Effects of gas escape and crystallization on the complexity of conduit flow dynamics during lava dome eruptions
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We investigated the coupled effects of gas escape and crystallization on the dynamics of lava dome eruptions using a one-dimensional conduit flow model. The relationship between chamber pressure $p_{ch}$ and mass flow rate $q$ for steady conduit flow commonly has a regime of negative differential resistance (i.e., $dp_{ch}/dq < 0$), which causes a transition from lava dome to explosive eruption. Two positive-feedback mechanisms that result in negative differential resistance have been identified. First, effective magma viscosity decreases with increasing $q$ because of a delay of crystallization, leading to reduced viscous wall friction (feedback 1). Second, magma porosity increases with increasing $q$ because of less efficient gas escape, leading to reduced gravitational load (feedback 2). For high-phenocryst-content magma (volume fraction $> 0.5$), feedback 1 is the main mechanism that forms negative differential resistance. In this case, the transition from lava dome to explosive eruption occurs when the magma supply rate exceeds a fixed critical value. For low-phenocryst-content magma (volume fraction $< 0.5$), feedback 2 plays a key role so that the transition is controlled by the permeability of the surrounding rocks; the critical magma supply rate remarkably decreases with decreasing permeability. Transition due to feedback 2 is associated with a change in the chemical composition of volcanic gas, a drastic increase in magma porosity from nearly 0 to greater than 0.8, and overpressure at a shallower level, which can be detected from geochemical and geophysical field observations.


1. Introduction

As volatile-rich magmas ascend to the Earth’s surface and decompress in volcanic conduits, the magmas exsolve gas bubbles and the exsolved gas expands. The dynamics of the gas-liquid mixture are governed by competition between gas expansion and gas escape from the magma. If gas escapes efficiently from the liquid magma, the gas-liquid mixture passively effuses as a dense lava dome [e.g., Eichelberger et al., 1986]; if exsolved gas remains trapped in the magma, an overflow of the inflating gas-liquid mixture can result in an explosive eruption [e.g., Jaupart and Allegre, 1991; Woods and Koyaguchi, 1994; Slezin, 2003]. The mechanism of gas escape, therefore, is a key factor that controls the transition of eruption styles.

The dynamics of conduit flow are also controlled by magma viscosity. For example, during lava dome eruptions driven by pressure change at magma chamber surrounded by elastic rocks, when effective viscosity of magma decreases as magma flow rate increases in the conduit, the magma flow and the chamber pressure tend to oscillate [e.g., Whitehead and Helfrich, 1991; Iida, 1996; Wylie et al., 1999; Maeda, 2000; Melnik and Sparks, 2005; Costa et al., 2007]. The effect of viscosity change on conduit flow dynamics is complex, because the viscosity of magma (gas-crystal-liquid mixture) is governed by many physical factors (e.g., temperature [Hess and Dingwell, 1996], dissolved volatile content of the magma [Richet et al., 1996], distribution and shape of bubbles [Llewellin et al., 2002; Pal, 2003], and degree of crystallization [Lejeune and Richet, 1995; Costa, 2005] (further comments available at http://arxiv.org/abs/physics/0512173). Furthermore, change in conduit radius modifies the viscous resistance of conduit flow [Slezin, 2003; de’Michieli Vitturi et al., 2008]. Among these factors, the kinetics of crystallization has been considered the primary factor that accounts for the complex features of conduit flow, because a delay of crystallization decreases the effective viscosity of liquid-crystal mixture with increasing mass flow rate [e.g., Melnik and Sparks, 1999, 2002; Barmin et al., 2002].

Although the individual effects of the above mechanisms were studied in detail in previous works, how these mechanisms interplay in conduit flow dynamics has not been clarified. Recently, Kozono and Koyaguchi [2010] has...
2.1. Basic Equations

The dynamics of lava dome eruptions of silicic volatile-rich magmas have been successfully modeled by a 1-D conduit flow model [Melnik and Sparks, 2002; Kozono and Koyaguchi, 2009a, 2009b]. The governing equations are composed of the mass conservation of volatile component (i.e., gas phase plus dissolved volatiles in the liquid)

\[
\frac{\partial}{\partial t} \left\{ (1 - \phi)(1 - \beta) \rho_c \phi + \phi \rho_w \right\} + \frac{\partial}{\partial z} \left\{ (1 - \phi)(1 - \beta) \rho_c \frac{\partial \phi}{\partial z} + \phi \rho_w \frac{\partial \phi}{\partial z} \right\} = -Q_w, \tag{1}
\]

the mass conservation of the non-volatile components (i.e., crystals plus non-volatile component in the liquid)

\[
\frac{\partial}{\partial t} \left\{ (1 - \phi)(1 - \beta) \rho_l + \beta \rho_c \right\} + \frac{\partial}{\partial z} \left\{ (1 - \phi)(1 - \beta) \rho_l \frac{\partial \phi}{\partial z} + \beta \frac{\partial \phi}{\partial z} \right\} = \rho g + \rho_f \nabla \cdot \mathbf{F}, \tag{2}
\]

and the momentum conservation

\[
\frac{\partial}{\partial t} \left[ (1 - \phi)(1 - \beta) \rho_l + \beta \rho_c \right] g + \rho_f \nabla \cdot \mathbf{F} = 0, \tag{3}
\]

where \( z \) is the vertical coordinate, \( w \) is the vertical velocity, \( \rho \) is density, \( p \) is pressure, \( g \) is acceleration due to gravity, \( c \) is the mass concentration of dissolved volatiles, \( \beta \) is crystal volume fraction, \( \phi \) is gas volume fraction (i.e., porosity), and the subscripts \( g \), 1 and \( c \) denote the gas, liquid and crystal, respectively. We assume that there is no pressure difference between gas and liquid (i.e., \( p_g = p_l \)) in this study [cf. Melnik and Sparks, 2005]. In these equations, the rate of lateral gas escape at each depth is expressed by \( Q_w \) (kg m\(^{-3}\)s\(^{-1}\)) and the viscous force per unit volume of liquid due to wall friction is expressed by \( F_{w} \), the details of which are described below. For lava dome eruptions, the inertia terms in the momentum conservation equations are negligible. The equation of state for the gas is \( \rho_g = p/(RT) \), and that for liquid is \( \rho_l = \text{constant} (2500 \text{ kg m}^{-3}) \). Here, \( R \) is the gas constant (462 J kg\(^{-1}\) K\(^{-1}\)) and \( T \) is the magma temperature (set to be 1123 K in this study). We assume that the exsolution of gas follows the equilibrium solubility law:

\[
c = \min \left[ \sqrt{sp} \left\{ \frac{\rho_l}{\beta} \right\} \right]. \tag{4}
\]
where $n_0$ is the initial H$_2$O content and $s$ is the solubility constant of H$_2$O, which is given by $s = 4.11 \times 10^{-6}$ Pa$^{-1/2}$ for typical silicic magmas.

In the governing equations, the effect of vertical gas escape is expressed by the relative motion between gas and non-gas phases. During lava dome eruptions, the magma may ascend as a bubbly flow at depth; as a network of gas forms, the gas escapes as a permeable flow through the liquid magma. Here, for simplicity, we treat the gas-liquid mixture as a permeable flow throughout the conduit; namely, it follows Darcy’s law as

$$k_w \frac{\partial p}{\partial z} + \rho_w g + w_g - w_l = 0,$$

where $k_w$ is the permeability for vertical gas escape and $\eta_w$ is the viscosity of the gas phase ($\sim 10^{-5}$ Pa s). Following Costa [2006], we tentatively assume that $k_w$ has a form of $k_w = k_w \phi^2/(1 - \phi)$, where $k_w$ is a constant (typically $10^{-12} - 10^{-11}$ m$^3$). It is also assumed that the friction between wall and gas phase is negligible. On the other hand, the effect of lateral gas escape is modeled by the term of $Q_w$ in equation (1) as [e.g., Jaupart and Allegre, 1991; Woods and Koyaguchi, 1994; Diller et al., 2006; Taisne and Jaupart, 2008]

$$Q_w = \frac{2\rho_w k_w [\rho - \{\rho_w g(-z) + \rho_s\}]}{\eta_w r^2},$$

where $k_w$ is the permeability of the wallrocks for lateral gas escape, $\rho_s$ is the density of the surrounding rocks, $\rho$ is atmospheric pressure, and $r$ is the conduit radius. It is assumed that the permeability of the wallrocks decreases with increasing depth as $k_w = k_w_0 \exp[-(\rho_g g(-z) + \rho_s)/P^2]$, where $k_w_0$ and $P^2$ are constants. Because the values of both $k_w_0$ and $P^2$ are poorly constrained, wide ranges of values are assumed for these parameters ($k_w_0 = 0 - 10^{-10}$ m$^3$ and $P^2 = 2.5 - 20$ MPa) in this study. [7] For lava dome eruptions, the viscous force per unit volume of liquid due to wall friction ($F_{lw}$) is evaluated from the viscous term of the Navier-Stokes equation for unidirectional cylindrical Poiseuille flow as

