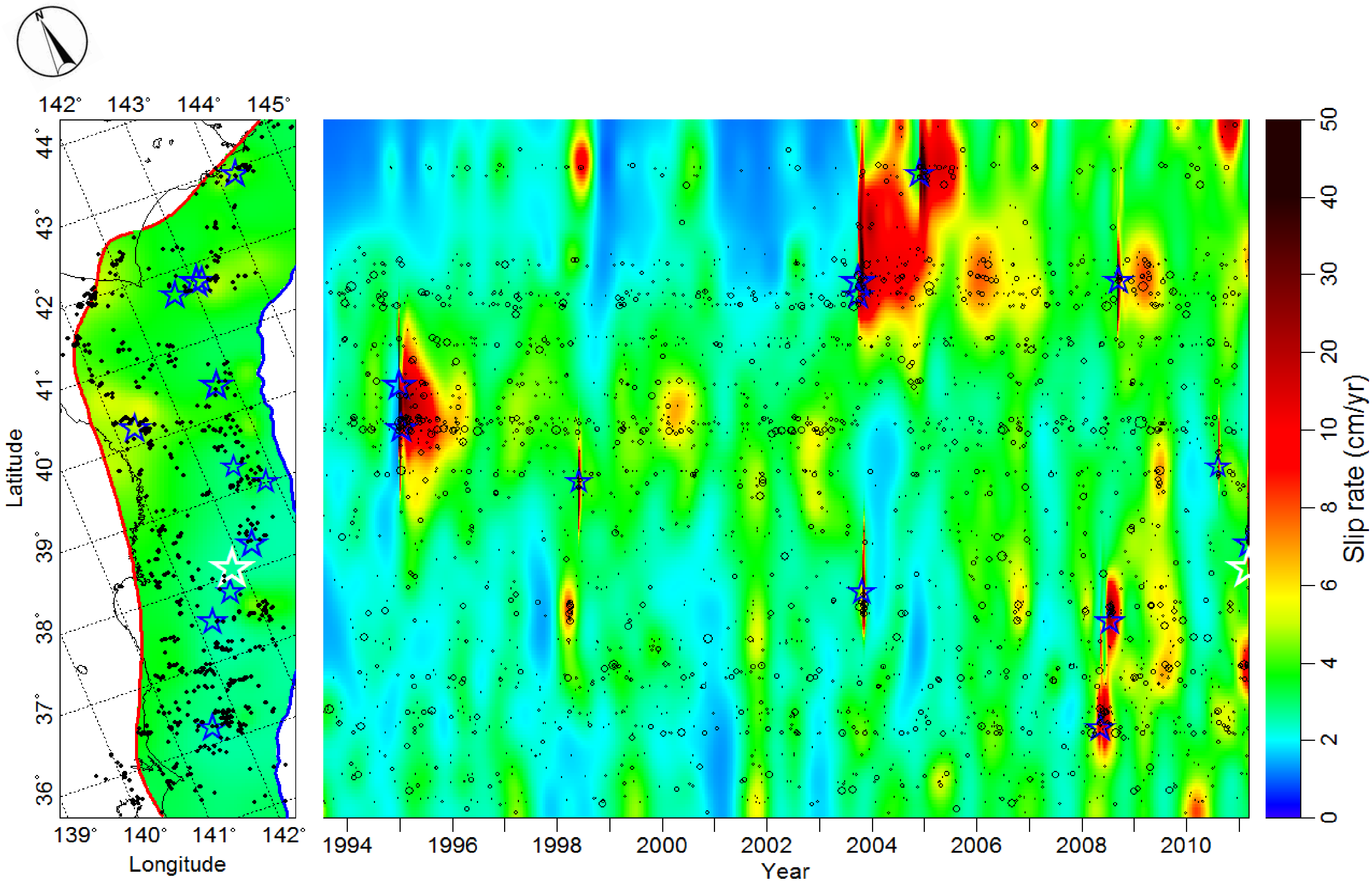
Settings

- Estimate spatio-temporal variation in slip rate by B-spline curve whose nodes are arranged by 50km and 3 months.
- Allocate discontinuous points (4 knots) at occurrence time of large earthquakes in their afterslip regions
- Allocate additional knots at 1, 3, 10 and 30 days after large earthquakes

