Geodynamo Simulations and Electromagnetic Induction Studies

Project Representative
Yozo Hamano Japan Agency for Marine-Earth Science and Technology

Authors
Yozo Hamano [*1, 2], Ataru Sakuraba [*2], Masaki Matsushima [*3], Futoshi Takahashi [*4], Hiroaki Matsui [*5], Makoto Uyeshima [*6], Masahiro Ichiki [*1], Takao Koyama [*1], Fumiko Tajima [*7], Yusuke Oishi [*2] and Hiroki Ichikawa [*6]

*1 Japan Agency for Marine-Earth Science and Technology
*2 Department of Earth and Planetary Science, University of Tokyo
*3 Department of Earth and Planetary Sciences, Tokyo Institute of Technology
*4 Japan Aerospace Exploration Agency
*5 Department of Geophysical Sciences, The University of Chicago
*6 Earthquake Research Institute, University of Tokyo
*7 Department of Earth and Planetary System Sciences, Hiroshima University

Numerical simulations of convective dynamos and electromagnetic induction in a 3-D heterogeneous earth have been performed by using the Earth Simulator (ES). For the geodynamo simulations, the ES is capable to calculate high resolution models with low viscosity. A number of dynamo simulations were performed this year based on a Spectral Transforma Method (STM), confirming that self-exciting dynamo action occurs at the Ekman number as low as 10^{-6} and the magnetic Prabdtl number less than unity. They also indicate the importance of MHD turbulence in dynamo action. Numerical simulations of nonlinear magnetoconvection were also performed for fundamental understanding of the dynamo process in the Earth's core. A new Fourier spectral Transform Method (FTM) was developed and tested with a standard benchmark model. As for the electromagnetic induction study, the electrical conductivity structure in the mid-mantle beneath Europe was estimated based on the geomagnetic depth sounding (GDS) method by using a non-linear iterative inversion code installed on the ES.

Keywords: Geodynamo, Geomagnetic field, Electromagnetic induction, Electrical conductivity structure, Earth's core

1. Introduction

The main aim of our project is to understand the dynamics of the core and the mantle through information on the geomagnetic field and its variations. In particular, numerical simulations of the dynamo process and the electromagnetic induction process in the Earth have been performed with the use of the Earth Simulator (ES). The geodynamo simulations are intended to improve our understanding of the fluid motion and the generating process of the geomagnetic field in the core. Among the dimensionless parameters prescribing the dynamical state of the core fluid, the Ekman (Ek) and Rayleigh (Ra) numbers are most important. Due to extremely low viscosity of the core fluid, the Ek in the core is estimated as low as 10^{-15}, whereas Ek > 10^{-5} in most of previous studies because of limitation of computer capability. With the use of ES, we intend to decrease Ek down to 10^{-6} and Ra up to 10^{8}. In 2004, our numerical geodynamo models, based on the Spectral Transformation Method (STM), confirm dynamo action at Ek as low as 10^{-6}. These high resolution simulations revealed the importance of MHD turbulence in generating magnetic field at low Ek dynamos. Comparison of the magnetic and kinetic energy spectra clearly shows the existence of inertial range of MHD turbulence and equipartition of energies at the higher wave numbers. Numerical simulations of nonlinear magnetoconvection were also performed for fundamental understanding of the dynamo process in the Earth's core. In addition to the STM models, a new simulation code based on FTM (Fourier Spectral Transformation Method) was developed and compared with the standard STM models.

Purpose of the electromagnetic induction study is to investigate the electrical conductivity structure of the Earth from the electromagnetic field variations observed at the surface. In order to obtain the deep structure of the Earth's mantle, induction effects of the strong surface heterogeneity due to the distribution of the ocean and land should be evaluated. Because of the complexity of the surface heterogeneity, high-resolution models are required to simulate the EM induction in the actual Earth. In 2004, we installed a non-linear iterative inversion code on the Earth Simulator to estimate the three-dimensional (3-D) electrical conductivity structure in a global scale by the electromagnetic (EM) sounding methods. By using this code, the electrical conductivity structure in the mid-mantle beneath Europe was estimated.