$$F_{lw} = \frac{8 \eta \rho_1}{r^2},$$

where $\eta$ is magma viscosity. Generally, magma viscosity drastically increases near the surface because of volatile exsolution and crystallization. In this study, we assume that viscosity has a form of $\eta = \eta_0(c) f_\beta(\beta) f_\phi(\phi)$, where $\eta_0(c)$ is liquid viscosity expressed by a function of the mass fraction of dissolved water in the liquid $c$, and $f_\beta(\beta)$ and $f_\phi(\phi)$ represent the influences of crystal and gas volume fractions on viscosity. The form of $\eta_0(c)$ is determined from the model by Hess and Dingwell [1996], and that of $f_\beta(\beta)$ is determined from the model by Costa [2005]. The form of $f_\phi(\phi)$ depends on the magnitude of the static capillary number ($\eta_\gamma \gamma /\sigma$ during bubble deformation, where $\gamma$ is the deformation rate of the bubble, $\gamma$ is the bubble radius and $\sigma$ is the surface tension). For the bubbles in dome lavas, the capillary number is likely to exceed 1 (typically of order $10^2$) [Massol and Jaupart, 2009]. In this case the effective shear viscosity of the gas-liquid mixture may decrease, due to bubble deformation during flow ($f_\phi(\phi) = (1 - \phi)^{5/3}$) [Llewellyn et al., 2002].

The increase in $\beta$ due to crystallization is calculated from a simple crystal growth model as

$$\frac{\partial \beta}{\partial t} + \frac{w_\beta}{\partial z} = \Gamma \beta^{2/3} (\beta_{eq} - \beta),$$

where $\Gamma$ is a constant that represents the inverse of the timescale of crystal growth, which is set to be $10^{-5} - 10^{-4}$ s$^{-1}$ in this study based on the experiment data of crystallization kinetics by Couch et al. [2003a], and $\beta_{eq}$ is the equilibrium crystal content based on the experiment data by Couch et al. [2003b] expressed as a function of pressure:

$$\beta_{eq} = \beta_{ph} + (1 - \beta_{ph}) \left[ C_{10} + C_{11} \ln(10^{-6} p) + C_{12} \{ \ln(10^{-6} p) \}^2 \right]$$

$$C_{10} = 0.842; C_{11} = -9.54 \times 10^{-2}; C_{12} = -1.16 \times 10^{-2}.$$ (9)

The volume fraction of phenocrysts $\beta_{ph}$ is given as the initial value of this equation.

The above equations are solved as a two-point boundary value problem for a given conduit of length $L$ to obtain the variables $\phi, w_g, w_l, \rho$, and $\beta$ as a function of depth $z$ and time $t$. The mass flow rate of magma $q$ (kg m$^{-2}$ s$^{-1}$) is calculated from these quantities as

$$q(z, t) = \phi \rho_g w_g + (1 - \phi) [(1 - \beta) \rho_l + \beta \rho_s] w_l.$$ (10)

The boundary condition at the bottom end of the conduit is that the pressure is equal to the pressure at the magma chamber ($P_{ch}$), and the boundary condition at the vent is that the pressure is equal to the atmospheric pressure.

Generally, the dynamics of conduit flow depend on how the magma chamber pressure accommodates the balance between magma influx and outflux. Here, we consider a model of the magma plumbing system where a magma chamber is located in elastic rocks and is continuously fed from below with new magma (Figure 1a) [cf. Melnik and Sparks, 2005]. In modeling the transition of conduit flow, $P_{ch}$ is assumed to change in response to the balance between magma influx $q_{in}$ from depth to the magma chamber and magma outflux $q_{out}$ to the conduit [Melnik and Sparks, 2005]:

$$\frac{dp_{ch}}{dt} = \frac{4(K) \pi r^2}{\langle \rho \rangle V_{ch} (3K + 4E)} (q_{in} - q_{out}),$$

where $E$ is the elastic modulus of surrounding rocks ($\sim 10^{10}$ Pa), $\langle \rho \rangle$ is the average magma density in the chamber, $\langle K \rangle$ is the average bulk modulus of magma in the chamber with the assumption of magma compressibility [Melnik and Sparks, 2005], and $V_{ch}$ is the chamber volume (set to be $10^2$ m$^3$). Although some previous studies revealed that conduit geometry also influences the dynamics of conduit flow [e.g., Costa et al., 2007; de Michieli Vitturi et al., 2008, 2010], in this study we assume a simple conduit with constant radius and length ($r = 15$ m and $L = 4000$ m) in
order to extract the effects of gas escape and crystallization. In the analyses of the time-dependent model, $q_{in}$ is given as a parameter, and $p_{ch}$ is calculated from equation (11) as a function of time. The dynamics of conduit flow are calculated under the condition where $q(−L, t) = q_{out}$ (see Figure 1a). The magma discharge rate is obtained as $\pi r^2 q(0, t)$ in kg s$^{-1}$. Our calculations indicate that $q(0, t)$ can be approximated by the flow rate at the conduit inlet (i.e., $q(0, t) \sim q_{out}$) in most cases. We followed the numerical method by Melnik and Sparks [2005] for the integration of these equations.

In this study, we introduce a non-dimensional parameter to express the degree of lateral gas escape at each depth as

$$E_w = 1 - \frac{dp_w}{n q_{out}},$$

where $n$ is defined by

$$n = \frac{n_0 - (1-n_0)}{(1-n_0) + n_0^c} = \frac{(1-n_0)}{(1-n_0) + n_0^c}.$$  

For steady states, the denominator of the second term of the right-hand-side equation of equation (12) means the mass-flow-rate of the gas for the reference state where no lateral gas escape is allowed; therefore, $E_w = 0$ when there is no lateral gas escape, and $E_w$ increases from 0 to 1, as more lateral gas escapes (see Appendix B). For the time-dependent model, the meaning of $E_w$ is less clear. However, $E_w$ is still useful as a measure of the degree of lateral gas escape, because the spatial variation of $q(z, t)$ is generally small even during the transitional state of conduit flow.

### 2.2. Basic Idea of Analyses

Generally, the dynamics of steady viscous pipe flow are characterized by the relationship between source (chamber) pressure $p_{ch}$ and mass flow rate $q$ (the steady $p_{ch}$-$q$ relationship). In stable steady flows, the $p_{ch}$-$q$ relationship has positive values of $dp_{ch}/dq$ (positive differential resistance); mass flow rate increases as source pressure increases, and vice versa. For conduit flow involving gas escape and crystallization, however, $dp_{ch}/dq$ can have negative values (negative differential resistance), which leads to complex dynamics, such as abrupt change and/or cyclic change of the magma discharge rate [e.g., Melnik and Sparks, 2005; Costa et al., 2007; Nakanishi and Koyaguchi, 2008].

In this study, we investigate the complex dynamics of conduit flow using the steady $p_{ch}$-$q$ relationship. This relationship is calculated from the conduit flow model in which the time-dependent terms in equations (1), (2), and (8) are omitted. In the steady conduit flow model, when the boundary conditions of pressures at the both ends are given, the mass flow rate $q$ will be determined. Otherwise, when the value of $q$ at the inlet of the conduit is given, the value of $p_{ch}$ will be determined. Consequently, we can obtain a relationship between $p_{ch}$ and $q$ for steady states. By definition, the values of $p_{ch}$ and $q$ for steady solutions will not change with time; however, when the value of $p_{ch}$ or $q$ slightly deviates from this curve, the subsequent evolution of the flow is different depending on whether $dp_{ch}/dq$ of this curve has a positive or negative value. Around the steady solution with $dp_{ch}/dq > 0$, the flow evolves toward the steady solution (i.e., a stable solution). Around the steady solution with $dp_{ch}/dq < 0$, on the other hand, the deviation grows with time and a completely different flow can result (i.e., an unstable solution).

