Geodynamo Simulations under Earth-like Conditions

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A number of dynamo simulations were performed during 2005 based on a Spectral Transform Method (STM). In one series of geodynamo simulations, the Ekman (Ek) and the Rayleigh (Ra) number dependences of the dynamo action were examined in detail. The change of the pattern of the fluid motion was observed as a function of Ra at Ek = 4×10^{-5}. The dynamo simulations at the lower value of Ek = 4×10^{-6} found the existence of dynamo action in a quasi-Taylor state, where the operating viscous force is negligibly small compared to the magnetic force. In the other series of geodynamo simulations, observation of the turbulent motion in the dynamo models was made based on the wave-number spectrum of the kinetic and magnetic energies and the time spectrum of the temporal variation of the external magnetic field. The observed linear relation between the frequency, f, and the wave number, k, may enable us to estimated the turbulent state in the earth's core from the observed time spectrum of the dipole field. In addition to these STM dynamo simulations, a new simulation code based on the Fourier spectral Transform Method (FTM) was developed, and the dynamo action at Ek =10^{-4} has been confirmed by the FTM.

Keywords: Geodynamo, Geomagnetic field, Earth's core

1. Introduction

The main aim of our project is to understand the dynamics of the core and the mantle through information on the geomagnetic field and its variations. For the purpose, numerical simulations of the dynamo process have been continued with the use of the Earth Simulator (ES) during the academic year of 2005. The geodynamo simulations are intended to improve our understanding of the fluid motion and the generating process of the geomagnetic field in the core. Among the dimensionless parameters prescribing the dynamical state of the core fluid, the Ekman (Ek) and the Rayleigh (Ra) numbers are most important. Due to extremely low viscosity of the core fluid, the Ek in the core is estimated as low as 10^{-15}, whereas Ek > 10^{-5} in most of previous simulation studies because of the limitation of computer capability. With the use of the ES, we intend to decrease Ek down to 10^{-6} and Ra up to more than 10^{8}.

In 2005, numerical simulations of geodynamo have been performed by using the two simulation codes based on the Spectral Transformation Method (STM). Both simulation results confirm dynamo action at Ek as low as 10^{-6}. The results of the geodynamo simulations are summarized in chapters 2 and 3. Development of the new approach by the Fourier Transform Method (FTM) is explained in chapter 4.

2. Geodynamo simulation varying the Rayleigh and Ekman numbers

We describe the results obtained from numerical simulation of the geodynamo in a rotating spherical shell at the Ekman number, Ek as low as 4×10^{-6} and the Rayleigh number, Ra up to 2.35×10^{8}. We have developed and used the code adopting pseudo-spectral method optimized for the Earth Simulator. Numerical simulations were carried out using 64 nodes of the Earth simulator.

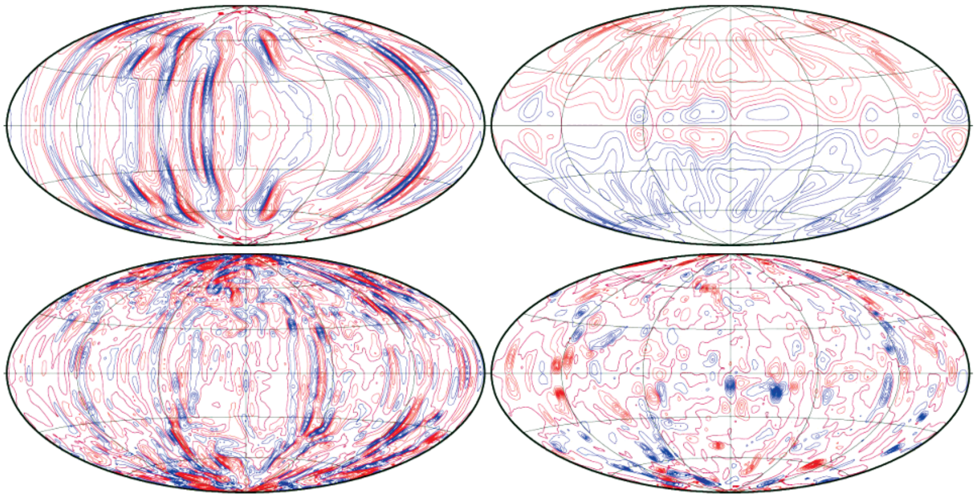

Fig. 1 The radial component of the velocity field at the mid-depth (left) and the magnetic field at the CMB (right). The Rayleigh number is 2.35×10^6 for the low-Ra-dynamo (top), and 3.52×10^7 for the high-Ra-dynamo (bottom). Red (blue) color represents the outward (inward) component.