2. Geodynamo simulations

2.1. Geodynamo simulations at low Ekman number

We consider time-dependent, three-dimensional magnetic field generation by thermal convection in the Earth's outer core. Our numerical models are based on the STM with the spherical harmonic expansion in the angular direction. A finite difference method or Chebyshev spectral expansions are utilized to resolve radial structures. The numerical code, a part of which was originally developed by Takahashi et al. (2003), has been implemented and optimized to perform high resolution simulations on the ES. We carried out geodynamo simulations at the Ekman number, Ek, as low as 10^{-6} with up to 256 × 384 × 768 grid points in the spherical coordinate system.

Summary of the simulation results are given in Fig. 1, which shows that dynamo action at low *Ek* facilitates maintenance of the magnetic field even at low magnetic Prandtl numbers, *Pm*. Structures of the generated magnetic field and convective motions at $Ek = 10^{-6}$, $Ra = 5 \times 10^8$ and $Pm = 0.5$ are shown in Fig. 2. It is found that the length scale of convection is very small, whereas the counterpart of the magnetic field is much larger and the axial dipole component prevails. It is suggested that the small-scale convection vortices generate global scale dipole field through backward energy cascade. In fact, significant fraction of the magnetic energy is generated by the small-scale convective motion. This is the main difference from the high *Ek* dynamos, in which large-scale flows generate most part of the magnetic energy in the high *Ek* dynamos. Such a feature is first found in the low *Ek* dynamo models by the extensive use of the ES.

The kinetic and magnetic energy spectra as functions of the azimuthal wave number *m* show characteristics of turbulence. In the case of slower rotation at $Ek = 10^{-5}$ the spectral curve of the kinetic energy has a broad peak around $m = 6$ and decreases with increasing the wave number in proportion to $m^{-3/2}$, well representing the inertial range of MHD turbulence. At the higher wave numbers greater than $m = 40$ the kinetic energy steeply decreases together with the magnetic energy, which indicates the equipartition of the energies. Similar behavior is seen in the cases of faster rotation. The equipartition of kinetic and magnetic energies implies that the small-scale flows are suppressed by magnetic diffusion even if molecular viscosity is extremely small and numerical simulations with a magnetic Prandtl number of order unity is still applicable to the actual geodynamo.

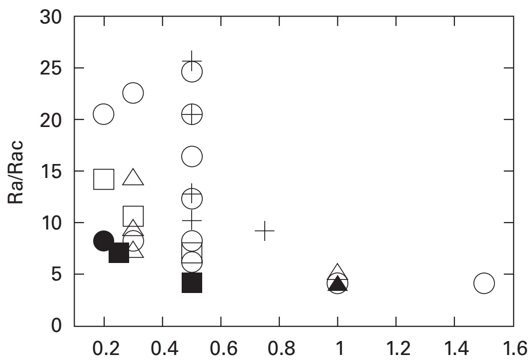

Fig. 1 Diagram of dynamo solutions as functions of *Pm* and *Ra* for each *Ek*, where *Pm*, *Ra*, and *Rac* denote the magnetic Prandtl number, the Rayleigh number, and the critical Rayleigh number for the onset of convection. Pluses are for $Ek = 4 \times 10^{-5}$, circles are for $Ek = 10^{-5}$, triangles are for $Ek = 4 \times 10^{-6}$, and boxes are for $Ek = 10^{-6}$. Filled symbols denote the failed dynamos.