In the model of Figure 1a, how $q_{out}$ (and hence, magma discharge rate) responds to $q_{in}$ depends on the range of $q_{in}$ in the steady $p_{ch}$-$q$ relationship. Let us consider the cases where the values of $q_{in}$ gradually increase (Figure 1b). While $q_{in}$ is within the range of $dp_{ch}/dq > 0$, a steady evolution with $q_{out} = q_{in}$ results ($S_2$ in Figure 1b). This eruption is stable as long as $q_{in}$ is fixed. As $q_{in}$ increases from the range of $dp_{ch}/dq > 0$ to that of $dp_{ch}/dq < 0$, the steady solution tends to become unstable and $q_{out}$ abruptly increases from $S_2$ to $S_3$ in Figure 1b. Strictly speaking, the steady solution can be stable even when $q_{in}$ is within the range of $dp_{ch}/dq < 0$ under some conditions [e.g., Melnik and Sparks, 2005; Nakanishi and Koyaguchi, 2008]. Nevertheless, the condition for the abrupt increase of $q_{out}$ can be roughly inferred from whether $q_{in}$ is within the range of $dp_{ch}/dq > 0$ or $dp_{ch}/dq < 0$. In other words, the mechanisms that control the complex dynamics of conduit flow can be specified by systematically understanding the factors that govern the features of the steady $p_{ch}$-$q$ relationship.

In the following sections we are particularly concerned with the effects of gas escape and crystallization kinetics on the dynamics of conduit flow. First, we describe how the steady $p_{ch}$-$q$ relationship changes with magma properties and geological conditions when these effects are taken into account. Second, we describe typical features of stable steady solutions (i.e., steady solutions within the range of $dp_{ch}/dq > 0$). Third, we investigate how the stable steady solutions become unstable and how the abrupt increase of $q_{out}$ results from a gradual increase of $q_{in}$, based on the time-dependent model. The main purpose of this study is to specify the mechanisms that control the complex dynamics of conduit flow by means of a systematic classification of the steady $p_{ch}$-$q$ relationship, rather than to describe the details of the transitional state during the abrupt increase of magma discharge rate. The results of a more extensive parametric study for the time-dependent model will be published elsewhere.

In the following analyses, in order to obtain clear insight into the mathematical structure of the steady $p_{ch}$-$q$ relationship, we assume a very wide range of chamber pressure, including conditions far from the lithostatic pressure. Geological implications of such unrealistic results will be assessed as necessary.

### 3. Results

#### 3.1. Steady $p_{ch}$-$q$ Relationship

The steady $p_{ch}$-$q$ relationship depends on the permeabilities for vertical and lateral gas escapes $k_{v0}$ and $k_{l0}$, the phenocryst content $\beta_{ph}$, and the growth rate of crystal $\Gamma$ (Figures 2 and 3). Our model considers both vertical and lateral gas escapes; the steady $p_{ch}$-$q$ relationship for this general case is referred to as “the vertical and lateral gas escapes (VLGE) curve”. The diversity of the VLGE curve can be systematically described using steady $p_{ch}$-$q$ relationships for three
reference states: the EGE curve, the NGE curve, and the NLGE curve. The EGE curve is the steady \( \rho - q \) relationship when the gas escape is so efficient (in the limit of the relationship for \( k_v \to \infty \) or \( k_w \to \infty \)) that no gas phase remains in the conduit. The NGE curve is the relationship with no gas escape (\( k_v = k_w = 0 \)), and the NLGE curve is the relationship with no lateral gas escape (\( k_w = 0 \)). For a given \( q \), the EGE curve has the highest \( \rho \), and the NGE curve has the lowest \( \rho \). The NLGE curve has an intermediate \( \rho \) between the EGE and the NGE curves; it asymptotically approaches the EGE curve as \( q \) decreases, and it approaches the NLGE curve as \( q \) increases. The value of \( q \) for the transition from EGE to NLGE curves increases with \( k_w \) (Figure 2).

Figure 2. Steady \( \rho - q \) relationship for \( n_0 = 0.05 \), \( T = 875 \) (C°), \( r = 15 \) (m), \( L = 4000 \) (m), \( k_v = 10^{-12} \) (m²), and \( P = 20 \) (MPa) with varying \( k_w \). Results for (a) \( \beta_{ph} = 0.6 \) and \( \Gamma = 10^{-4} \) (s⁻¹), (b) \( \beta_{ph} = 0.6 \) and \( \Gamma = 10^{-5} \) (s⁻¹), (c) \( \beta_{ph} = 0.4 \) and \( \Gamma = 10^{-4} \) (s⁻¹), and (d) \( \beta_{ph} = 0.4 \) and \( \Gamma = 10^{-5} \) (s⁻¹). The dotted curve indicates efficient gas escape (EGE), the short-dashed curve indicates no gas escape (NGE), and the long-dashed curve indicates no lateral gas escape (NLGE). The points of E, E’, N1, N2, N3, and N4 in (b), (c), and (d) represent the typical stable solutions described in Table 1; they correspond to the results in Figure 6.

For a given \( q \), the EGE curve has the highest \( \rho \), and the NGE curve has the lowest \( \rho \). The NLGE curve has an intermediate \( \rho \) between the EGE and the NGE curves; it asymptotically approaches the EGE curve as \( q \) decreases, and it approaches the NLGE curve as \( q \) increases. The value of \( q \) for the transition from EGE to NLGE curves increases with \( k_w \) (Figure 2).

Figure 2 indicates that the steady \( \rho - q \) relationship strongly depends on phenocryst content. For high phenocryst...
content ($\beta_{\text{ph}} = 0.6$), the difference in $p_{\text{ch}}$ for a given $q$ is relatively small between the EGE and NGE curves. When the growth rate of crystal is high ($\Gamma = 10^{-4}$ s$^{-1}$; Figure 2a), $p_{\text{ch}}$ of the EGE, NGE, and NLGE curves monotonically increases with $q$. Because of these features of the EGE, NGE, and NLGE curves, $p_{\text{ch}}$ of the VLGE curve also monotonically increases with $q$. When the growth rate of crystal is relatively low ($\Gamma = 10^{-5} - 10^{-4}$ s$^{-1}$), the variation in $p_{\text{ch}}/dq$ has negative values in the intermediate regimes of $q$, but $dp_{\text{ch}}/dq > 0$ in the low-$q$ and high-$q$ regions. As a result, the VLGE curve between these curves also has a sigmoidal shape.

[21] For low phenocryst content ($\beta_{\text{ph}} = 0.4$; Figures 2c and 2d), the variation in $p_{\text{ch}}$ between the EGE and NGE curves is much wider than for high phenocryst content. The EGE and NGE curves commonly have a sigmoidal shape for a wide range of the growth rate of crystal ($\Gamma = 10^{-5} - 10^{-4}$ s$^{-1}$). Because of these features of the EGE and NGE curves, the VLGE curve has complex features with $dp_{\text{ch}}/dq < 0$. The VLGE curves for extremely low-$k_{\text{w}0}$ (i.e., those around the NLGE curve) have negative $dp_{\text{ch}}/dq$ in a wide range from low to intermediate $q$. This feature implies that lateral gas escape must play a role for stable solutions of low-$q$ lava dome eruptions with low phenocryst content to exist. For $k_{\text{w}0} = 10^{-13}$ m$^2$, the VLGE curve shows an “overfold” in the intermediate regime (Figures 2c and 2d). It should also be noted that, for high $\Gamma$, the NLGE and the VLGE curves with low-$k_{\text{w}0}$ have two minima at $q = 10^{-1} - 10^{-2.3}$ kg m$^{-2}$ s$^{-1}$ (Figure 2c).

[22] The effect of variations in the permeability for vertical gas escape ($k_{\text{w}0}$) on the VLGE curve is minor compared with that of lateral gas escape ($k_{\text{w}0}$); it influences the VLGE curve for only very small $k_{\text{w}0}$ (Figure 3). For $k_{\text{w}0} < 10^{-16}$ m$^2$, the VLGE curves shift upward with increasing $k_{\text{w}0}$ in the low-$q$ range. For $k_{\text{w}0} > 10^{-14}$ m$^2$, however, this effect of variation in $k_{\text{w}0}$ becomes negligible. From these results, it is inferred that the variation in $k_{\text{w}0}$ controls mainly the NLGE curve (Figure 2). For example, the minimum point of $p_{\text{ch}}$ at $q = 10^{-1} - 10^{-2}$ kg m$^{-2}$ s$^{-1}$ for low $\beta_{\text{ph}}$ and high $\Gamma$ (e.g., Figure 2c) may disappear for relatively large $k_{\text{w}0}$ (see also Figure 3).