First, we show the results at $Ek = 4 \times 10^{-5}$ in the definition of Kono and Roberts (2001). We examine the effects of varying the Rayleigh number on the flow and magnetic field structure and their dynamics. Fig. 1 shows the radial components of the velocity field at the mid-depth of the core and the magnetic field at the Core-Mantle Boundary (CMB) for low and high-Ra-dynamos (Takahashi and Matsushima, 2005). The low-Ra-dynamo is characterized by the two-dimensional columnar flow and the dipolar magnetic field with strong flux patches at high-latitudes. The high-Ra-dynamo shows notable polar convection in addition to less columnar flow. The magnetic field is less dipolar and many strong flux patches are irregularly distributed. The flow is strongly disturbed by the inertial term represented by the non-linear Reynolds stress due to high Ra. Then, two-dimensional columnar convection is broken as the inertial term strongly relaxes the Proudman-Taylor's constraint. As a result, the strong dipolar magnetic field collapses and is replaced by the weak non-dipolar field. This result indicates that different force balances in the core determined by Ra, which is directly related to the mantle dynamics, can give birth to diverse behavior of dynamo action.

Next, dynamo simulations at lower value of the Ekman number, $E = 4 \times 10^{-6}$, were carried out. This is an essential step to scrutinize the influence of the viscosity. The dynamo models we have obtained are found to be similar to the geodynamo in terms of not only spatial and temporal features but also the dynamic regime. We have found that the present dynamo models are in a quasi-Taylor state. In other words, they are asymptotically in the dynamic regime corresponding to that in the Earth's core. Axial torque balance clearly indicates that the viscous torque is negligible compared with the magnetic torque (Fig. 2).

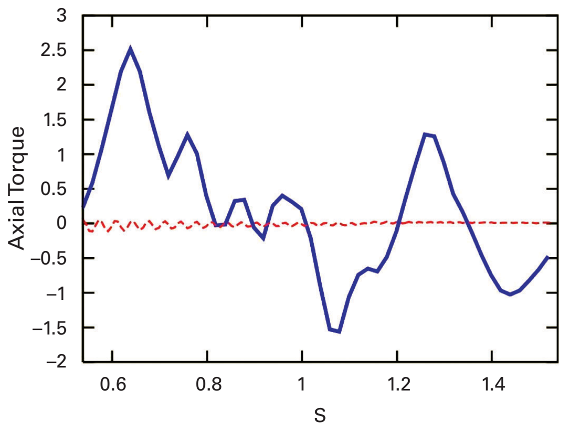

Fig. 2 The surface averaged axial magnetic (the blue line) and viscous (the red line) torques acting on cylindrical surface at a distance s ($r_i \le s \le r_o$) from the rotation axis (modified from Takahashi et al. 2005), where r_i and r_o represent the radii of the inner and outer cores, respectively.

In the quasi-Taylor state dynamo models, the magnetic field often undergoes polarity reversals (Takahashi et al., 2005). Time evolution of magnetic lines of force outside the core before, during and after a polarity reversal is shown in Fig. 3. Before and after the reversal, the magnetic field is strongly dominated by the dipolar component, whereas the quadrupolar component becomes significant due to the weakened dipolar component during the transition. This fact is consistent with the knowledge from the paleomagnetic study.