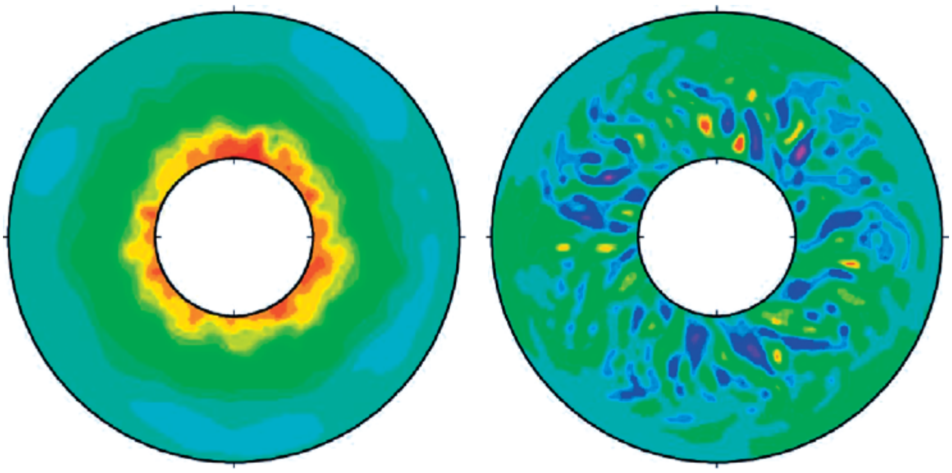

Fig. 2 Structures of the magnetic field and convective motion in the fluid outer core on the equatorial plane for $Ek = 10^{-6}$, $Ra = 5 \times 10^8$, $Pm = 0.5$, and $Pr = 1$. The axial component of the magnetic field (left) and the streamlines (right) are drawn. Red (blue) regions represent positive (negative) values of the magnetic field and clockwise (counter-clockwise) flows.

2.2. Magnetoconvection at low Ekman number

Numerical simulations of nonlinear magnetoconvection are performed for fundamental understanding of the dynamo process in the Earth's core. The numerical code is essentially the same as that of the geodynamo simulation but a uniform magnetic field parallel to the spin axis is externally applied. The Ekman number is lowered down to 2×10^{-6}, one order of magnitude smaller than previous studies (Sakuraba and Kono; 2000, 2002), so that we can investigate magneto-convective flows peculiar to a rapidly rotating system. At a moderate Rayleigh number the convection pattern is characterized by a few convection rolls superimposed on a strong westward zonal flow circulating around the inner core. Magnetic field lines near the equatorial plane are concentrated in anti-cyclonic vortices as is well known from previous studies. In this case, however, the induced magnetic field becomes so strong that the circulating flow around the anti-cyclone tends to be concentrated in a narrow band along its outer rim, essentially because the Ekman number is very low. Under the influence of the zonal flow, the circulating flow is modified and a thin jet ensues at the downstream side of the vortex. An electric current sheet also forms parallel or anti-parallel to the jet because the magnetic field induced inside the anti-cyclone sharply decreases across the jet region. We designate this jet-like structure as a magnetic front by analogy to fronts in mid-latitude cyclones. Substituting parameters relevant to the Earth's core, we can estimate a magnetic front of width 15 km and of flow speed 1 cm/s. In spite of such a thin structure, a viscous force is totally negligible and an inertial force is also of secondary importance in the equation of motion. Therefore, the magnetic pressure (Lorentz force) at the front is compensated not by the inertial force but by the Coriolis force which cannot be given until a thin jet forms parallel or anti-parallel to the electric current sheet. Since the ratio of the inertial to the Lorentz forces is exactly the same as the ratio of kinetic to magnetic energy densities which is proportional to the Ekman number, use of a low Ekman number is crucial for simulating a magnetic front and this may be the reason why such jet-like flows have never been found in previous studies.

2.3. Benchmark test of Fourier Spectral Transformation Method (FTM)

In this FTM method, velocity and magnetic fields are transformed to the Fourier spectral space in longitudinal direction and the variables in the spectral space are solved by the two-dimensional finite difference method in the latitudinal and radial directions. Since the time-consuming Legendre transformation is not employed in this method, higher computation efficiency is expected than the standard STM simulation code. To test this method, we solved the dynamo benchmark problem advocated by Christensen et al. (2001). The benchmark was based on a quasi-stationary solution that is stationary aside from azimuthal drift, and defined some values for quantitative comparison. We compared these values with the standard solution suggested in the benchmark. The error levels of the mean kinetic and magnetic energy densities and some values at a defined local point were within 3% and that of the drift frequency was about 9%. Fig. 4 compares the results of the FTM simulation with that of the standard STM method as functions of the average spatial resolutions per one-dimension. It is consequently found that the present FTM model with sufficient resolution can simulate a self-exciting dynamo with an acceptable accuracy. The code is being optimized for higher resolution calculations which are critical to simulate the Earth-type dynamos with low Ekman numbers.

