

Energy Cascade Processes in Stratified Turbulence

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In order to investigate the formation process of the $k_H^{-5/3}$ energy spectrum observed in the atmospheric mesoscales, we perform numerical experiments on forced turbulence in a rotating stratified fluid and examine the energy cascade processes. When the energy injection by the dynamical forcing is concentrated in small scales, upscale energy cascade is expected to form a $k_H^{-5/3}$ spectrum. However, our result shows that the upscale cascade is not enough to form this spectrum for the terrestrial parameter range. On the other hand, the spectral slope generated by downscale energy cascade from energy injection in larger scales is close to -2 and is not sensitive to static stability when the Coriolis parameter is greater than the terrestrial angular velocity.

Keywords: Stratified turbulence, energy cascade

1. Introduction

The atmospheric energy spectrum as a function of horizontal wavenumber k_H over the range from a few kilometers to synoptic scales was obtained by Nastrom *et al.* [1984] and Nastrom and Gage [1985]. They found that the spectra follow the -3 power law in the range from 1000 to 3000 km and the $-5/3$ power law in the horizontal scales less than a few hundred km (about 400-500 km). More recent observations also support the $-5/3$ power law in the mesoscales [Cho *et al.* 1999]. Numerical simulations using the GFDL-SKYHI GCM reproduced these spectral slopes [Koshyk *et al.* 1999, Koshyk and Hamilton 2001]. While the formation mechanism of the k_H^{-3} spectrum can be interpreted as enstrophy downscale cascade in quasi-geostrophic turbulence [Charney 1971], an interpretation of the $k_H^{-5/3}$ spectrum in the mesoscales is more complicated. It can be speculated as it is created by energy cascades like 3D turbulence or inverse energy cascades like 2D turbulence. Lilly [1983] attempted to understand the energy spectrum in the mesoscales from inverse energy cascades in stratified turbulence. However, numerical simulations of stratified turbulence indicate that inverse energy cascades do not occur unless Rossby number is less than unity [Metais *et al.* 1994].

In this study, numerical experiments on stratified turbulence are conducted with a simple dynamical model. We investigate downscale energy cascades from energy injection at large scales as well as inverse energy cascades from small scales. We examine whether or not an inverse energy cascade process makes a $-5/3$ spectral slope in smaller wavenumber. Further, energy spectra formed by the energy cascade are also discussed.

2. Model description

We assume a nonhydrostatic, incompressible Boussinesq fluid on an f -plane. The domain is set to 400×400 km² in a horizontal plane and 10 km vertically. The number of the computational grids is $200 \times 200 \times 40$, which corresponds to 2 km resolution in the horizontal direction and 250 m in the vertical one. The boundary conditions are assumed to be cyclic in the horizontal direction and rigid at the top and bottom. Time-integration is carried out for 15 days with a time interval of 50 seconds. We analyze the results for the last five days where turbulence is speculated to be in quasi-equilibrium in our calculations.

The dependence of the forcing amplitude on the total horizontal wavenumber k_H is given by the following formulation:

$$|F(k_H)|^2 = F_0^2 \frac{k_H^{\gamma/2}}{(k_0 + k_H)^\gamma}.$$

Here, k_0 and γ characterize a spectral peak wavenumber and a spectral band width, respectively. F_0 determines a forcing amplitude. We examine two types of the forcing distribution: $(k_0, \gamma) = (20, 100)$ (referred as Type I hereafter) and $(k_0, \gamma) = (1, 20)$ (Type II). In experiments with the Type I forcing, we expect upscale energy cascades from the peak wavenumber of the forcing and intend to examine energy spectrum in the wavenumbers smaller than it. On the other hand, the Type II forcing distribution has a peak at the domain size in order to examine energy cascade processes to smaller scales. The vertical distribution of the forcing is assumed to have the first baroclinic structure. Following the traditional studies in forced turbulence [*e.g.* Lilly, 1969], a random Markovian formulation is used for time evolution of the forcing function.