[23] Our preliminary studies indicate that the detailed features of the steady $p_{\text{ch}}$-$q$ relationship also depend on parameters related to lateral gas escape ($k_{\text{w}0}$), such as the constant for the permeability of the wallrocks $P^*$ in equation (6). As $P^*$ decreases, the magnitude of $k_{\text{w}0}$ decreases; as a result, the VLGE curve shifts toward the NLGE curve (Figure 4). Despite these complex dependences on the mechanisms of vertical and lateral gas escapes, the fact that the characteristics of the steady $p_{\text{ch}}$-$q$ relationship are determined by at which $q$ the VLGE curve transits from the EGE curve to the NLGE curve is robust.

[24] The diverse features of the steady $p_{\text{ch}}$-$q$ relationship are schematically summarized in Figure 5. They are classified into four groups (A, B, C, and D) according to the features of the EGE and NLGE curves, which depend on the phenocryst content $\beta_{\text{ph}}$ and the growth rate of crystal $\Gamma$. For high $\beta_{\text{ph}}$, both the EGE and the NLGE curves are monotonically increasing functions for high $\Gamma$ (group A), or both have sigmoidal shapes for low $\Gamma$ (group B). With low $\beta_{\text{ph}}$, for high $\Gamma$, the EGE curve is a monotonically increasing function, and the NLGE curve has two minima (group C); in contrast, for low $\Gamma$, the EGE curve has a sigmoidal shape, and the NLGE curve has a single minimum point (group D). In each group, the features of the VLGE curve are primarily controlled by the permeability for lateral gas escape $k_{\text{w}0}$. The VLGE curve is approximated by the NLGE curve for low $k_{\text{w}0}$ and by the EGE curve for high $k_{\text{w}0}$. As $k_{\text{w}0}$ increases, the
value of $q$ for the transition from the EGE curve to the NLGE curve increases.

3.2. Stable Steady Solutions of Conduit Flow

[25] Among all the steady solutions, stable solutions (i.e., solutions in the range of $\frac{dp_{ch}}{dq} > 0$ in the $p_{ch}$-$q$ relationship) are assumed to characterize the eruptions that are continuously observed in nature. We can identify at least six types of stable solutions in Figure 5: E, E', N1, N2, N3, and N4. Solutions E are stable solutions along the EGE curve, and solutions N are those along the NLGE curve. Solutions N are subdivided into four types according to phenocryst content and mass flow rate: N1 and N3 for high phenocryst content (i.e., groups A and B in Figure 5) with low and high mass flow rate, respectively, and N2 and N4 for low phenocryst content (i.e., groups C and D in Figure 5) with low and high mass flow rate, respectively. In addition, stable solutions exist between the EGE and NLGE curves in the slightly higher-$q$ side of the EGE curve (solutions E').

[26] Solutions E and N have distinct features in the distribution of $E_w$ (the degree of lateral gas escape; see equation (12)). In solutions E and E', $E_w$ approaches 1 throughout the conduit except for the deepest level because of efficient lateral gas escape, whereas $E_w \sim 0$ in solutions N1, N2, N3 and N4 (Figure 6a). The distributions of crystal content and viscosity inside the conduit are closely related to the mass flow rate ($q$). In the solutions with high $q$ (N3 and N4), the crystal content does not increase during ascent (Figure 6e), so that magma viscosity increases only slightly toward the surface, due to the exsolution of volatile components (Figure 6f). In solutions with low $q$ (E, E', N1, and N2), the crystal content increases during ascent, which causes a remarkable increase in the viscosity of the crystal-magma mixture near the surface (see Figures 6e and 6f).

Figure 5. Classification of steady $p_{ch}$-$q$ relationships. The relationships are classified into four groups: A for high $\beta_{ph}$ and high $\Gamma$, B for high $\beta_{ph}$ and low $\Gamma$, C for low $\beta_{ph}$ and high $\Gamma$, and D for low $\beta_{ph}$ and low $\Gamma$. Typical stable solutions (solutions E, E', N1, N2, N3, and N4 in Table 1) are also schematically shown in these diagrams.
These variations in degree of gas escape and in viscosity lead to the variations of porosity and pressure distributions inside the conduit (Figures 6b, 6c, and 6d and Table 1).

[27] Solutions E and E’ have low porosity (<0.2) throughout the conduit because of efficient lateral gas escape (i.e., $E_w \sim 1$ in Figure 6a), whereas solutions N have relatively higher porosity (>0.2). There is a systematic difference between solutions N with low q (N1 and N2) and those with high q (N3 and N4): low-q N solutions (N1 and N2) are characterized by decreased porosity at a shallower level. The difference in porosity distribution at a shallower level between low-q and high-q N solutions is explained by the

Figure 6. Distributions of (a) parameter $E_w$, (b) porosity, (c) pressure, (d) overpressure (the difference between magma pressure and lithostatic pressure), (e) crystal content, and (f) magma viscosity inside the conduit of six typical stable solutions (the solutions E, E’, N1, N2, N3, and N4 in Figure 2).
effects of magma viscosity on vertical gas escape [Kozono and Koyaguchi, 2010]. Generally, magma viscosity suppresses liquid ascent by viscous wall friction. When magma viscosity is so high that the effect of wall friction exceeds that of gas drag force, efficient vertical gas escape is enhanced, which leads to decrease in porosity. In low-\(q\) N solutions (N1 and N2), the effect of wall friction due to crystallization is so large near the surface (Figures 6e and 6f) that porosity decreases toward the surface near the vent. In high-\(q\) N solutions (N3 and N4), porosity increases upward throughout the conduit and exceeds 0.8 at the vent because the viscosity increase due to crystallization is negligible.

[25] The above variations in magma viscosity and porosity influence the pressure distribution inside the conduit (Figure 6c). The pressure distributions in solutions E, N1, and N2 are characterized by overpressure (the difference between magma pressure and lithostatic pressure; \(p - (\rho g f(z) + p_s)\)) at the shallower level of the conduit, whereas overpressure is less remarkable in solutions E, N3, and N4 (Figure 6d). Overpressure at the shallower level results from a steep pressure gradient near the surface, as well as a relatively gentle pressure gradient at the deeper level. Such pressure distribution is caused by the coupled effects of increased porosity at the deeper level and increased magma viscosity at the shallower level [e.g., Sparks, 1997; Kozono and Koyaguchi, 2009a; Burgisser et al., 2010]. Generally, the pressure gradient in the conduit is determined by the sum of gravitational load of magma and viscous wall friction (equation (3)). Because magma viscosity is relatively low at the deeper level (Figure 6f), the pressure gradient is primarily determined by the gravitational load (i.e., \(-\rho g\)). At the shallower level, a drastic increase in magma viscosity due to crystallization (Figures 6e and 6f) can lead to a steep pressure gradient due to wall friction. Overpressure at the shallower level in solutions E, N1, and N2 is explained by very high viscosity due to crystallization near the surface (Figures 6e and 6f) and relatively high porosity (hence, relatively small \(\rho\)) at the deeper level (Figure 6b). Slight overpressure in solutions N3 and N4 results from their low crystal content (hence, relatively low viscosity) near the surface (Figures 6e and 6f). In solution E, no localized overpressure is possible at the shallower level because of its steep pressure gradient at the deeper level due to extremely low porosity (Figure 6b).

[29] In Figure 6c, the values of \(p_{ch}\) in solutions E, N1, and N3 are much higher, and that in solution N2 is much lower than lithostatic pressure. These results are presented here in order to emphasize the typical features of different types of steady solutions. We confirmed that the absolute values of \(p_{ch}\) for E, N1, and N3 solutions can be tuned to realistic values by changing assumed parameters related to viscosity without modifying the qualitative features of the present results (see Appendix A). On the other hand, solution N2 is unlikely to occur under realistic geological conditions for lava dome eruptions. Very low \(p_{ch}\) is an inherent feature of solution N2 because this solution has a gentle pressure gradient throughout the conduit due to low \(\rho\) (or high \(\phi\)) and low \(q\) (or small \(w_1\)) in equations (3) and (7). Although very low \(p_{ch}\) may be possible during a caldera-forming eruption, such a situation is beyond the scope of this study. In addition, solution N2 (i.e., the range of \(dp_{ch}/dq > 0\) with low \(p_{ch}\) along the NLGE curve) tends to diminish with a slight increase in \(k_{v0}\) (see Figure 3). For the same reason, we will not discuss the steady solutions with positive \(dp_{ch}/dq\) resulting from the overfold of the \(p_{ch}-q\) relationship for low \(\beta_{ph}\) and \(k_{v0} = 10^{-13}\) \(m^2\) (Figures 2c and 2d); these solutions also occur only when \(p_{ch}\) is very low and they diminish with a slight change in \(k_{v0}\).