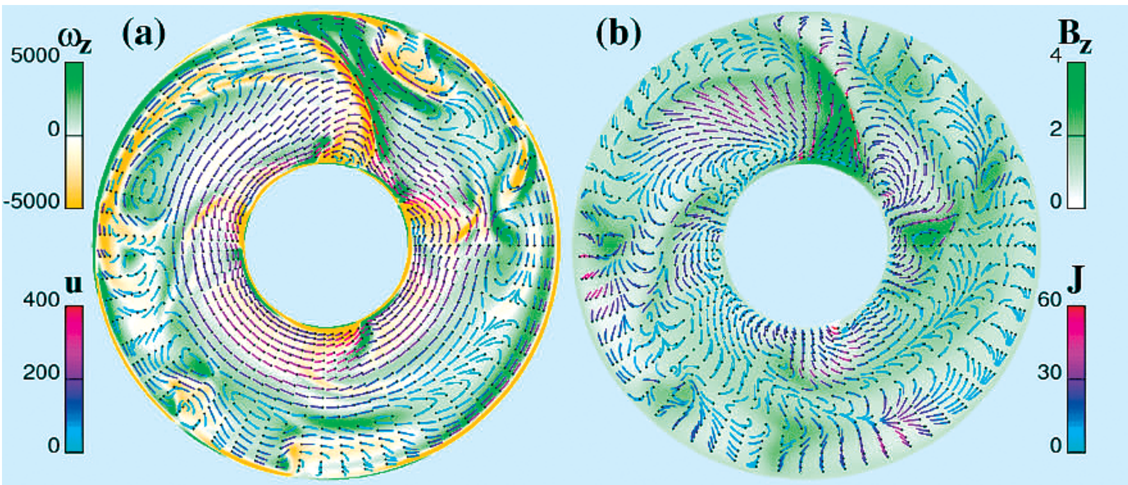

Fig. 3 Magneto-convective patterns on the equatorial plane viewed from the north. (a) The axial vorticity is represented by a color map and the velocity by instantaneous streamlines starting from dots. The color of the streamline indicates the flow speed, while its length is arbitrary. (b) The axial magnetic field is shown by a color map and the electric current by streamlines.

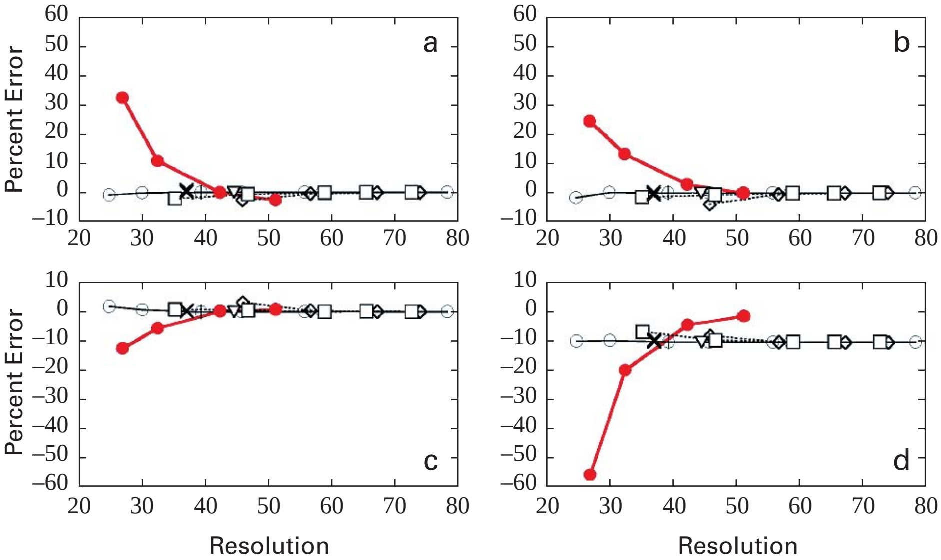

Fig. 4 Convergence of results for dynamo benchmark problem. Black = benchmark contributors, Red = this study. a) Mean kinetic energy density, b) mean magnetic energy density, c) local magnetic field, d) drift frequency.