The eddy viscosity and thermal diffusivity forms are assumed for the dissipation terms. In order to determine the eddy viscosity and thermal diffusion coefficients, we adopt the formulation based on the Smagorinsky-Lilly parameterization [Smagorinsky 1963, Lilly 1962], which is well known as a parameterization of LES. Namely,

$$\nu_{(h,v)} = (C_S \Delta_{(h,v)})^2 \sqrt{\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2} f_v(\text{Ri}), \quad \kappa_{(h,v)} = \nu_{(h,v)} \frac{f_\kappa(\text{Ri})}{f_v(\text{Ri})},$$

where $\Delta_{(h,v)}$ is the horizontal and vertical grid intervals and C_S is called the Smagorinsky constant, which is set to 0.21. In this formulation, effects of stratification are explicitly expressed as functions of Richardson number Ri. In our numerical experiments, the following empirical formulae based on laboratory experiments and the measurements in atmospheric boundary layer by Ueda *et al.* [1981] are adopted:

$$f_v(\text{Ri}) = \begin{cases} (1 + 2.5\text{Ri})^{-1} & (\text{Ri} > 0), \\ \min((1 - 25\text{Ri})^{1/3}, 6) & (\text{Ri} < 0), \end{cases}$$

$$f_\kappa(\text{Ri}) = \begin{cases} (1 + 5.625\text{Ri})^{-2} & (\text{Ri} > 0), \\ 1 + 5(1 - \exp(12\text{Ri}))^2 & (\text{Ri} < 0). \end{cases}$$

3. Results

Figure 1 indicates energy spectra obtained for Type I forcing and atmospheric stratification with 20 minutes of Brunt-Vaisala period. Amplitude of the vortical mode (solid line) is dominant especially in the lower wavenumbers. However, a $-5/3$ slope in the lower wavenumbers cannot be seen in the range $0 \leq f \leq 2\Omega$ (Ω is the terrestrial angular velocity) due to too small amplitude of the energy in the large scales. This energy distribution shows that upscale energy cascades are not sufficient to form the $-5/3$ slope. It is suggested that the inverse cascade process like 2D turbulence cannot be appropriate for the formation mechanism of the spectrum in the atmospheric mesoscales. On the other hand, the distribution close to $-5/3$ spectral slope is obtained in the lower wavenumbers than the forcing scale in the high rotation cases, $f = 20\Omega$. In this case, Rossby and Froude numbers averaged over the domain are about 0.3 and these values are consistent with the criterion for predominance of inverse

energy cascade shown by Metais *et al.* [1994].

In the experiments using Type II forcing, the spectral slope largely depends on rotation and stratification. In order to summarize this dependency, we estimate the slopes by the least-squares method in the range from 10 to 100 km and illustrate the results in Figure 2. In all the cases, error bars are small enough to discuss the dependency. In the cases without stratification, the slope is about -1 and the dependence on rotation is weak, while the classical theory of 3D isotropic turbulence predicts that the energy spectrum can be expressed by a $k_H^{-5/3}$ (k_H : horizontal wavenumber) [Lilly 1983]. This discrepancy would be attributed to the aspect ratio of the computational domain in our experiments. In the experiments including stratification, the slope becomes steeper as Brunt-Vaisala period is shorter, but its dependence is not so sensitive in the $f = 2\Omega$ cases. For the rotation rate more than Ω , the slope of the total energy is within the range from -1.9 to -2.1 .

We calculate total energy flux for Type II forcing cases (Figure 3). In neutral stratification cases, the total energy flux has a positive constant value in the horizontal scale of 20–100 km. This fact indicates that an inertial subrange appears in the range of this scale. The value of the flux is $7 \times$

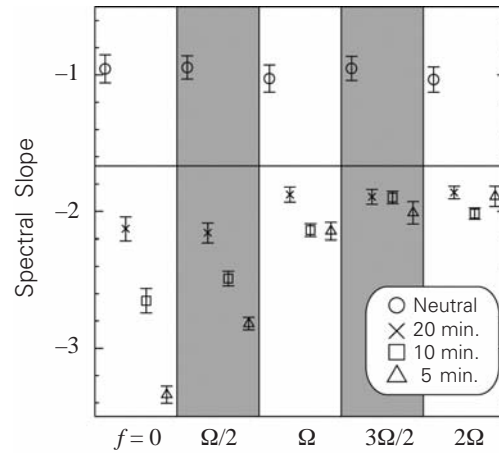


Fig. 2 The spectral slopes estimated by the least-squares method in the range from 10 to 100 km. The horizontal reference line denotes $-5/3$. Error bars are taken as two times a standard deviation.