### 3.3. Characteristics of Transitional State

[30] In the model of Figure 1b, as \(q_{in}\) increases from the range of \(dp_{ch}/dq > 0\) to that of \(dp_{ch}/dq < 0\) in the steady \(p_{ch}-q\) relationship, \(q_{out}\) abruptly increases from the low-\(q\) solution (S2) to the high-\(q\) stable solution (S3). In this section, we investigate the behavior of the transition from low-\(q\) to high-\(q\) solutions based on the time-dependent conduit flow model. Here, we focus on how the qualitative features of transition depend on the types of the steady \(p_{ch}-q\) relationship (Figure 5) and the stable steady solution (Figure 6 and Table 1), rather than the details of each transitional state such as its timescale and/or associated oscillation behavior.

[31] In this analysis, we initially set \(q_{in}\) at a stable steady solution in the low-\(q\) regime around the maximum of the \(p_{ch}-q\) relationship, and then, we investigate how the conduit flow responds when \(q_{in}\) slightly increases from the range of \(dp_{ch}/dq > 0\) to that of \(dp_{ch}/dq < 0\). We classify the transitional states into five types based on the types of low-\(q\) and high-\(q\) stable solutions: transitions EE, N1N3, EE’N1, EE’N3, and EE’N4 (Table 2). Transition from

### Table 1. Features of Typical Stable Solutions

<table>
<thead>
<tr>
<th>Name</th>
<th>Curve</th>
<th>(\beta_{ph})</th>
<th>(q)</th>
<th>(E_s)</th>
<th>(\phi_d)</th>
<th>(\phi_s)</th>
<th>(\Delta p_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>EGE</td>
<td>low</td>
<td>~1</td>
<td>~0</td>
<td>~0</td>
<td>low</td>
<td></td>
</tr>
<tr>
<td>E’</td>
<td>near EGE</td>
<td>low</td>
<td>~1</td>
<td>~0</td>
<td>~0</td>
<td>low</td>
<td></td>
</tr>
<tr>
<td>N1</td>
<td>NLGE</td>
<td>high</td>
<td>~0</td>
<td>0.2-0.4</td>
<td>increase → decrease</td>
<td>high</td>
<td></td>
</tr>
<tr>
<td>N2</td>
<td>NLGE</td>
<td>low</td>
<td>~0</td>
<td>0.2-0.4</td>
<td>increase → decrease</td>
<td>high</td>
<td></td>
</tr>
<tr>
<td>N3</td>
<td>NLGE</td>
<td>high</td>
<td>~0</td>
<td>0.2-0.4</td>
<td>increase (&gt;0.8)</td>
<td>low</td>
<td></td>
</tr>
<tr>
<td>N4</td>
<td>NLGE</td>
<td>low</td>
<td>~0</td>
<td>0.2-0.4</td>
<td>increase (&gt;0.8)</td>
<td>low</td>
<td></td>
</tr>
</tbody>
</table>

*Here \(\phi_d\) indicates magma porosities at a deeper level, \(\phi_s\) indicates those at a shallower level, \(\phi_s\) indicates those at the vent, and \(\Delta p_s\) indicates overpressure at a shallower level.

### Table 2. Features of Transition Patterns of Conduit Flow Among the Stable Solutions

<table>
<thead>
<tr>
<th>Transition</th>
<th>(\beta_{ph})</th>
<th>(E_s)</th>
<th>(\phi)</th>
<th>(\Delta p_s)</th>
<th>Group in Figure 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>E → E (EE)</td>
<td>-</td>
<td>~1</td>
<td>~0</td>
<td>no change</td>
<td>B, D</td>
</tr>
<tr>
<td>N1 → N3 (N1N3)</td>
<td>high</td>
<td>~0</td>
<td>increase at a shallower level</td>
<td>decrease</td>
<td>B</td>
</tr>
<tr>
<td>E → E’ → N1 (EE’N1)</td>
<td>high</td>
<td>1 → 0</td>
<td>0 → high-(\phi) propagates upward</td>
<td>increase</td>
<td>B</td>
</tr>
<tr>
<td>E → E’ → N3 (EE’N3)</td>
<td>high</td>
<td>1 → 0</td>
<td>0 → high-(\phi) propagates upward</td>
<td>increase</td>
<td>D</td>
</tr>
<tr>
<td>E → E’ → N4 (EE’N4)</td>
<td>low</td>
<td>1 → 0</td>
<td>0 → high-(\phi) propagates upward</td>
<td>increase</td>
<td>C, D</td>
</tr>
</tbody>
</table>

*\(\Delta p_s\) indicates overpressure at a shallower level.*
stable solution with unrealistically low $p_{ch}$ (e.g., N2) will not be discussed here.

[32] Transition EE occurs between the solutions along the EGE curve. During this transition, $q_{out}$ increases without remarkable changes in $E_{w}$, porosity, or overpressure (Figure 7a). The conduit flow is characterized by low porosity ($\phi \sim 0$), due to efficient lateral gas escape ($E_{w} \sim 1$) throughout the transition, although $\phi$ increases slightly and $E_{w}$ decreases slightly with time at the deeper level. Overpressure at the shallowest level decreases and that at the middle level increases with time; however, the magnitude of the pressure change is less than a few MPa.

[33] Transition N1N3 occurs between the solutions along the NLGE curve for high $\beta_{ph}$. This transition is associated with increased porosity at the shallower level (Figure 7b). Because lateral gas escape is inefficient along the NLGE curve, $E_{w}$ has low values ($\sim 0$) throughout the conduit during the transition. A transition of this type may be counter-intuitive in the sense that overpressure at the shallower level decreases with increasing $q_{out}$. The decrease in overpressure is due to the pressure gradient at the shallower level becoming gentle as porosity increases and viscosity decreases from N1 to N3.

[34] Transition from a solution on the EGE curve to a stable solution on the NLGE curve (i.e., transitions EE’N1 and EE’N3 for high $\beta_{ph}$, and EE’N4 for low $\beta_{ph}$) indicates complex changes in porosity and pressure distributions in the conduit (Figures 7c, 7d, and 7e). By definition, $E_{w}$ decreases from 1 to 0 with increasing $q_{out}$ during these transitions. Since these transitions pass through solution E’, overpressure at the shallow level increases from E to E’ at the beginning, and then decreases with time from E’ to N3 or N4 toward the end of the transition. While $q_{out}$ is small around solution E, porosity is close to 0 throughout the conduit. In the transitional state, as $q_{out}$ increases, porosity begins to increase in the deepest part of the conduit, and the region of high porosity ($\phi \geq 0.2$) propagates upward. The final porosity at the shallower level depends on whether the high-$q$ solution is solution N1, N3, or N4. In transition EE’N1, the maximum porosity is 0.4 (Figure 7c). On the other hand, in transitions EE’N3 and EE’N4, porosity near the surface increases drastically and can sometimes exceed 0.8 (Figures 7d and 7e).

3.4. Comparison With Other Conduit Flow Models

[35] Complex features of steady $p_{ch}$-$q$ relationships and transitional patterns of eruption styles have been reported in many previous studies on conduit flow models [e.g., Woods and Koyaguchi, 1994; Slezin, 2003; Melnik and Sparks, 1999, 2005; de’Michieli Vitturi et al., 2008, 2010]. We will briefly compare our results with some representative results in these previous studies. Here, we focus on the effects of vertical and lateral gas escapes on conduit flow during lava dome eruptions.

[36] Melnik and Sparks [1999, 2005] have developed steady and time-dependent conduit models in which vertical effects of gas escape from magma and crystallization kinetics are taken into account in a sophisticated way, in order to reproduce the complex features of the 1995–99 eruption of Soufrière Hills Volcano, Montserrat. These models assume that the effect of lateral gas escape is negligible, and that phenocryst content at the magma chamber is relatively high ($\beta_{ph} \sim 0.6$), following petrological data of Soufrière Hills Volcano. Judging from these assumptions, the steady $p_{ch}$-$q$ relationship in their model represents the NLGE curve for high $\beta_{ph}$ (Figure 5). It is also suggested that the transition pattern between eruptions with different $q$ in their model corresponds to transition N1N3 (Table 2). Indeed, our results for transition N1N3 are consistent with their results (e.g., porosity and pressure distributions with remarkable overpressure at the shallower level in the steady solutions [Melnik and Sparks, 1999, Figure 2], and drastic increase in porosity at the shallower level during the transition [Melnik and Sparks, 2005, Figure 6b]).