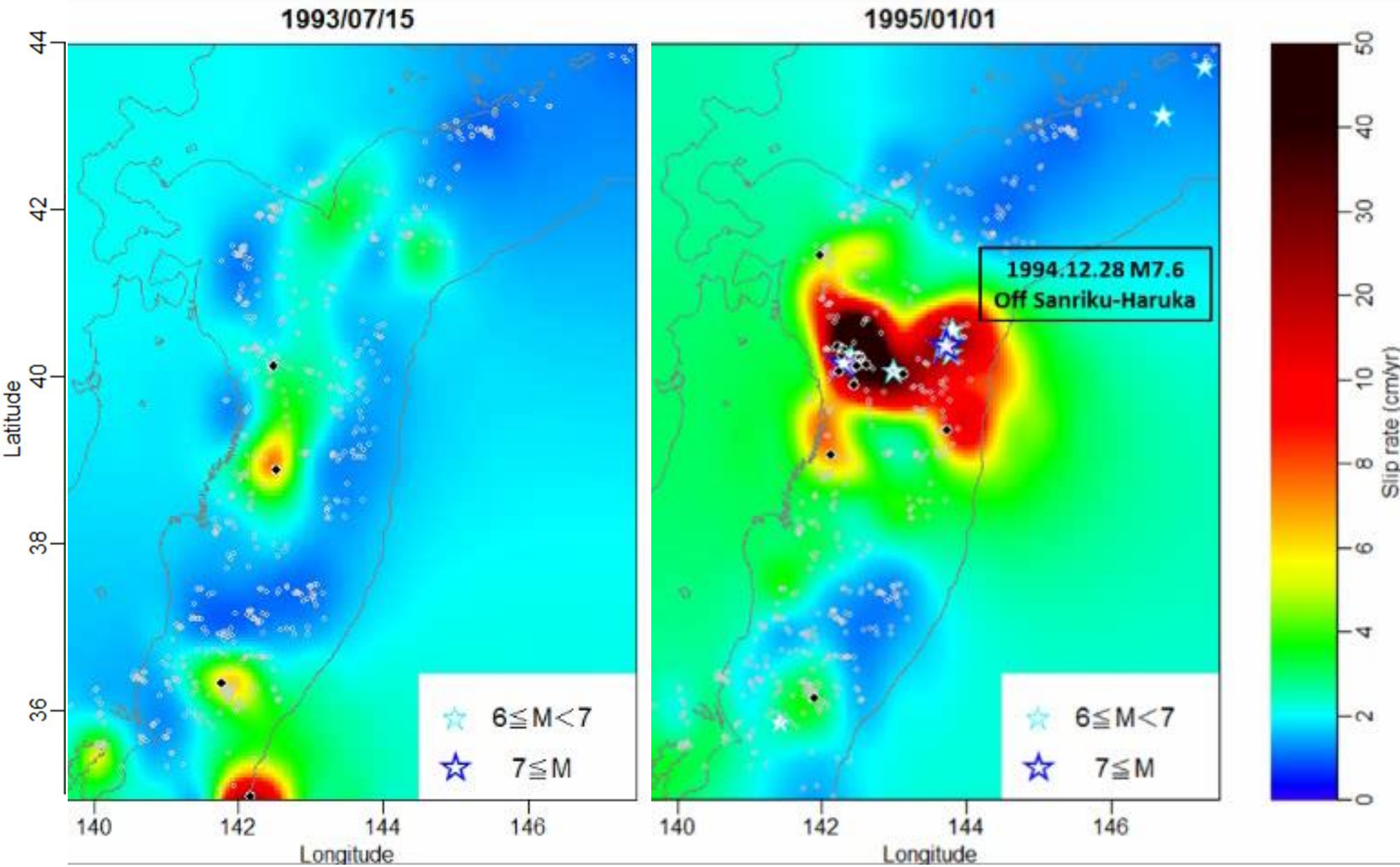
Map of Repeating Sequences



