Numerical Simulations of Convective Dynamos and Electromagnetic Induction in the Three-Dimensional Heterogeneous Earth

Group Representative

Yozo Hamano   Institute for Frontier Research on Earth Evolution

Authors

Yozo Hamano *1, 2 ・ Ataru Sakuraba *2 ・ Masaki Matsushima *3 ・ Futoshi Takahashi *3
Hiroaki Matsui *4 ・ Makoto Uyeshima *5 ・ Masahiro Ichiki *1 ・ Takao Koyama *1
Kiyoshi Baba *1 ・ Fumiko Tajima *6 ・ Ryoji Matsumoto *7 ・ Hiroaki Toh *8
Yusuke Oishi *2

*1 Institute for Frontier Research on Earth Evolution

*2 Department of Earth and Planetary Science, University of Tokyo

*3 Department of Earth and Planetary Sciences, Tokyo Institute of Technology

*4 Department of Geophysical Sciences, The University of Chicago

*5 Earthquake Research Institute, University of Tokyo

*6 Department of Earth and Planetary System Sciences, Hiroshima University

*7 Faculty of Science, Chiba University

*8 Faculty of Science, Toyama University

Numerical simulations of convective dynamos and electromagnetic induction (EM) in a 3-D heterogeneous earth have been performed by using the Earth Simulator. In the geodynamo simulation, dynamo action was confirmed at the Ekman number, Ek = 10^{-5}, 10^{-6}, and the Rayleigh number (Ra) up to 10Rc, where Rc is the critical Rayleigh number for the onset of convection. Spherical Harmonic Spectral Transformation Method (STM) is used for this simulation, where the spherical harmonics up to degree 255 are used. Results of the simulation indicate the transition of the dynamo state from no magnetic field to unstable polarity reversing dynamos through stable dipole-dominant states as Ra increases. With the FEM (Finite Element Method) simulation code, dynamo simulations were performed with Ek as low as 5×10^{-5}. Besides these two approaches, a new method was developed in order to decrease the Ek less than the possible range of the previous methods. As for the electromagnetic induction study, forward modeling of electromagnetic induction in a 3-D heterogeneous earth have been conducted in time and frequency domains. In the time-domain method, the effects of the surface heterogeneity due to the distribution of the lands and oceans are well reproduced by using the spherical harmonic expansion up to degree 128. In the frequency domain analysis, a non-linear iterative inversion code on the ES was developed to estimate the 3-D electrical conductivity structure.

Keywords: Geodynamo, Geomagnetic field, Electromagnetic induction, Electrical conductivity structure, Earth's core

1. Introduction

Main aim of our project is to investigate the dynamics of the core and the mantle by using the geomagnetic field and its variations. For the purpose, numerical simulations of geodynamos and electromagnetic induction in the Earth have been performed by using the Earth Simulator (ES).

The geodynamo simulations are intended to improve our understanding of the fluid motion and the generating process of the geomagnetic field in the core. In most of the previous dynamo simulation studies, the low viscosity of the core fluid could not be well accounted because of the limitation of the computer capability. Among the dimensionless parameters prescribing the dynamical state of the core fluid, the Ekman (Ek) and Rayleigh (Ra) numbers are most important. Due to the extremely low viscosity of the core fluid, the Ek in the core is estimated as low as 10^{-9}, whereas Ek > 10^{-5} in most of the previous studies. With the ES, we intend to decrease Ek down to 10^{-6}, and Ra up to 10^{8} by using several simulation codes. In 2003, numerical simulations with the Spectral Transformation Method (STM) and the Finite Element Method (FEM) have been continued, and a new method was developed in addition to these methods. The newly developed FTM (Fourier Spectral Transformation Method) is a modification of the STM and is expected to be more efficient for high-resolution simulations (harmonic degrees higher than 512).

Purpose of the electromagnetic induction study is to investigate the electrical conductivity structure of the Earth from the electromagnetic field variations observed at the surface. In order to obtain the deep structure of the Earth's mantle, induction effects of the strong surface heterogeneity due to the distribution of the ocean and land should be evaluated. Because of the complexity of the surface heterogeneity, high-resolution models are required to simulate the EM induction in the actual Earth. In 2003, forward modeling studies of the EM induction in a 3-D heterogeneous earth by using several simulation codes in time and frequency domains were made. The frequency domain approaches have been commonly used in most of the previous induction studies, and are suitable for analyzing long-period signals to detect the deep structure of the Earth. On the other hand, the time domain approaches are suitable for analyzing the transient signals, and may provide new information on the electrical conductivity structure from the analysis of magnetic disturbances during large magnetic storms. Considering the complementary roles of the both approaches, we developed the simulation codes both in time and frequency domains on the ES. In the following, we report the progress of the various studies of our project during 2003.

