Simulation Study on the Generation and Distortion Process of the Geomagnetic Field in Earth-like Conditions

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Numerical simulations on the generation process of the geomagnetic field in the core and the electromagnetic induction process through the laterally heterogeneous mantle were performed by using Earth Simulator (ES). For the geodynamo modeling, two types of simulation codes were installed to ES. One is constructed based on a Spectral Transform Method (STM) and the other code uses a Finite Element Method (FEM). The highest resolution model of STM (with spherical harmonics of degrees up to 256 and Chebyshev polynomials up to 128) runs on 64 nodes of ES with the speed of 0.16 sec/step (1.96 TFlops; 48% of peak performance). The resolution is sufficient for geodynamo modeling at the Ekman number $E = 10^{-5}$ and will be enough to decrease E by one order towards the Earth condition with slightly finer models. For the induction study, several codes were installed to ES. These models calculate the electromagnetic induction in a spherical earth with a 3-D structure in time and frequency domains. One of the time-domain codes (with spherical harmonics of degrees up to 128) runs on 160 nodes of ES with the speed of 0.008 sec/step (6.1 TFlops; 60% of peak performance), and simulate a high resolution pattern of the induced field corresponding to actual magnetic disturbances during a large magnetic storm.

Keywords: Geomagnetic field, Dynamo action, Electromagnetic induction, Electrical conductivity structure, Earth's mantle, Earth's core

1. Overview of the project

Purpose of this project is to investigate the dynamics of the core and the mantle by using the geomagnetic field and its variations. Numerical simulations on the generation processes of the geomagnetic field (MHD dynamo simulations) and the electromagnetic (EM) induction process in a 3-D heterogeneous earth (Electromagnetic induction modelings) are performed for understanding the dynamics of the mantle and the core of the Earth.

The geodynamo simulations are intended to improve our understanding of convection and a process of magnetic field generation in the Earth's core. It is known that the Ekman and Rayleigh numbers (E and Ra hereafter) are the most important dimensionless parameters that characterize geodynamo models. In the previous studies E and Ra are restricted, respectively, to larger than some factor of 10^{-5} and smaller than 10^{8}, while $E < 10^{-9}$ and $Ra > 10^{12}$ in the Earth's core. In particular, decreasing the Ekman number is quite important because a large E leads to a significant viscous effect which may destroy characteristic features in a rapidly rotating system. Our aim is to decrease E down to 10^{-6} and increase Ra up to 10^{9} by high resolution numerical simulations. Such expansion of parameter space is only possible by Earth Simulator (ES) at this moment.

Forward modeling of electromagnetic induction in a spherical earth with a 3-D electrical conductivity structure has also been an active field of investigation. Purpose of the modeling study is to estimate the 3-D electrical conductivity structure of the mantle and clarify the dynamical state of the mantle by jointly interpreting the seismic and electromagnetic structures. In order to obtain the deep structure of the mantle, effects of the strong electrical conductivity heterogeneity of the surface layer caused by the distribution of the ocean and land should be examined, for which high resolution computation is necessary. Moreover, for the inversion of the structure, repetitive computation of the forward modeling is required, and the computation time has to be reduced as short as possible. With the ES, we investigate the induction process of the 3-D heterogeneous earth and obtain the response functions of the Earth in frequency and time domains in order to detect the lateral heterogeneity of the deep mantle and infer the dynamics of the mantle convection.

2. Geodynamo simulations

Most of the previous geodynamo models are based on a spectral transform method that is also used in this study. However, the spectral method has a serious problem in its computational cost. In fact, total operation counts per one time step is proportional to N^4 provided the total grid points in the domain are N^3, while other local methods such as a Finite Element Method (FEM) require $O(N^3)$ operations. For this reason we adopt other two methods; a FEM and a hybrid method based on both Fourier expansions and finite differences, which are expected to show good performance in a very high resolution simulation.

2.1. Spectral method

The Earth's outer core is mimicked by a rotating fluid spherical shell. The equations to be solved are induction equation for a magnetic field, Navier-Stokes equation for fluid motion and equation of heat conduction. Toroidal/poloidal decompositions are applied to velocity and magnetic fields and all scalar unknowns are expanded in spherical harmonics and Chebyshev polynomials to resolve their horizontal and radial structures. The time integration by a predictor-corrector method is performed in the spectral space, while the nonlinear terms are evaluated in the physical space. The Fourier spectral space (azimuthal wavenumber) is decomposed into subspaces to be distributed among the nodes of ES available, while the latitude is decomposed in the physical space. Therefore we need an all-to-all memory transfer in the process of spectral transform.