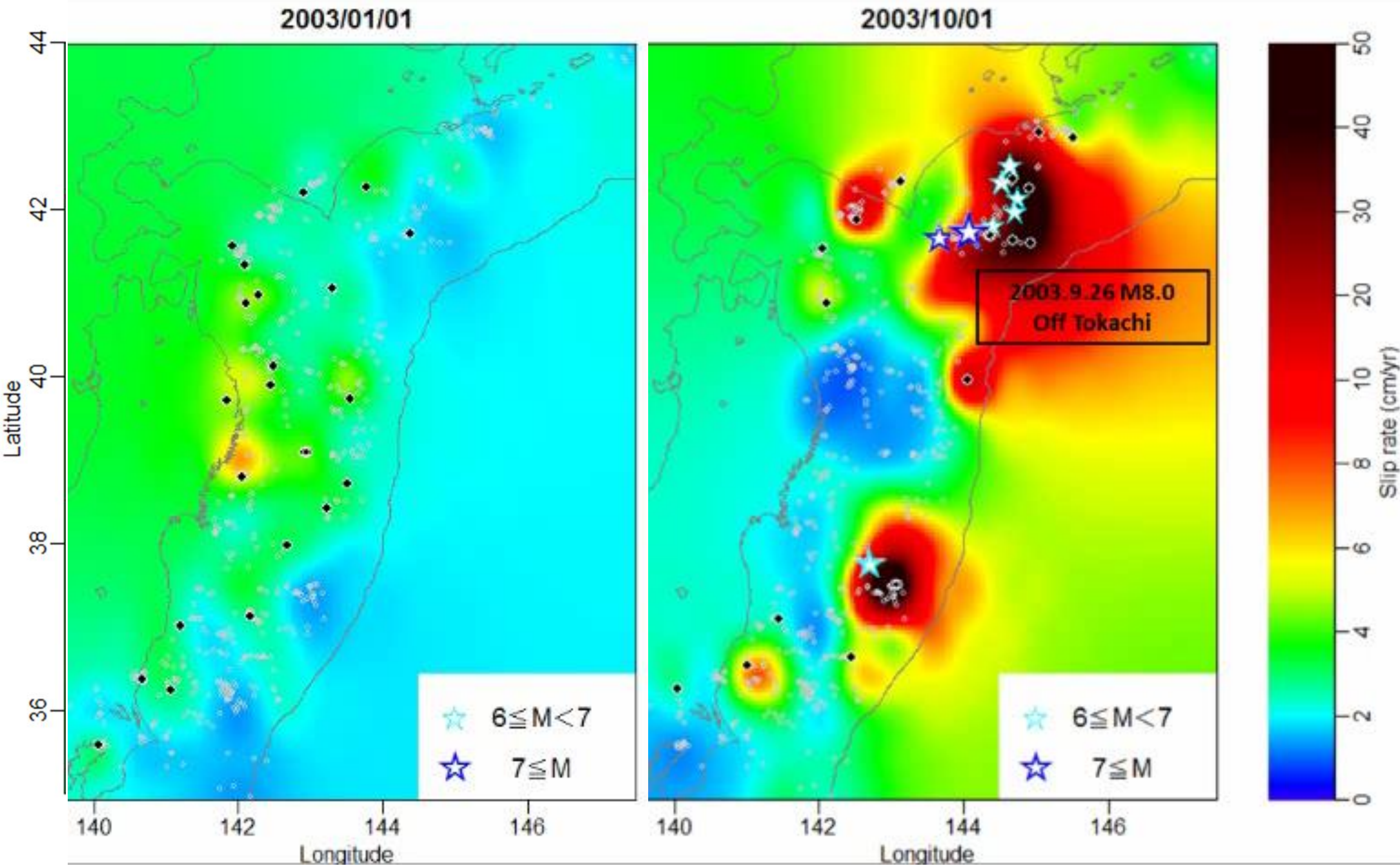
Mean (left) and temporal change (right) of slip rate