[37] The effects of lateral gas escape have rarely been studied in the literature [e.g., Woods and Koyaguchi, 1994; de’Michieli Vitturi et al., 2008, 2010]. Woods and Koyaguchi [1994] demonstrated that lateral gas escape induces a stable solution of low-$q$ effusive eruption with extremely low porosity in the conduit [Woods and Koyaguchi, 1994, Figure 2a]. De’Michieli Vitturi et al. [2008, 2010] also showed a drastic decrease in porosity results from lateral gas escape in their model where the effect of lateral gas escape is considered in only the shallowest part of the conduit. These features in the previous studies are consistent with our results for solution E (Table 1).

[38] In this study, we applied some simplifications to our model. For example, we used a more simplified equation for crystallization kinetics (equation (8)) than the model by Melnik and Sparks [2005], and we did not take into account the effects of conduit geometry [e.g., Costa et al., 2007; de’Michieli Vitturi et al., 2008, 2010]. Nevertheless, the above comparisons indicate that our simplified model can successfully reproduce the results of more sophisticated models for crystallization kinetics. We therefore consider our model sufficient for systematic analysis of the effects of both vertical and lateral gas escapes, and those of viscosity change due to crystallization for a wide range of phenocryst content.

4. Discussion

4.1. Origin of Negative Differential Resistance

[39] Thus far, we have demonstrated that the presence of negative differential resistance in the steady $p_{ch}$-$q$ relationship causes complex features in conduit flow. In conduit flow involving gas escape and crystallization, two positive-feedback mechanisms generate negative differential resistance. One is a change in magma viscosity. Magma viscosity drastically increases near the surface as crystal content exceeds the critical value $\beta_{cr}$ [Lejeune and Richet, 1995; Costa, 2005]. As the mass flow rate increases, the increase of magma viscosity becomes less remarkable because of delayed crystallization, which causes an increase in mass flow rate (“positive feedback 1” in Figure 8a). The other mechanism is the change in porosity. Generally, as mass flow rate increases, upward gas drag force against the liquid phase increases, which makes vertical gas escape less efficient [e.g., Kozono and Koyaguchi, 2010]; as a result, porosity in the conduit increases with increasing mass flow rate. Increased porosity reduces the gravitational load in the conduit, which causes an increase in mass flow rate (“positive feedback 2” in Figure 8b). The increase in porosity can
Figure 7. Numerical results of conduit flow during the transitions listed in Table 2: transitions (a) EE, (b) N1N3, (c) EE’N1, (d) EE’N3, and (e) EE’N4. (i) The transition path in steady $p_{ch}-q$ relationship, the distributions of (ii) the parameter $E_w$, (iii) porosity, and (iv) overpressure at points 0 to 4 in (i), and the variations of (v) maximum value of $E_w$, (vi) porosity at vent, and (vii) maximum value of overpressure as a function of time.
also reduce the effective magma viscosity because of bubble deformation, which can strengthen the positive feedback 2.

We can distinguish the negative differential resistance caused by these two feedback mechanisms in the steady $p_{ch}=q$ relationship. In Figure 2, the EGE and NGE curves have regions of $dp_{ch}/dq < 0$. These negative differential resistances are explained by only feedback 1 because the degree of gas escape cannot change with $q$ in these extreme cases. On the other hand, feedback 2 plays a role in forming negative differential resistance associated with the transition from the EGE curve to the NGE curve (i.e., those in the NLGE and VLGE curves). Judging from the ranges of $q$ for $dp_{ch}/dq$ to have negative values in Figure 2, negative differential resistance in the VLGE curves generally results from the combination of feedbacks 1 and 2.

The EGE and NGE curves in Figure 2 indicate that whether negative differential resistance due to feedback 1 is present or not depends on phenocryst content ($\beta_{ph}$) and/or the growth rate of crystal ($\Gamma$). Feedback 1 plays a role only when the crystal content exceeds a critical value for a drastic increase of effective viscosity ($\beta_{cr} \sim 0.6$) during magma ascent. Whether or not this condition is satisfied is inferred by comparing the timescale for the crystal content to reach the critical value ($\tau_{cr} \equiv (\beta_{cr} - \beta_{ph})/\Gamma$) with the timescale for magma ascent ($\tau_{ma} \equiv \rho L/q$). When $\tau_{cr} \gg \tau_{ma}$, viscosity does not drastically increase in the conduit. When $\tau_{cr} \ll \tau_{ma}$, however, crystal content reaches the equilibrium crystal content ($\beta_{eq}$), so that the effect of feedback 1 becomes insignificant. Consequently, feedback 1 can play a role only when $\tau_{cr} \sim \tau_{ma}$ (i.e., $q/\Gamma \sim \rho L/(\beta_{cr} - \beta_{ph}) \sim 10^6$ kg m$^{-2}$).

Figure 8. Two positive feedback mechanisms causing negative differential resistance.
The ranges of $q$ with $d\rho_{ch}/dq < 0$ in the EGE and NGE curves (Figure 2) are explained by this condition for feedback 1 to occur. Figure 9 indicates the value of $q$ at which $d\rho_{ch}/dq$ changes from positive to negative (i.e., the local maximum) in the NGE curve as a function of $\Gamma$: we call this value $q_{\text{max}}$. For $\beta_{ph} = 0.4$, the value of $q_{\text{max}}/\Gamma$ ranges from $10^3$ to $10^6$ kg m$^{-2}$ s$^{-1}$. For $\beta_{ph} = 0.6$, $q_{\text{max}}/\Gamma$ has a slightly greater value ($\sim 10^6$ kg m$^{-2}$ s$^{-1}$), because $(\Delta \gamma - \beta_{ch})$ has lower values for higher $\beta_{ph}$. Generally, the EGE and NGE curves indicate a drastic increase in $\rho_{ch}$ with $q$ in the range of $q > 10^2$ kg m$^{-2}$ s$^{-1}$, due to high wall friction (equations (3) and (7)). Under such conditions, the effect of a delay of crystallization is not sufficient for $d\rho_{ch}/dq$ to become negative; for this reason, the region of $d\rho_{ch}/dq < 0$ disappears as $\Gamma$ exceeds a certain value ($\Gamma_{cr}$ in Figure 9). This explains the absence of the region of $d\rho_{ch}/dq < 0$ for $\Gamma = 10^{-4}$ and $\beta_{ph} = 0.6$ in Figure 2a.

Figure 2c illustrates that the range of $q$ for the transition from EGE curve to NGE curve to occur, and hence that for feedback 2 to cause negative differential resistance, depends on the permeability for lateral gas escape ($k_{w0}$) and phenocryst content ($\beta_{ph}$). For low $\beta_{ph}$, the transition from EGE curve to NGE curve always leads to negative differential resistance in the VLGE curve (Figures 2c and 2d). For high $\beta_{ph}$, because $p_{ch}$ increases with $q$, due to high wall friction, the transition from the EGE curve to the NGE curve does not necessarily result in negative differential resistance in the VLGE curve; $d\rho_{ch}/dq$ can be negative only when the transition from the EGE curve to the NGE curve (i.e., feedback 2) is coupled with feedback 1 (Figure 2b). It is also notable that the NLGE curve has negative $d\rho_{ch}/dq$ in the low-$q$ part of the steady $p_{ch}$-$q$ relationship for low $\beta_{ph}$, whereas it has positive $d\rho_{ch}/dq$ there for high $\beta_{ph}$.

The above suggests that the physics behind the complex features in the steady $p_{ch}$-$q$ relationship differs between high $\beta_{ph}$ and low $\beta_{ph}$. We discuss the geological implications of this difference below.

### 4.2. Effect of Phenocryst Content on Transition From Lava Dome to Explosive Eruptions

The results in section 3 indicate that transitions N1N3, EE′N3, and EE′N4 are characterized by a drastic increase in porosity inside the conduit during the transition (Figures 7b, 7d, and 7e). Considering that magma tends to fragment as porosity exceeds a critical value ($\sim 0.8$) [Sparks, 1978; Proussevitch et al., 1993], this porosity increase may lead to an explosive eruption. On the other hand, the discussion in section 4.1 suggests that these transitions are driven by different mechanisms between high $\beta_{ph}$ (i.e., transitions N1N3 or EE′N3) and low $\beta_{ph}$ (transition EE′N4). For high $\beta_{ph}$, the transition is driven by feedback 1 or the combination of feedbacks 1 and 2. For low $\beta_{ph}$, the transition can be driven by feedback 2 alone. Let us consider how these differences in driving mechanisms influence the critical conditions for transition from lava dome to explosive eruptions.