Our finest model at present is S256C128 (with spherical harmonics of degrees up to 256 and Chebyshev polynomials up to 128), which runs on 64 nodes of ES with the speed of 0.16 sec/step (1.96 TFlops; 48% of peak performance). This model is enough to perform a numerical simulation of magnetoconvection at $E = 10^{-5}$ and $Ra = 2 \times 10^8$ (Fig. 1), in which we find a fine structure of turbulent magnetoconvection in a rapidly rotating system (Fig. 2). In order to decrease E down to 10^{-6} and increase Ra further, we need finer models such as S512C256. The execution time of the S256C128 model is 2 days for one million time steps which correspond to one diffusion time of a magnetic dipole and will be halved if we use 128 nodes. We expect that our development of the spectral code enables researches covering a new parameter space and will open a possibility to explore the dynamics in the deep interior of the Earth.

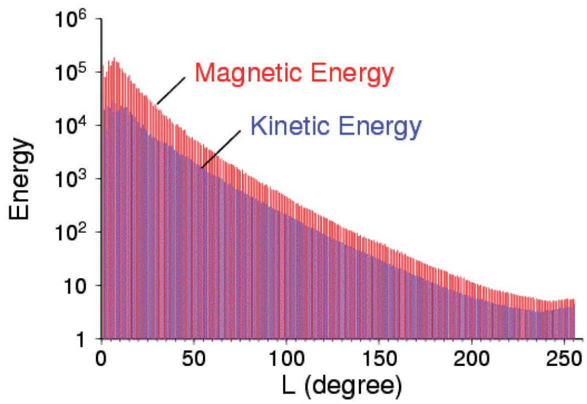

Fig. 1 Energy spectra obtained from a numerical simulation of magnetoconvection at $E = 10^{-5}$ and $Ra = 2 \times 10^8$. Red and blue bars show magnetic and kinetic energies in the fluid shell as functions of degrees of spherical harmonics.

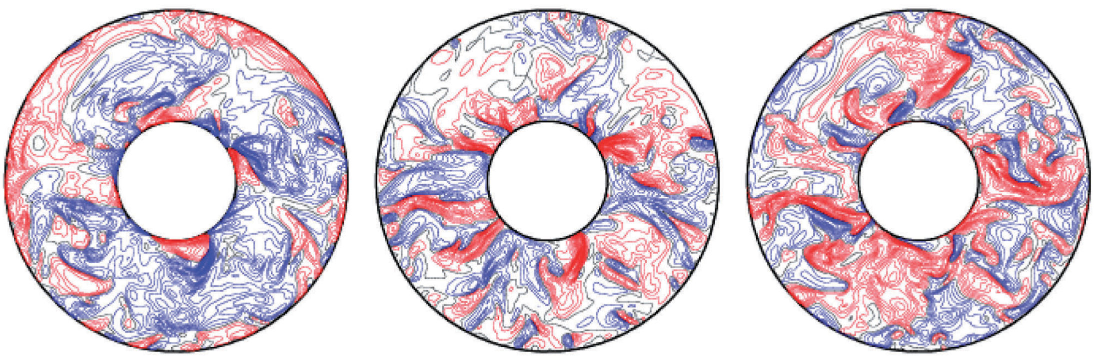

Fig. 2 A snapshot of magnetoconvection at $E = 10^{-5}$ and $Ra = 2 \times 10^8$. From left to right, zonal velocity, radial velocity and axial magnetic field in the equatorial plane are mapped as viewed from the north, a red (blue) contour line showing a positive (negative) value.

2.2. Hybrid method

The hybrid model is basically the same as the spectral model except that expansions by Legendre functions and Chebyshev polynomials are not performed but a Finite Difference Method is used in radial and latitudinal directions instead. The magnetic field in the insulating mantle does not need to be explicitly calculated owing to Fourier expansions. The numerical code is under construction but some important parts are completed and optimized on ES. Test calculations suggest that the hybrid model of F512L1024R512 (with Fourier modes up to degree 512 and a 1024×512 grid in a meridional plane) is significantly faster than S512C256, while F256L512R256 is comparable to S256C128. We aim to execute F1024L2048R1024 model using 512 nodes of ES in future.

2.3. Finite Element Method

The FEM model is based on the GeoFEM thermal-hydraulic subsystem, which is designed for simulating thermally driven convection by a parallel FEM. The velocity, the temperature and the magnetic vector potential are computed as functions of time and space. The simulation domain (the outer and inner cores and the mantle of finite conductivity) is divided into tri-linear hexahedral elements, and the physical parameters are defined at each node and interpolated by a tri-linear function within each hexahedron. We use a fractional step scheme for time integration in which the Crank-Nicolson scheme is used for solving the diffusion terms and the 2nd-order Adams-Bashforth scheme for the other terms.