3. Three-dimensional electrical conductivity structure in the mid-mantle beneath Europe estimated by geomagnetic depth sounding method

We installed a non-linear iterative inversion code on the ES to estimate the three-dimensional (3-D) electrical conductivity structure in a global scale by the electromagnetic (EM) sounding methods. By using this code, the electrical conductivity structure in the mid-mantle beneath Europe was estimated by using the geomagnetic depth sounding (GDS) method which is a kind of EM sounding methods. The geomagnetic field data were used for the data analysis from twelve permanent observatories in mid latitude of European regions. The GDS responses are estimated in a frequency domain between 0.018 to 0.19 cpd, which are ratios of vertical and horizontal components of the geomagnetic field and are data parameters of our analysis hereafter. A total number of data parameters are 264. The external EM source is ring currents in the magnetosphere, and then the source field is approximated to P10 distribution over a frequency range of our analysis.

At the beginning, a one-dimensional (1-D) structure of the electrical conductivity was estimated, which is radially symmetric and can fit all the data parameters averagely. The estimated 1-D structure in the mid-mantle beneath Europe is very similar to the 1-D reference model beneath the north Pacific by Utada et al. (2003). It might mean that the electrical conductivity is almost radially symmetric in a whole mid-mantle of the Earth.

Next we estimated a 3-D anomalous structure of the electrical conductivity from the 1-D reference model. The model parameters are $\log(\sigma_{3D}/\sigma_{1D})$ in each model grid, where σ_{3D} and σ_{1D} are values of 3-D and 1-D reference model of the electrical conductivity, respectively. The surveyed area is between 30N, 15W and 75N, 30E. The depth range is between 350 and 850 km. The grid size is 15 degrees × 15 degrees × 100 km, and then number of model parameters are 405. The inverse calculations were executed on the ES. The fixed ocean-land contrast is included at the surface layer. The other regions are fixed to the 1-D reference model. Each MPI rank is distributed to each frequency to parallely solve independent EM induction equations. The vector operation ratio is about 98%.

Fig. 5(a) shows the 3-D anomalous model in the mid-mantle of the electrical conductivity estimated by the inversion. This indicates that the northern part is more conductive and the southern part is more resistive than the 1-D reference structure. Focusing on the southern part, resistive anomaly is embedded beneath and northern part of the plate boundary. The African plate subducts northward from this boundary beneath Eurasian plate, and the resistive part might be a stagnant slab which has low temperature. There is the high velocity anomalies in both of bulk and shear modulus in Fig. 5(b) and (c) (Gorbatov, personal communication), which is consistent with anomaly by low temperature. Therefore the 3-D inversion solver of the electrical conductivity on the ES can reveal the anomalous structure such as a stagnant slab which is very important to elucidate the mantle dynamics.

References

Christensen U.R., Aubert J., Cardin P., Dormy E., Gibbons S., Glatzmaier G.A., Grote E., Honkura Y., Jones C., Kono M., Matsushima M., Sakuraba A., Takahashi F., Tilgner A., Wicht J., Zhang K., "A numerical dynamo benchmark". *Phys. Earth Planet. Inter*., 128, 25-34, 2001.