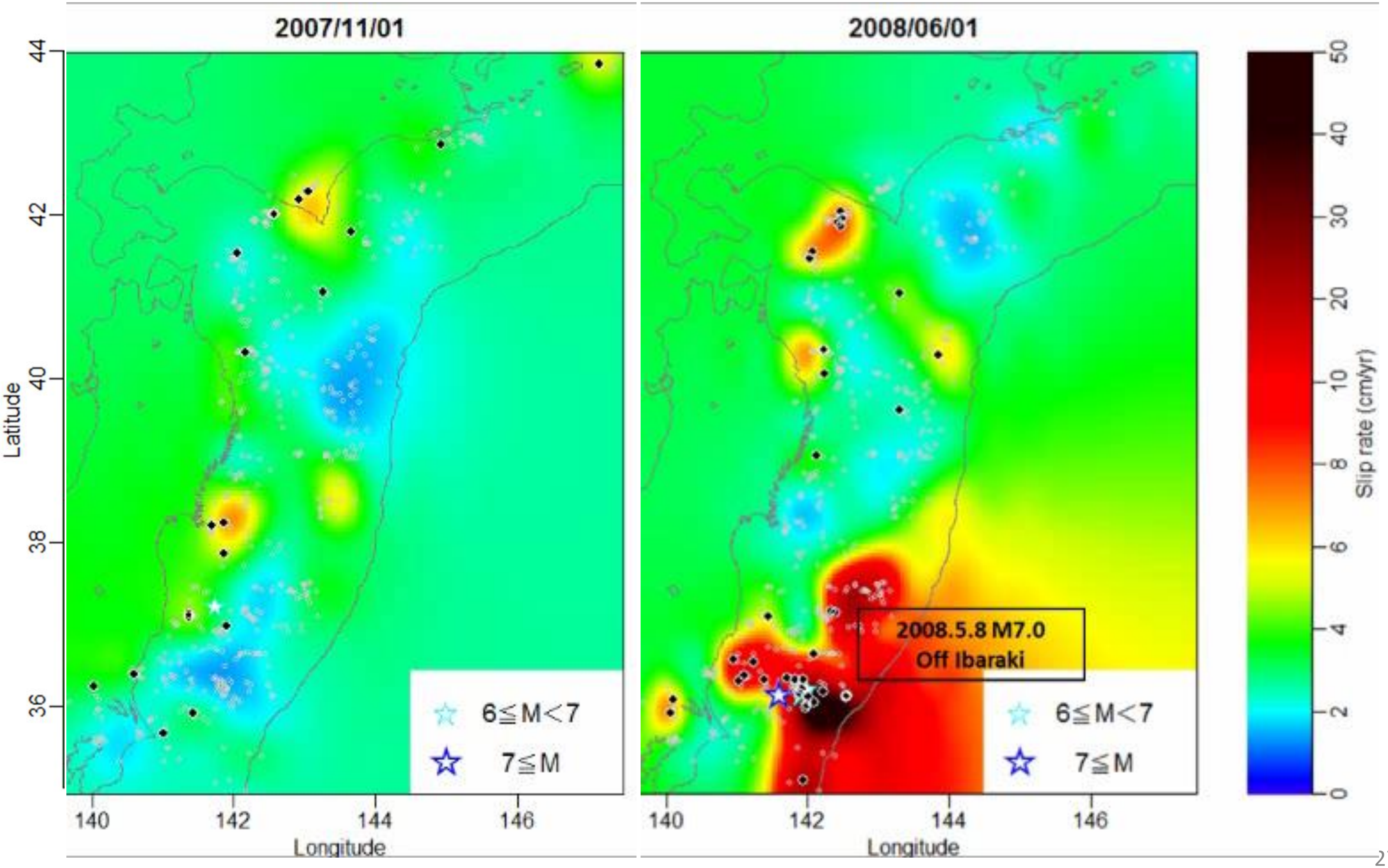
Spatial Distribution of Slip Rate (before and after Off Sanriku-Haruka M7.6)