We consider three levels of optimization on ES: (1) vectorization in each processor, (2) parallelization by OpenMP in each SMP node, and (3) domain decomposition and message passing among nodes. We use a Conjugate Gradient solver using PDJDS (Parallel Descending-order Jagged Diagonal Storage) and Multicolor ordering by Nakajima and Okuda (2002) for solving the Poisson equations and the Crank-Nicolson scheme. To obtain the right-hand vector, we use an ordering technique based on PDJDS ordering. Owing to these ordering techniques, the vector operation ratio becomes approximately 99%, and parallel efficiency reaches 99.8% in the case of a 5-million-element mesh (see Table 1).

Our simulation code is verified by comparing our results to the benchmark solution by Christensen et al. (2001). We have performed this test on SR2201 with 7.8×10^4 elements and on SR8000 with 1.4×10^5 elements. On ES, by using 8 nodes, we can perform the same test with the meshes of 3.2×10^5, 5.0×10^5, and 6.2×10^5 elements. The solutions approach to the one which has obtained by a high resolution spectral model. It is found that our finest model is in accordance with the spectral model within an error of 2%.

3. Electromagnetic induction models

For the EM induction modeling, several simulation codes were installed to the ES. These induction models calculate the electromagnetic induction in a spherical earth with a 3-D electrical conductivity structure in time and frequency domains. Frequency domain responses are commonly used to analyze a steady state part of the geomagnetic variations to detect the conductivity structure of deep interiors of the earth, whereas time-domain approaches are also useful for analyzing the transient responses of the induction due to the external magnetic disturbances observed during large magnetic storms, and the sudden change of the geomagnetic field of core origin such as jerk.

Among the installed codes, the time-domain approach based on the formulation given by Hamano (2002) achieved a high performance on the ES. This method utilizes the standard decomposition of the magnetic field into toroidal and poloidal parts, and spherical harmonic expansions of both the magnetic fields and the conductivity heterogeneity. A finite difference approximation was used to solve the set of diffusion equations for the spherical harmonics. This method can be efficiently used to analyze transient geomagnetic variations to estimate the 3-D conductivity structure of the earth. Since the method directly solves the temporal variation of the expansion coefficients corresponding to the source field, which is also expanded by the spherical harmonics, the method can work together with the results of the spherical harmonic analysis on the observed record of the geomagnetic field.

This code was installed to the ES, and optimized for the architecture of the ES. The optimization was made in three levels, i.e., vectorization in each CPU, shared-memory parallel processing within each node, and inter-node parallel processing for distributed memory architecture. The inter-node parallelization is MPI-based, where the number of MPI processes is equal to that of nodes. The parallelization is made in spherical harmonics spectral domain. After each time step of the calculation, MPI_Allgather is used for inter-node communication of the spherical harmonic coefficients. Starting from the spherical

Table 1 Performance of the FEM model.

	Number of elements	$\Delta\phi$ (degree)	Min. Δr (km)	Number of nodes	Parallel ratio (%)	Elapsed time (sec/step)
Mesh(1)	331,008	2.5	7.1km	4	98.20	3.06
Mesh(2)	995,328	2.5	3.1km	8	99.10	5.17
Mesh(3)	2,239,488	1.87	1.8km	20	99.50	7.22
Mesh(4)	5,163,264	1.25	1.8km	72	99.83	8.15

harmonics of degree 20, the truncation level was increased up to 128. With this increase of the truncation level, the horizontal spatial resolution decreases from about 10 degrees to 1.5 degrees. Although the amount of computation increases by about 2000 times due to this change, the increased computing efficiency keeps the computation time in a reasonable range. For the highest resolution model, the speed of the calculation is 0.008sec/step. Since the time step is as small as 0.1 sec for suppressing computational instability, it takes about 6000 seconds for calculating a 1-day length response. With the maximum degree model, the vector operation ratio achieves 99.7% and the resulting computing efficiency is about 60% of the peak performance on 160 nodes.

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実地球環境での地球磁場・変動シミュレーション

利用責任者

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本プロジェクトでは、コアでの磁場生成に寄与するダイナモ過程の高精度シミュレーション、及び3次元的な電気伝導度不均質構造をもつ地球の電磁誘導プロセスのシミュレーションを実施している。ダイナモシミュレーションに関しては、本年度はスペクトル変換法と有限要素法による2種類の計算コードを地球シミュレータに移植し最適化を行った。スペクトル変換法に基づく最も解像度の高いモデルでは、従来のダイナモ計算に比較して、エクマン数を1桁減少、レーリー数を1桁増加させ、より地球環境に近い計算が可能となる。電磁誘導プログラムに関してはいくつかのコードが地球シミュレータに移植され最適化がはかられた。これらは不均質構造をもつ地球の電磁応答を時間領域または周波数領域において計算するプログラムであり、マントルの電気伝導度構造探査に用いられる。今年度、時間領域で応答関数を計算するコードでは、最適化により高い実行性能を達成した。

キーワード：地球磁場、ダイナモ作用、電磁誘導、電気伝導度構造、マントル、コア