Sakuraba, A. and M. Kono, "Effect of a uniform magnetic field on nonlinear magnetoconvection in a rotating

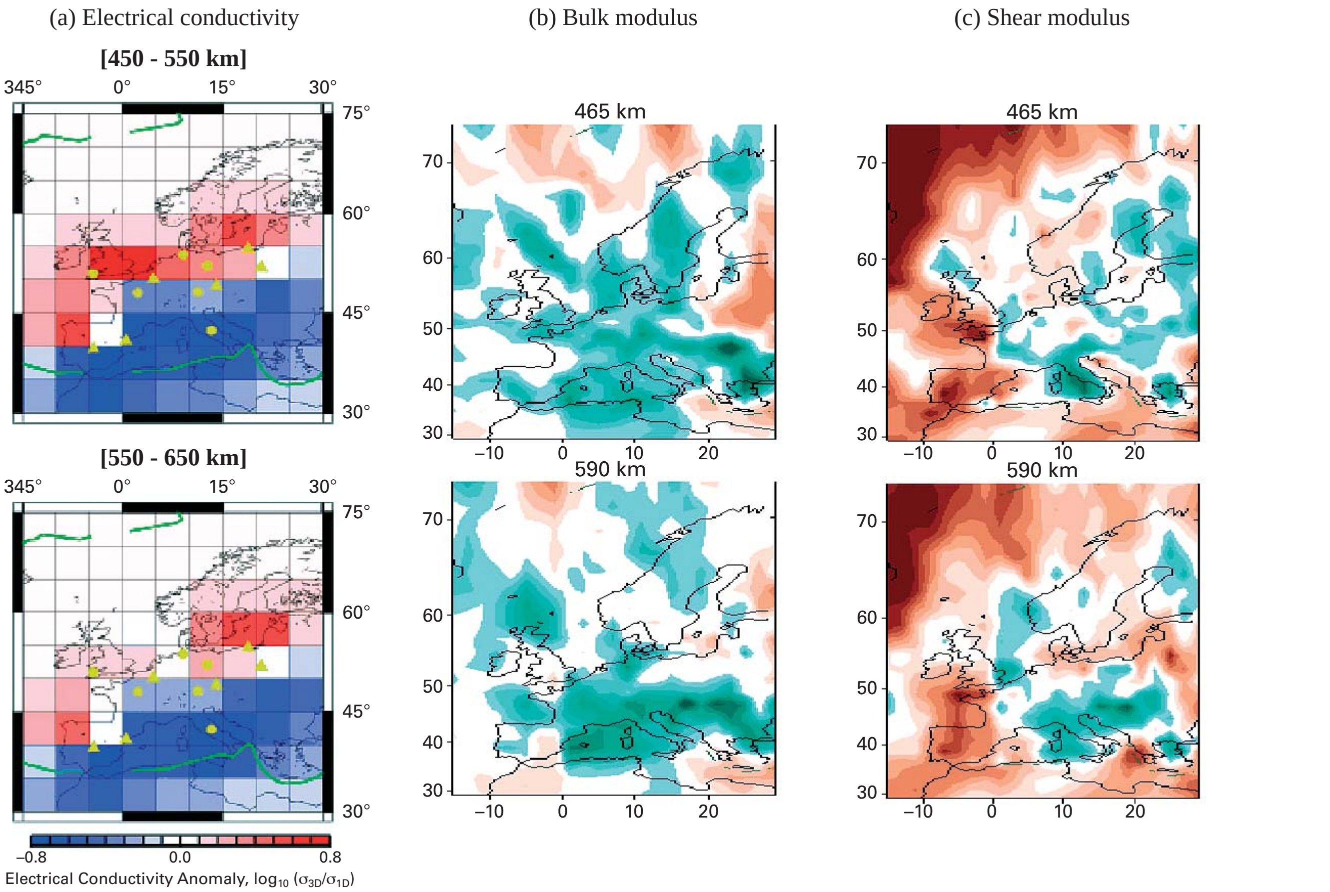

Fig. 5 The anomalous structure in the mid-mantle beneath the Europe. (a): electrical conductivity. Yellow marks and green curves are positions of geomagnetic observatories and plate boundaries, respectively. (b), (c): bulk and shear modulus estimated by seismic joint tomography both of P and S wave travel times by Alexei Gorbatov (personal communication).

fluid spherical shell, Geophysical and Astrophysical Fluid Dynamics, 92, 255-287, 2000.