2. Geodynamo simulation

2.1. Spherical Harmonic Spectral Transformation Method

In the STM dynamo simulation, we model the Earth's core by a rotating spherical shell filled with an electrically conducting Boussinesq fluid, and the induction equation, Navier-Stokes equation and heat transfer equation are solved simultaneously. The velocity and magnetic fields are decomposed into the toroidal and poloidal parts to automatically satisfy their solenoidal condition. All variables are expanded in terms of spherical harmonics in the angular direction, and a finite difference method is used in the radial direction. The spectral transform method is applied to evaluate non-linear terms in 3-D physical grid space. The Fourier spectral space is divided into subspace for inter-node parallelization, whereas the co-latitude is divided in physical space. In order to perform low Ekman number (Ek) geodynamo simulation, we ensure high spatial resolution in terms of spherical harmonics up to degree 255 and in the radial direction up to 256 grid points.

In 2003, we carried out many numerical simulations of the geodynamo models at Ek smaller than or equal to 10^{-5}, in some of which polarity reversal of the dipole component was found at high Ra models (Fig. 1). These results were obtained using 64 nodes, at maximum, of the ES at a performance of 1.18 TFLOPS, that is, 29% of the peak performance. Due to the ES, we got able to explore the parameter space at smaller Ek by one order of magnitude compared to the previous models. Furthermore, as shown in Fig. 2, we obtained an example of polarity reversal, which encourages further investigation in parameter space closer to the Earth's condition.

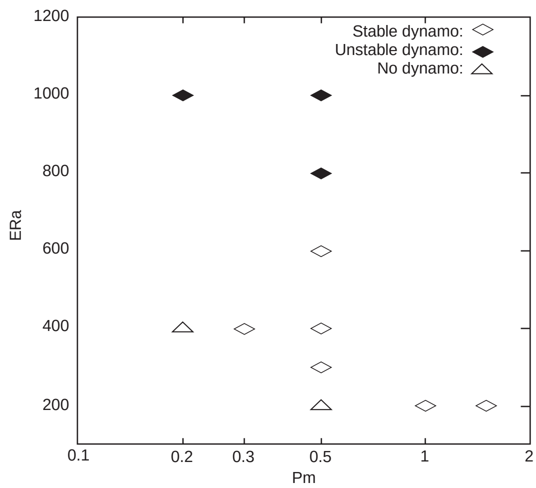

Fig. 1 Diagram of dynamo simulation results. Ordinate represents *ERa*, and abscissa *Pm*, where *E*, *Ra* and *Pm* are the Ekman number, the Rayleigh number and the magnetic Prandtl number, respectively. Open diamond represents stable dipolar dynamo, and filled diamond reversing dipolar dynamo, and triangle no dynamo.

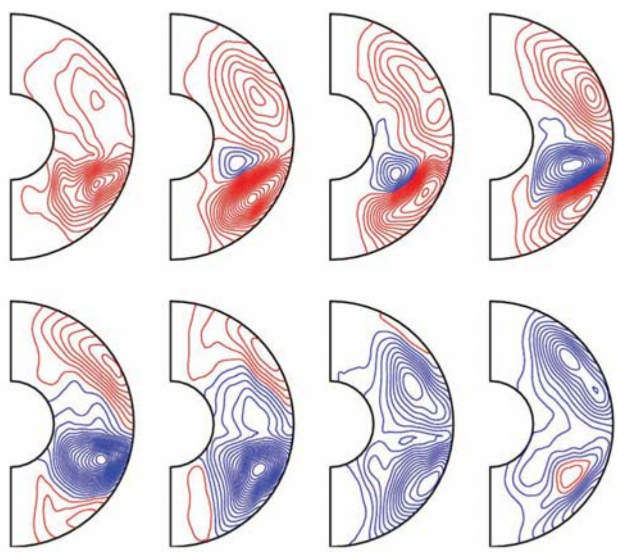

Fig. 2 Time sequence of the dipole field reversal from left to right in top low, then bottom left. Magnetic lines of force are drown. Parameter values are Ek = 10^{-5}, $Ra = 8 \times 10^7$ and $Pm = 0.5$. Red (blue) lines represent counter-clockwise (clockwise) fields.