Figure 10 indicates the value of $q$ at which $d\rho_{ch}/dq$ changes from positive to negative (i.e., $q_{\text{max}}$) in the VLGE curve (Figure 2) as a function of permeability for lateral gas escape ($k_{w0}$). $q_{\text{max}}$ in the VLGE curve approximately represents the critical magma supply rate ($q_{in}$ in Figure 1b) above which a stable lava dome eruption becomes unstable and transition N1N3, EE′N3, or EE′N4 occurs (when there are two local maxima in the curve, the maximum which directly results in transition N1N3, EE′N3, or EE′N4 is chosen as $q_{\text{max}}$). The relationship between $q_{\text{max}}$ and $k_{w0}$ strongly depends on phenocryst content.

For high $\beta_{ph}$, $q_{\text{max}}$ for transition N1N3 or EE′N3 is insensitive to $k_{w0}$ and depends mainly on $\Gamma$ (Figure 10a), because feedback 1 (delay of crystallization) is a necessary condition for negative differential resistance to be present for high $\beta_{ph}$. Generally, transition N1N3 occurs when $k_{w0} > 10^{-16}$ m$^2$, and transition EE′N3 occurs when $k_{w0} > 10^{-16}$ m$^2$ in this diagram because the low-$q$ part of the VLGE curve is approximated by the NLGE curve for small $k_{w0}$ and by the EGE curve for large $k_{w0}$ (Figure 2). For low $\beta_{ph}$, $q_{\text{max}}$ for transition EE′N4 decreases remarkably with decreasing $k_{w0}$ (Figure 10b) because feedback 2 (porosity change) is the main mechanism for this transition. This result also implies that lava dome eruptions with low $\beta_{ph}$ tend to be unstable without lateral gas escape. This reflects the fact that the NLGE curve for low $\beta_{ph}$ has negative $d\rho_{ch}/dq$ in the low-$q$ part of the steady $p_{ch}$-$q$ relationship in Figure 2.

For both high $\beta_{ph}$ and low $\beta_{ph}$, the VLGE curve approaches the EGE curve in the limit of $k_{w0} \rightarrow \infty$. For this reason, when $k_{w0} > 10^{-14}$ m$^2$ for high $\beta_{ph}$ and $k_{w0} > 10^{-12}$ m$^2$ for low $\beta_{ph}$, transition N1N3, EE′N3, or EE′N4 no longer occurs; only transition EE is possible in this range of $k_{w0}$. Because negative differential resistance of the EGE curve can result from only feedback 1, $q_{\text{max}}$ in the limit of $k_{w0} \rightarrow \infty$ depends on only $\Gamma$.

The value of $q_{\text{max}}$ for $k_{w0} \rightarrow \infty$ roughly represents the maximum value of $q_{\text{max}}$ for given $\beta_{ph}$ and $\Gamma$. The results in
Figure 10 imply that lava dome eruptions become unstable for both high $\beta_{ph}$ and low $\beta_{ph}$, and the transition to explosive eruptions occurs as the magma supply rate ($q_m$) exceeds this maximum value. The maximum $q_{max}$ in Figure 10 is consistent with the range of mass flow rate for typical lava dome eruptions ($10^{-2} - 10^2$ kg m$^{-2}$ s$^{-1}$) [e.g., Newhall and Melson, 1983]. In addition, for low $\beta_{ph}$, the permeability of the surrounding wallrocks ($k_{w0}$) controls the transition from lava dome to explosive eruptions; the transition can occur for $q_m$ much smaller than $q_{max}$ for $k_{w0} \to \infty$. Furthermore, even for a constant $q_m$, a slight decrease in $k_{w0}$ may cause a transition from lava dome eruption to explosive eruption.

4.3. Detecting Transitions EE’N3 and EE’N4 From Field Observations

[50] We have demonstrated that the transition from lava dome eruptions to explosive eruptions (transitions N1N3, EE’N3 and EE’N4) is governed by viscosity change due to crystallization (feedback 1) or porosity change due to lateral gas escape (feedback 2). The transition driven by feedback 1 (transition N1N3) has been investigated in previous studies based on conduit models without the effect of lateral gas escape as well as field data [e.g., Melnik and Sparks, 1999, 2005]. Detailed studies of transitions driven by feedback 2 (transitions EE’N3 and EE’N4), however, are still limited.

[51] Field data indicate that lateral gas escape can play an essential role at least in some lava dome eruptions. Figure 11 indicates the variations in phenocryst content for representative lava dome eruptions. For high $\beta_{ph}$ like the 1995–99 eruption of Soufrière Hills Volcano ($\beta_{ph} > 0.5$), lava dome eruption can be stable under a low-$q$ condition even without lateral gas escape, and the effect of feedback 1 would be essential for the transition of eruption styles to occur [e.g., Melnik and Sparks, 1999, 2005]. However, our results suggest that the presence of relatively low-$\beta_{ph}$ lava domes (commonly $\beta_{ph} < 0.5$) in other volcanoes requires efficient lateral gas escape beneath these volcanoes. It is important to evaluate the efficiency of lateral gas escape, because the critical flow rate for the transition to explosive eruptions depends on $k_{w0}$ in this case (Figure 10b). We will suggest below some possible methods to evaluate the effect of lateral gas escape on the transition of eruption styles, based on field

![Figure 10](image-url) Critical mass flow rate for (a) transitions N1N3 and EE’N3 and (b) transition EE’N4 to occur as a function of permeability for lateral gas escape ($k_{w0}$) with varying crystal growth rate ($\Gamma$). This flow rate corresponds to the rate at the local maximum point of the sigmoidal steady $p_{ch} - q$ relationship in Figure 2. A stable lava dome eruption continues in the range below the curve, whereas transition to an explosive eruption can occur in the range above the curve.

![Figure 11](image-url) Phenocryst contents and total crystal contents of dome-forming lavas for the eruptions of Mount Pinatubo (1991) [Bernard et al., 1996; Hammer et al., 1999], Mt. St. Helens (1980–86) [Cashman, 1988, 1992], Soufrière Hills Volcano (1995–99) [Sparks et al., 2000], Shiveluch Volcano (2001–04) [Dirksen et al., 2006], Mount Unzen (1991–95) [Nakada and Motomura, 1999], and Merapi Volcano (1986–95) [Hammer et al., 2000].
observations such as petrological, geophysical, and geochemical data. We focus on how transitions EE’N3 and EE’N4 are distinguished from transition N1N3.

[52] One of the most distinguishing features between transition N1N3 and transition EE’N3 or EE’N4 is the behavior of localized overpressure at the shallower level; transitions EE’N3 and EE’N4 are characterized by a strong increase in overpressure, whereas such an increase in overpressure is absent (or at least less remarkable) during transition N1N3 (Figure 7). These variations of overpressure inside the conduit may be detected from geodetic observations such as tilt and GPS measurements. In particular, the measurements around the summit area would be useful for detecting change in the localized overpressure at the shallower level [e.g., Widiwijayanti et al., 2005; Albino et al., 2011].

[53] Another feature that separates transitions EE’N3 and EE’N4 from transition N1N3 is the porosity distribution inside the conduit; magma has relatively high porosity (φ > 0.2) during transition N1N3, whereas porosity is extremely low (φ ~ 0) in the initial stage of transition EE’N3 or EE’N4 (Figure 7). Change in porosity at the vent can be measured by petrological methods such as density measurements of lava [e.g., Kueppers et al., 2005]. Porosity at a shallower level may also be estimated from petrological data of Vulcanian eruptions associated with dome growth [e.g., Clarke et al., 2007; Burgisser et al., 2010]. In addition, recent progress in cosmic ray muon radiography of volcanoes may enable real-time monitoring of density (and hence, porosity) distributions inside the conduit from the shallower level to the vent [Tanaka et al., 2007, 2009]. Our calculations of transitions EE’N3 and EE’N4 indicate that localized overpressure reaches its maximum (point 2 in Figure 7d and point 3 in Figure 7e) just before porosity at the vent reaches its maximum (point 4 in Figures 7d and 7e). This implies that real-time monitoring of porosity distribution along with that of overpressure may be utilized for predicting the transition from a lava dome eruption to an explosive eruption.

[54] Finally, we suggest that geochemical features of volcanic gas differ between lava dome eruptions before transition EE’N3/EE’N4 and those before transition N1N3. Because of the difference in the efficiency of lateral gas escape, lava dome eruptions before transition N1N3 have small Ew (～0), whereas those before transition EE’N3/EE’N4 have large Ew (～1) (Figures 6 and 7). According to a geochemical model coupled with our model for steady conduit flow (Appendix B), the difference in Ew reflects geochemical features of volcanic gas: Ew controls the partitioning of volatile components between gas and melt during magma ascent. When Ew = 0, the compositional variation of volatile tiles is explained by batch fractionation. On the other hand, when Ew = 1, the compositional variation is explained by Rayleigh fractionation. It is suggested that, in principle, the efficiency of lateral gas escape can be evaluated from the field data of the chemistry of volcanic gas or melt inclusions.