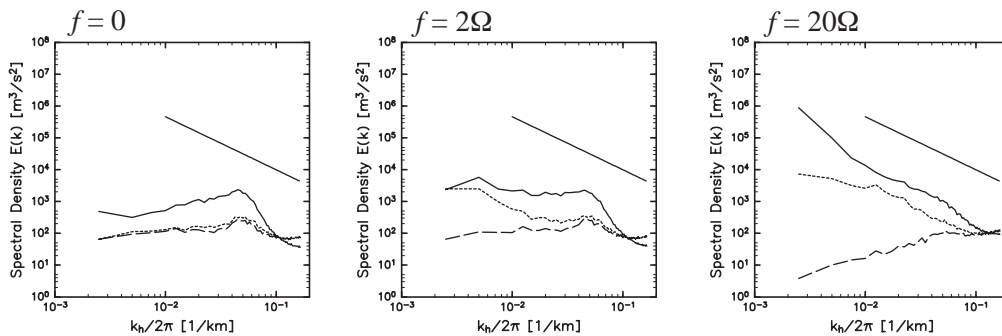


Fig. 1 Energy spectra as a function of horizontal wavenumber for the Type I forcing cases. Each panel represents $f = 0, 2\Omega$ and 20Ω from the left. Solid, dashed and dotted lines indicate rotational part and divergent part of kinetic energy and potential energy, respectively. The reference straight line represents a $-5/3$ power law.

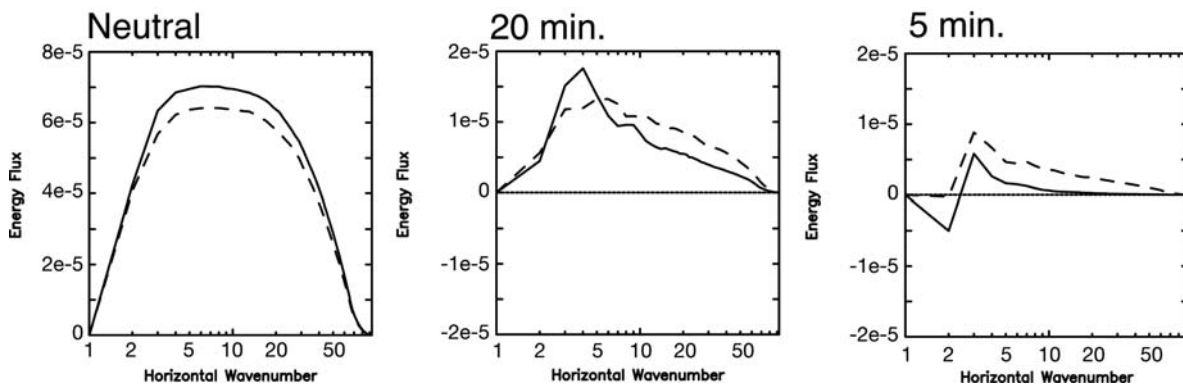


Fig. 3 Energy flux averaged over the last five days for the Type II forcing cases. Results for same stratification are drawn in a same panel. Solid and dashed lines indicate $f=0$ and 2Ω , respectively.

10^{-5} (m^2/s^3) for $f=0$ and 6×10^{-5} (m^2/s^3) for $f=2\Omega$. The flux in the case with stratification has a positive peak in the horizontal wavenumber 3–5 and decreases in the higher wavenumber. The flux is smaller for larger static stability and the smaller flux corresponds to the steeper slope of the energy spectrum (Figure 2). Effects of rotation appear in high wavenumbers; the increase of the rotation rate contributes to energy transfer to the high wavenumbers.

4. Summary

In the small-scale forcing cases, energy transfer to lower wavenumbers increases with static stability, but this upscale energy cascade is too small to form the $k_H^{-5/3}$ spectrum in the terrestrial parameter range of rotation and stratification. On the other hand, upscale energy cascade by the vortical mode is dominant in the case with an extremely rapid rotation rate. The spectral slope generated by downscale energy cascade is within the range from -1.9 to -2.1 for $f \geq \Omega$ and in the case close to the one observed in the mid- and high latitudes. However, it is sensitive to static stability without rotation, while the observed spectral slope is universal throughout all latitudes [Cho *et al.* 1999].

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成層乱流におけるエネルギーカスケード過程

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メソスケール大気にみられる $k_H^{-5/3}$ のエネルギースペクトルの形成過程を調べるため、回転成層流体における強制乱流の数値実験を行い、エネルギーカスケード過程について調べた。力学的強制によるエネルギー注入が小スケールに集中している場合、エネルギー逆カスケードによって $k_H^{-5/3}$ のスペクトルが形成されることが期待されるが、地球におけるパラメータレンジではこのエネルギースペクトルを得るのに十分な逆カスケードは起こらなかった。一方で、大規模なスケールからのエネルギーカスケードによって作られるエネルギースペクトルの傾きは-2程度となり、これはコリオリ因子が地球の回転角速度よりも大きな場合には大気安定度にほとんど依存しない。

キーワード：成層乱流, エネルギーカスケード