2.2. Fourier Spectral Transformation Method (FTM)

In this method, velocities and magnetic fields are transformed to the Fourier spectral space in longitudinal direction and the variables in the spectral space are solved by the two dimensional finite difference method in the latitudinal and radial directions. Since the time-consuming Legendre transformation is not employed in this method, higher computation efficiency is expected than the standard STM simulation code. Preliminary estimate of the computation time for one time step (Fig. 3) indicates that the time with the FTM becomes shorter than the STM when the degree of harmonic expansion exceeds 512, while, at degree 256, the calculation times are comparable with each other. Numerical simulation code in this method has been developed and completed on a workstation, and ported to the ES. Preliminary simulations of the benchmark test models were performed by using 1 node of the ES. The results indicate that the pattern and the magnitude of the fluid motion, and other state variables approach to the previous results as the increase of resolution of the FTM simulation.

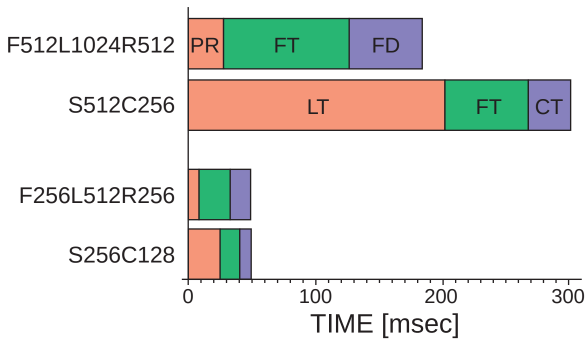

Fig. 3 Comparison of the one time-step computation time between the STM and FTM methods. S... denotes the degree of the spherical harmonic expansion, C... denotes the degree of the Chebyshev polynomial, F... denotes the order of the Fourier expansion, and L... and R... indicates the numbers of grid in latitudinal and radial directions, respectively. Estimated computation times for degrees 256 and 512 are shown.

2.3. Finite Element Method

In the FEM dynamo simulation, we performed the geodynamo simulations in a rotating spherical shell under the conditions of lower Ekman number and higher Rayleigh number than the dynamo benchmark test. The used simulation code is based on the GeoFEM thermal-hydraulic subsystem, which is designed for numerical simulation of thermally driven convection by a parallel FEM. We used FEM meshes as given in Table 1. The simulation domain is considered from the center to 7.9 Re, where Re is the Earth's radius in the present simulation because we can only treat finite size of simulation domain. To reduce data size, results in the equatorial plane and on the Core-Mantle boundary are obtained through the simulations by connecting GeoFEM visualization subsystem.

We performed three simulations as given in Table 2, and the magnetic energy grows to approximately three times of the kinetic energy in these cases. The convection pattern has columner structures in the all cases, but small-scale structure is generated in the lower Ekman number case.

To investigate generation processes of the magnetic field, we focus spatial distributions of intensity of the magnetic energy generation -u.(J × B). The results show that the magnetic energy is generated in convection columns that have negative z-component vorticity. Furthermore, negative magnetic energy generations are located in the boundary layer of the outer boundary. This result suggests that the magnetic field supports the Ekman pumping in the boundary layer.

Table 1 Spatial resolution of FEM meshes

	Number of Element	Nr	$\Delta\phi$	Min. Δr	Num of PE
Mesh(a)	2.2×10^6	96	1.87 Deg.	1.8 km	20×8
Mesh(b)	5.2×10^6	96	1.25 Deg.	1.8 km	72×8

Nr: Number of element in the outer core in the radial direction

Table 2 Dimensionless numbers for the simulations and obtained average kinetic and magnetic energies

	Mesh	Prandtl	Ekman	Rayleigh	Magnetic Prandtl		Kinetic Energy	Magnetic energy
Run(1)	(a)	1.0	1.0×10^{-4}	4.0×10^6	1.0		1.2×10^3	5.4×10^3
Run(2)	(a)	1.0	1.0×10^{-4}	6.0×10^6	1.0		2.8×10^3	1.1×10^4
Run(3)	(b)	1.0	5.0×10^{-5}	1.6×10^7	0.5		7.0×10^3	2.2×10^4

3. Electromagnetic induction

3.1. Forward modeling in time-domain

In this approach, the magnetic fields are decomposed into toroidal and poloidal parts, and the scalar variables and the electrical conductivity heterogeneity are expanded by the spherical harmonics. Time variations of the resulting variables in spherical harmonic spectral space are solved by the finite difference method in the radial direction. Simulation code based on this method was developed in the last year, and several simulations were performed. In the highest resolution model, spherical harmonics up to degree 128 are used and the computing performance is about 60% of the peak performance on 160 nodes of the ES.