3. Turbulent spectra in high-resolution dynamo models.

The Earth's magnetic field is generated by fluid motion in the liquid outer core, where highly turbulent flow is expected because of its extremely low viscosity. However, direct observation of the small scale features of the fluid motions are inherently impossible. In order to infer the turbulent state in the fluid core, we examined the relation between the ener-

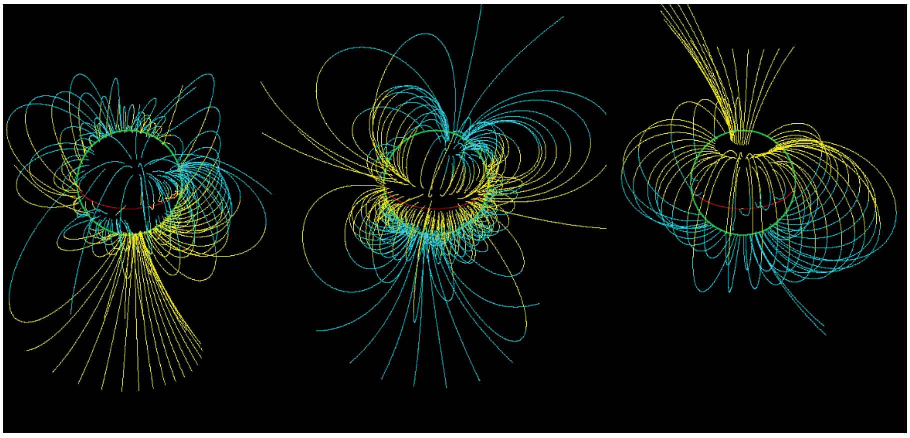

Fig. 3 Three bird's-eye snapshot views of the magnetic lines of force before (left), during (center) and after (right) the reversal. Yellow lines represent outward fields and blue lines inward fields.

gy spectra in the fluid core and the time spectrum of the dipole moment based on geodynamo simulation models.

We carried out numerical simulations of geodynamo employing a spectral transform method for solving the dynamo equations. The simulation code is described in Sakuraba and Kono (2000) and Sakuraba (2002), and was ported to the Earth Simulator for attaining the high resolution models. As for the non-dimensional parameters, the Ekman number is $Ek = 10^{-5}$, the Rayleigh number, $Ra = 4 \times 10^{7}$ for Model 1, and $Ek = 2 \times 10^{-6}$, $Ra = 4 \times 10^{8}$ for Model 2. The magnetic Prandtl number and the Prandtl number are both set to unity. The ratio of inner and outer shell radii is 0.35.

Both cases exhibit a self-exciting dynamo dominated by an axial dipole field. Fluid motion in the fluid outer shell is dominated by two-dimensional columnar flows in both cases, and the length scale of convection is much smaller in Model 2 than that in Model 1, whereas the scale of the magnetic field is much larger than the fluid flows in both models as shown in Fig. 4, suggesting that the small-scale convection vortices generate global scale magnetic dipole field through a dynamo action.

The time-averaged kinetic and magnetic energies contained in the fluid shell as functions of the angular order m were calculated for the two models and shown in Fig. 5. The kinetic energy spectrum shows the nature of magnetohydrodynamic (MHD) turbulence. In Model 1, the linear portion of the spectrum from m = 6 to 40 with a slope close to $m^{-3/2}$ suggests the inertial range, where the kinetic energy which is injected through a typical scale of around m = 6 is transferred to smaller scale flows without significant energy loss. At the higher wave numbers, higher than m~40, the kinetic energy rapidly decreases nearly coinciding with the magnetic one, showing equi-partition of the energies probably due to the Alfven effect in which a large-scale strong magnetic field causes equal excitation of small-scale kinetic and magnetic energies. At the lower wave numbers, the magnetic fields are effectively generated by dynamo action. In Model