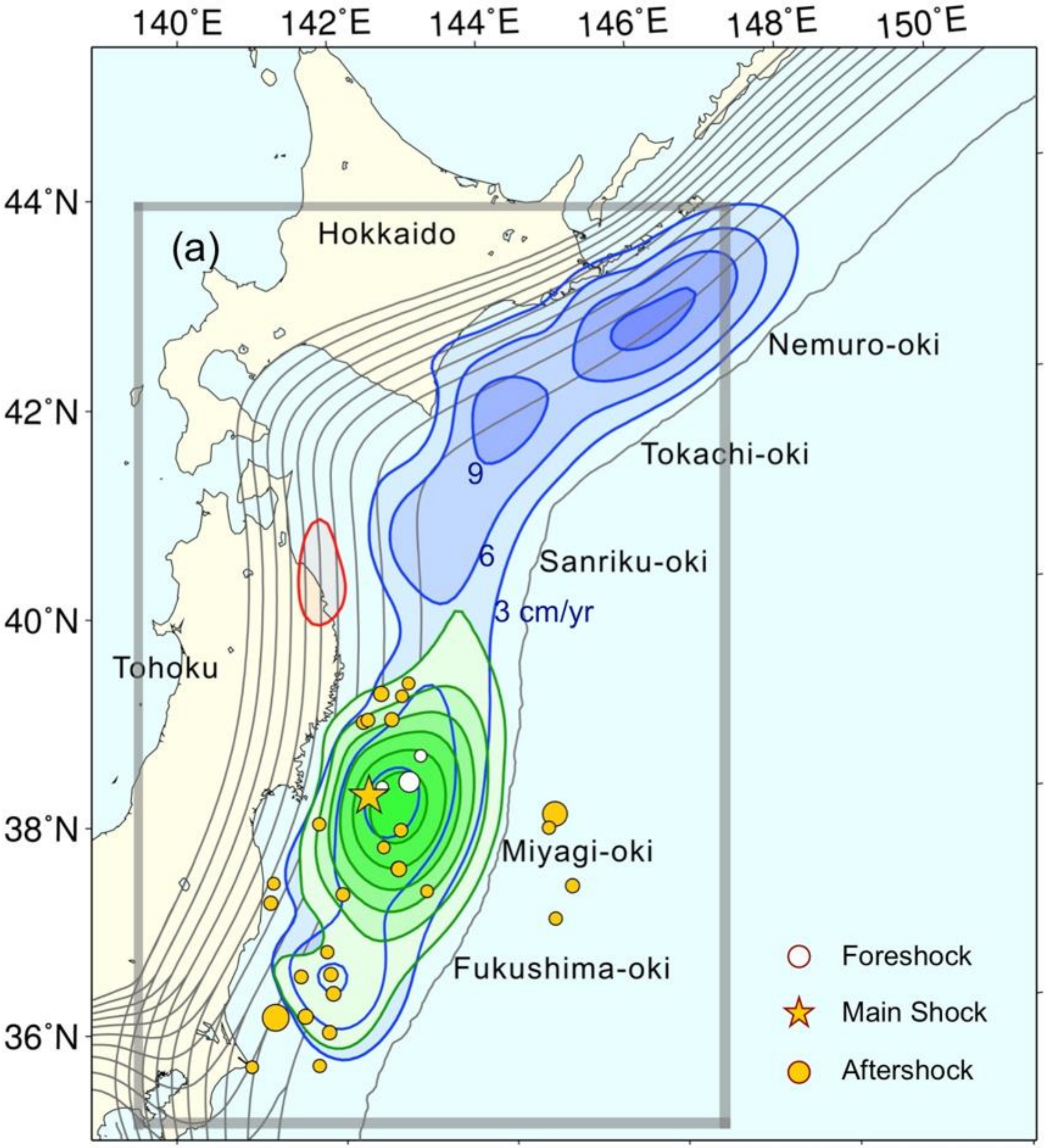
Spatial Distribution of Slip Rate (before and after Off Tokachi M8.0)