In 2003, the actual time variation of the external source field was obtained by the spherical harmonic analysis. For the analysis, we selected the variations during a large magnetic storm commenced at around midnight on April 7,2000. Continuous records of the hourly mean values (April 5–15,2002) and minutes values (April 6–9, 2002) reported from 57 magnetic observatories were used, and the external and internal Gauss coefficients up to degree 6 were separated. Results of the forward modeling of the EM induction in a 3-layer earth model with the realistic surface heterogeneity by using the separated external field, indicate that most of the short wave-length fields variations (spherical harmonic degrees higher than 3) are caused by the interaction between the dominant degrees 1 and 2 source fields and the surface heterogeneity. This result of the simulation suggests a new inversion technique to obtain the heterogeneous structure of the deep mantle.

3.2. Inversion code in frequency domain

We developed a non-linear iterative inversion code on the Earth Simulator to estimate the three-dimensional (3-D) electrical conductivity structure by the EM induction methods. We parallelize this code by distributing each MPI rank to each frequency of the EM field variations, in which the EM induction equation is independent of each other. The vectorization ratio of this code is 98%. In addition to this code, we developed the code to calculate of the sensitivity of each model parameter to data parameters. This calculation on the Earth Simulator is the first trial to estimate the 3-D sensitivity of the electrical conductivity structure, because the calculation of the sensitivity requires a huge amount of the 3-D forward calculation of the EM induction equations. Parallelization of the code was made by distributing each MPI node to a perturbation of each model parameter, and the vectorizaion ratio is 98%.

Finally, we applied these codes into the experimental EM data measured in the north Pacific region and estimated the electrical conductivity structure in the mantle transition zone. Fig. 4(a) shows a profile of the electrical conductivity model beneath a line through Philippine, Guam and Hawaii, which are sensitive and well resolved as the results of the model sensitivity check and checkerboard resolution test. We found out that the several features of the large-scale anomalies exist such as a conductive region beneath Hawaii and a resistive region beneath Philippine, and some features are consistent of the seismic wave velocity structure (Fig. 4(b)). By using these two physical parameters, we can theoretically separate the anomalies into one of the thermal origin and one by difference of the chemical composition. Therefore we can elucidate the Earth's interior more precisely by using the Earth Simulator.

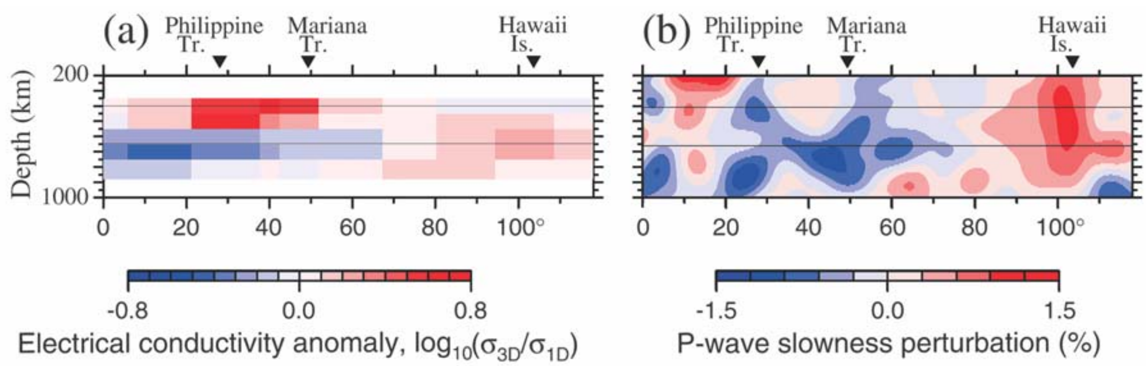

Fig. 4 Profiles of the mantle structure beneath the line through Philippine, Guam and Hawaii. (a) Electrical conductivity anomaly model obtained by using the Earth Simulator. (b) Seismic P-wave slowness perturbation model by Fukao et al. (2003).