Sakuraba, A., "Linear magnetoconvection in rotating fluid spheres permeated by a uniform axial magnetic field", Geophysical and Astrophysical Fluid Dynamics 96, 291-318, 2002.

Takahashi, F., M. Matsushima, and Y. Honkura, "Dynamo action and its temporal variation inside the tangent cylinder in MHD dynamo simulations", *Phys. Earth Planet. Inter*., 140, 53-71, 2003.

Utada, H., T. Koyama, H. Shimizu, and A. D. Chave, A semi-global reference model for electrical conductivity in the mid-mantle beneath the north Pacific region, *Geophysical Research Letters*, Vol.30, No.4, 1194, doi:10.1029/2002GL016092, 2003.

地球ダイナモと電磁誘導過程の数値シミュレーション

プロジェクト責任者
浜野　洋三　　海洋研究開発機構/東京大学

著者
浜野　洋三[*1,2]，桜庭　　中[*2]，松島　政貴[*3]，高橋　　太[*4]，松井　宏晃[*5]，上嶋　　誠[*6]，
市來　雅啓[*1]，小山　崇夫[*1]，田島　文子[*7]，大石　裕介[*2]，市川　浩樹[*6]
＊1　海洋研究開発機構
＊2　東京大学
＊3　東京工業大学
＊4　宇宙航空研究開発機構
＊5　シカゴ大学
＊6　東京大学 地震研究所
＊7　広島大学

本プロジェクトでは、地球磁場とその変動を手がかりとして地球内部のコアとマントルのダイナミクスを明らかにするため、コアでの磁場生成に寄与するダイナモ過程の高精度シミュレーション、及び3次元的な電気伝導度不均質構造を持つ地球での電磁誘導過程のシミュレーションを実施している。ダイナモシミュレーションに関しては、本年度はスペクトル変換法に基づく2種類のシミュレーションコードを用いて、地球ダイナモの数値シミュレーションを前年度に引き続いて行い、本プロジェクトの当初目標であったエクマン数 $Ek = 10^{-6}$ で、ダイナモ過程によって自発的に磁場が生成維持されることを確かめた。これらの高精度シミュレーションでは、小スケールでの流れ場の実態が捕らえられ、ダイナモ過程においてMHD乱流が重要な役割を果たしていることが示されている。運動エネルギー及び磁場エネルギーの波数スペクトルは、低波数領域での活発なダイナモ作用、中間波数領域でのMHD乱流の慣性領域の存在を示し、また高波数側では、運動エネルギーと磁場エネルギーのエネルギー等分配が成り立っていることを示す。この等分配領域の存在は、小スケールの流れが粘性拡散ではなく、磁気拡散によって抑制されていることを示し、ダイナモシミュレーションの結果から地球コアでのダイナモ作用を推定するために重要な示唆を与えるものである。また、同じシミュレーションコードを用いた低エクマン数($Ek = 2 \times 10^{-6}$)のマグネトコンベクションのシミュレーションでは、高エクマン数の計算では見られなかった、ジェット構造が観察された。このジェット構造は、地球磁場の短周期変動の起源として重要である。ダイナモシミュレーションでは、これまで行ってきたスペクトル変換法、有限要素法に加えて、あらたにフーリエスペクトル変換法に基づくシミュレーションコードを開発している。本年度はこの方法によるシミュレーションをベンチマークモデルについて実施し、スペクトル変換法と比較して計算が正しく行えていることを確かめた。電磁誘導シミュレーションでは、グローバルスケールで地球の3次元構造を探査するため、今年度は電磁誘導法に基づいて非線形逐次インバージョンを行う計算コードを地球シミュレータに移植し、ヨーロッパ地域の中深部マントルの3次元電気伝導度構造を推定した。

キーワード：地球ダイナモ，地球磁場，電磁誘導，地球流体核，電気伝導度構造