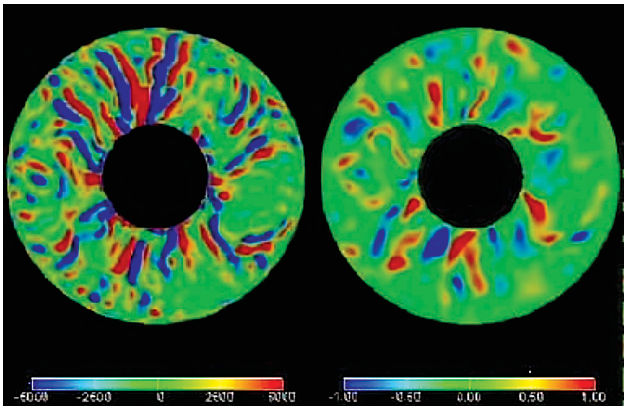

Fig. 4 Structures of the magnetic field and convective motion in the fluid outer core on the equatorial plane for *Model 2*. The vorticity (left) and the axial component of the magnetic field Bz (right) are drawn. In both figures, Red (blue) regions represent positive (negative) values of the magnetic field and clockwise (counter-clockwise) flows.

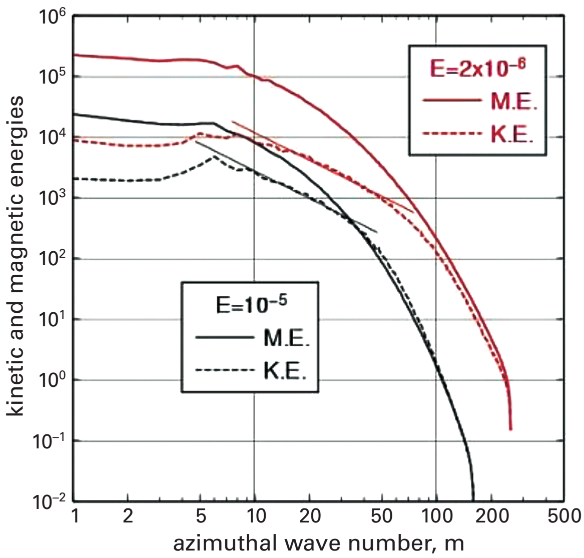

Fig. 5 Time averaged kinetic (dashed line) and magnetic (solid line) energies as functions of the azimuthal wave number ,m. The energy spectra for Model 1 (black) and Model 2 (red) are compared.

2, the magnetic and kinetic energies are significantly higher than Model 1 and the inertial range shifted to the higher wave number by about a factor of two due to the faster rotation and the higher Rayleigh number.

In Model 1, the numerical integration was continued to about 100 kyrs provided that the magnetic diffusivity of the fluid is 2 m^2s^{-1}, after a quasi-stable state is achieved. Time series of the Gauss coefficients at the core surface were calculated from the Model 1 results, and the power spectra of these sets of the time series were estimated by the periodogram method. Among the power spectra, the power spectral density of the time series of the time derivative of the axial dipole moment m[= g_1^0] is shown in Fig. 6. It is to be noted that the slope of the spectrum at the frequency ranging from ~0.00045 to ~0.003 is close to f^(–3/2). This frequency range covers the periods between about 2000 years and 300 years, and may reflect the inertial range of the turbulent motion in the core. The good correspondence between the time spectrum of the dipole moment and the kinetic energy spectrum in the fluid shell indicates a possibility that the kinetic energy spectrum in the core can be inferred from the time variation of the dipole moment.

4. Verifications of Fourier Spectral Transformation Method (FTM)

The Fourier Spectral Transformation Method (FTM) has been developed to aim much higher resolution geodynamo simulations than that based on the standard STM. In 2004, we performed relatively low resolution calculations of dynamo benchmark problem (Christensen et al., 2001) as the first step of verification of this method. In 2005, calculation code was optimized for the Earth simulator and (1) higher resolution benchmark calculations and (2) calculations at the higher Rayleigh and the lower Ekman numbers than the benchmark problems were implemented on the Earth simulator.