5. Conclusion

[55] We have investigated the effects of gas escape and crystallization on the dynamics of lava dome eruptions through systematic analyses of the steady pch–q relationship using a 1-D conduit flow model. We have identified two positive-feedback mechanisms that result in the complex features of conduit flow: increased effective magma viscosity due to crystallization (feedback 1) and increased average magma porosity in the conduit (feedback 2). We found that feedback 1 is the main mechanism for high-phenoocryst-content magma (volume fraction >0.5), whereas feedback 2 plays a key role in the transition of eruption styles for low-phenoocryst-content magma (volume fraction <0.5). In the latter case, the permeability of the surrounding wall rocks as well as the magma supply rate control the transition from a lava dome eruption to an explosive eruption. The transition due to this mechanism is characterizedly associated with an increased overpressure at a shallower level and a change in geochemical features of volcanic gas, which can be detected by multiple field observations.

[56] In previous works [e.g., Melnik and Sparks, 1999, 2005], the complex features of conduit flow have been mainly accounted for by viscosity change due to crystallization (i.e., feedback 1). Our results suggest that the effects of lateral gas escape cause a new positive feedback mechanism due to density change (i.e., feedback 2), and that the combination of the two mechanisms explains the diverse features of lava dome eruptions including transition to explosive eruptions.

[57] In our conduit flow model, we used a simplified crystal-growth model and assumed a cylindrical conduit with a constant radius and no pressure difference between gas and liquid. Parametric studies for time-dependent processes of conduit flow are still limited. In order to fully understand the dynamics of lava dome eruptions and the transition of eruption styles, we are conducting further analyses to better assess these additional effects.

Appendix A: Effect of Equilibrium Crystal Content on the pch–q Relationship

[58] In Figure 6, solutions E’, N1, and N3 have pch much higher than the lithostatic pressure. We demonstrate here that the absolute values of pch for E’, N1, and N3 solutions can be adjusted to realistic values by appropriately changing assumed parameters related to viscosity.

[59] The high absolute values of pch in these results are attributed to the steep pressure gradients in the conduit, which, in turn, result from the fact that the viscosity of the crystal-liquid mixture becomes extremely high near the vent in our model. Such high viscosity is calculated because we tentatively adopt equation (9) for equilibrium crystal content (βeq) in section 2.1. According to this formula, βeq becomes close to 1 near the vent under atmospheric pressure, which, together with the formula for the viscosity of crystal-liquid mixture by Costa [2005], yields very high viscosity. The formula of equation (9) for βeq is based on experiment data; however, it is not necessarily supported by the field data, indicating that actual dome-forming lavas have a wide range of total crystal contents from 0.5 to 0.9 (Figure 11).

[60] Figure A1 depicts the steady pch–q relationship in Figure 2b for k00 = 10^-17 – 10^-12 m^2/s using a more realistic formula for βeq instead of equation (9), as

$$\beta_{eq} = \beta_{ph} + 0.6(1 - \beta_{ph}) \cdot [C_{c0} + C_{c1} \ln(10^{-6}p) + C_{c2} \ln(10^{-6}p)]^2. \tag{A1}$$
Steady Hydrogen isotope ratio in liquid (n/C0) [e.g., Dobson et al. 1980; B08204 + (10p17–d1015 = n/C0 in equation (B5) as E1 = 0.76 E1q/C0 is the initial D/H ratio in liquid, n/C0 is the D/H ratio in gas and þ relationship are unchanged by this revision C1 = [ (bfr/C0 C1K(1) C0)] Cfr(ð is constant. þðq¼pcr þðw¼pcrðis the D/H ratio in f¼q¼c/C0 (equation (12)) influences the chemical composition of þw¼pcrð E¼q/C0 ¼ 0, the compositional variation is explained by batch fractionation: C1/C0 = [(1 – Ew)Kd – 1] f + 1 | m=nq=C1, (B5) where C1q is the initial D/H ratio in liquid, f (≡n/n0) is the degree of gas exsolution, and Kd(≡Cg/C1) is the partition coefficient of the D/H ratio between gas and liquid. For simplicity, we assumed that Kd is constant.

Equation (B5) indicates that when Ew = 0, the compositional variation of volatiles is explained by batch fractionation: C1/C0 = [(Kd – 1) f + 1]–1. On the other hand, when Ew = 1, the compositional variation is explained by Rayleigh fractionation: C1/C0 = (1 – f)Kd–1. The wide variation of the D/H ratio observed from geochemical measurements is accounted for by the variation of Ew from 0 to 1 (Figure B1).

In previous geochemical studies on volcanic gas, batch fractionation has commonly been interpreted as

\[
\frac{d}{dz}(q_g + q_h + q_v) = -Q_v C_g = C_g \frac{d}{dz}(q_g + q_h),
\]

where \( q_g, q_h, \) and \( q_v \) are the mass flow rates of gas (H₂O), H₂O in liquid, and the other components in liquid + crystal, respectively (cf. equations (1) and (2)). In order to complete chemical modeling, we add the mass conservation equation of the hydrogen isotopes [Iwamori, 1994]:

\[
\frac{d}{dz} \left( \frac{q_g C_g + q_h C_1}{C_1} \right) = -Q_v C_g = C_g \frac{d}{dz} \left( \frac{q_g + q_h}{C_1} \right),
\]

where \( C_g \) is the D/H ratio in gas and \( C_1 \) is the D/H ratio in liquid. From equations (B1), (B2), and (B4), \( C_1 \) is expressed as

\[
C_1 = \left[ \frac{1}{(1 - E_w)K_d - 1} + 1 \right] \frac{q_g}{q_h}.
\]

This formula yields an equilibrium microlite content lower than that of equation (9) by a factor of 0.6, such that total crystal content near the vent becomes within the range of field observations (\( \beta = 0.76 - 0.84 \) for phenocryst content \( \beta_{ph} = 0.4 - 0.6 \)). Applying the new formula (equation (A1)) leads to a significant decrease in \( \rho_{ch} \); as a result, transitions N1N3 and EEN3 occur under geologically realistic conditions. It is also suggested that the qualitative features of the original \( \rho_{ch}-q \) relationship are unchanged by this revision from equation (9) to equation (A1).

**Appendix B: Geochemical Modeling Using Parameter \( E_w \)**

[61] In this appendix, we demonstrate how the parameter \( E_w \) (equation (12)) influences the chemical composition of volcanic gas during steady lava dome eruption. As an example, we focus on the relationship that describes the partitioning of the hydrogen isotope ratio (the ratio of deuterium D to hydrogen H) between gas and liquid, which has been widely used for analyzing the degassing process during magma ascent [e.g., Taylor et al., 1983; Dobson et al., 1989].

[62] For steady conduit flow, the equations for mass conservation are expressed using \( E_w \) as

\[
q_g = \phi \rho_3 w_g = (1 - E_w) n q_{out},
\]

\[
q_h = (1 - \phi)(1 - \beta) c \rho w_1 = (n_0 - n) q_{out},
\]

and

\[
q_o = (1 - \phi)(1 - c) (1 - n_0) q_{out}.
\]

Figure A1. Steady \( \rho_{ch}-q \) relationship when equilibrium crystal content is expressed by equation (A1) (solid curves) for \( k_w = 10^{-17} - 10^{-15} \) (m²). The other parameters are the same in Figure 2b. The results for equation (9) (dashed curves) are also presented for comparison.

Figure B1. Hydrogen isotope ratio in liquid (δD) as a function of dissolved water content (n0 – n) with varying \( E_w \). The value of δD is obtained from \( C_1/C_0^d \) in equation (B5) as δD = δD(10δD + δD0)(C1/C0 – 1) [e.g., Dobson et al., 1989]. Following the 1980–85 eruptions of Mount St. Helens [Anderson et al., 1995], the parameters are set as \( K_d = 1.04, \delta D^0 = -34 \) (‰), and \( n_0 = 0.03 \); observed data for this eruption are also presented.
evidence of closed system degassing or no gas escape, and Rayleigh fractionation as evidence of open system degassing or gas escape. In the framework of steady conduit flow, however, such interpretations are no longer valid: compositional variations of batch fractionation result even when vertical gas escape occurs efficiently without lateral gas escape (i.e., $E_g = 0$).

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References


Iglesias, (1996), Cyclic fluid flow and compositional variation of batch fractionation result even when vertical gas escape occurs efficiently without lateral gas escape (i.e., $E_g = 0$).