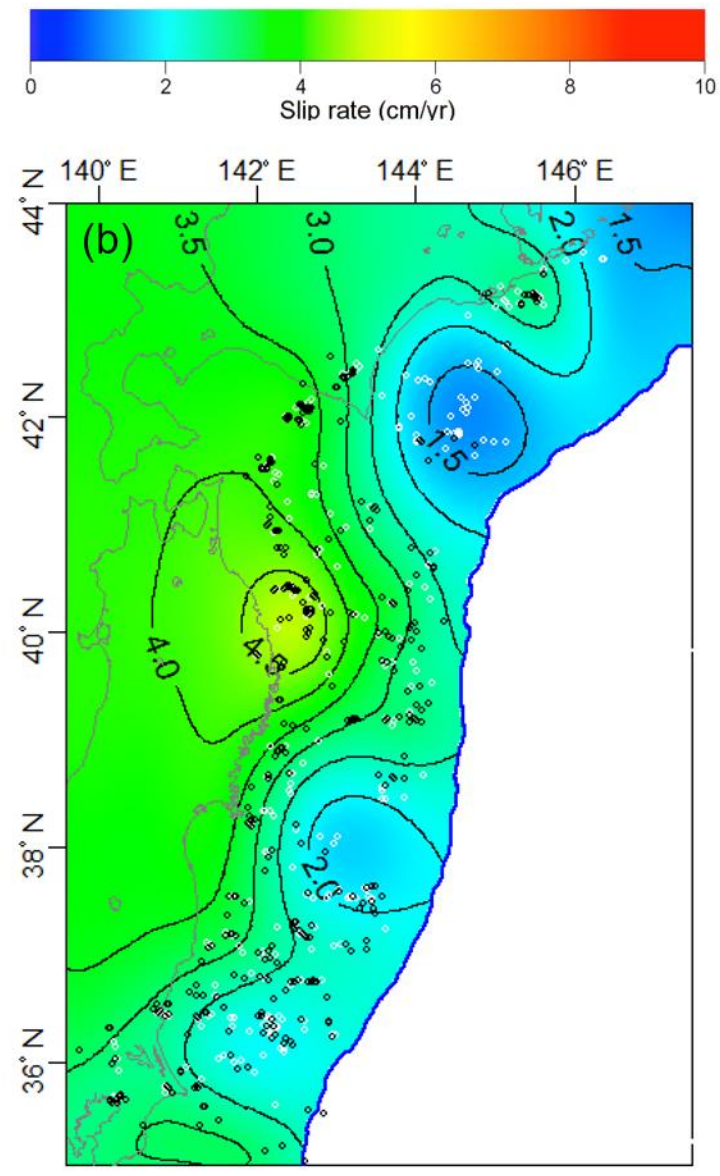
Spatial Distribution of Slip Rate (before and after Off Ibaraki M7.0)



Comparison with GPS inversion（1996/6～2000/5）



（Hashimoto et al., GJI, 2012）



（Nomura et al., GJI, 2017）

Summary

- We proposed a new stochastic model for repeating earthquake sequences to estimate spatio-temporal change in slip rate.
- Periodicity of repeating earthquakes was conserved in the transformed time even in rapid slip rate after the large earthquakes.
- Acceleration of slip and its propagation was found from the estimated spatio-temporal rate, which is difficult to see by other instruments. Such information is important to reveal mechanism of earthquakes in a geophysical view
- Estimated slip rate is basically consistent with slip-deficit rate inferred from GPS observation. Our method is especially useful for the region far from GPS network.