We solved Case 0 (rotating non-magnetic convection) and Case 1 (self-exciting dynamo with an insulating inner core) of the dynamo benchmark problems whose solutions are stationary aside from an azimuthal drift. The Ekman number is Ek = 2.1 × 10^{-4} and the Rayleigh number is Ra = 2.0 × 10^5 in the definition of Kono and Roberts (2001) in both cases. Some values which indicate the properties of the solutions were defined for the quantitative comparisons in the benchmark (see Fig. 7). All of these values of our results for Case 0 and Case 1 except for the angular frequency of the azimuthal drift for Case 0 were in agreement to the standard solutions of the benchmark within about 0.5%. The error

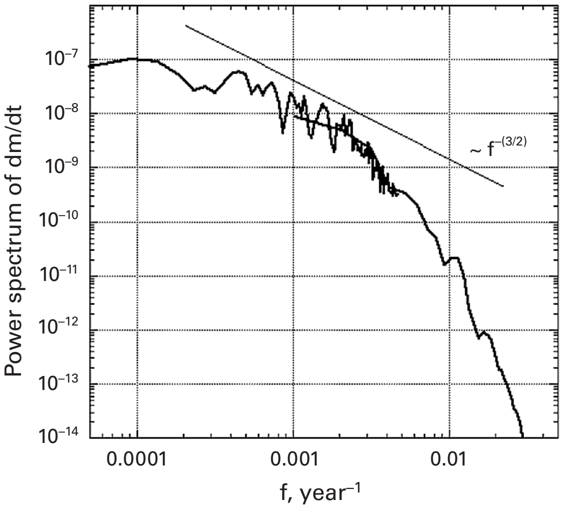

Fig. 6 Power spectrum of the temporal variation of the time derivative of the axial dipole moment from Model 1.

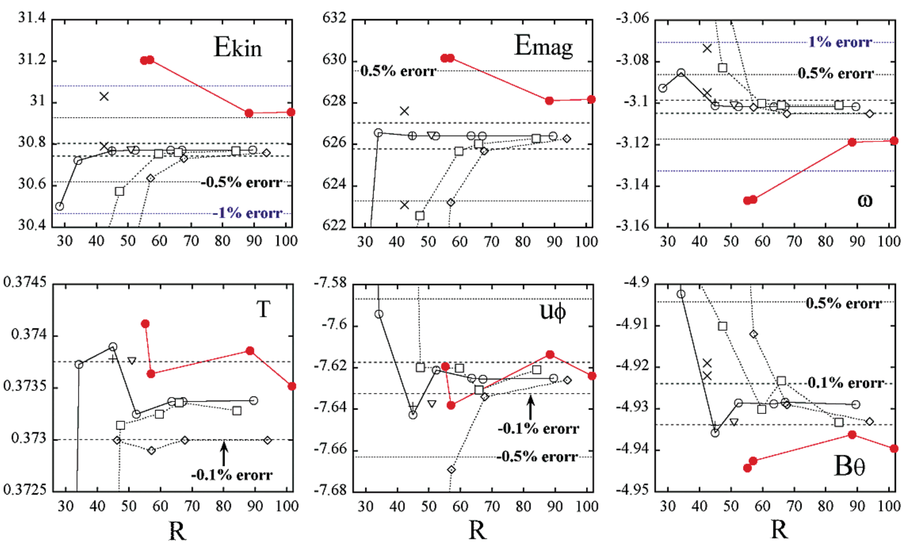

Fig. 7 Convergences of results for Case 1. Ekin and Emag: mean kinetic and magnetic energy density, respectively; ω: angular drift frequency; u_ϕ, B_θ and T: velocity and magnetic fields and temperature at a local point, respectively. Red symbols: results of the FTM; black symbols: results of the STM by the benchmark contributors (Christensen et al., 2001). R: mean one-dimensional resolution. +/–1, +/–0.5, and +/–0.1% error levels from the standard solution of benchmark are indicated by dotted lines.

level of the drift frequency for Case 0 was about 2%. Fig. 7 shows the convergences of the solutions of the FTM with those of the STM by the benchmark contributors as functions of the average spatial resolutions per one-dimension.

For further verification, we also performed dynamo simulations with higher Rayleigh (Ra up to 4 times critical Rac) and lower Ekman ($E = 6.3 \times 10^{-5}$ or 2.1×10^{-5}) numbers than those of the benchmark, and compared the solutions with that of our STM model. This comparison also showed an acceptable consistency. The representative examples of time evolution of magnetic energies by the FTM and the STM simulations are in Fig. 8. Through the above verification tests, it is consequently found that the FTM can simulate a self-exciting dynamo consistently with the STM.

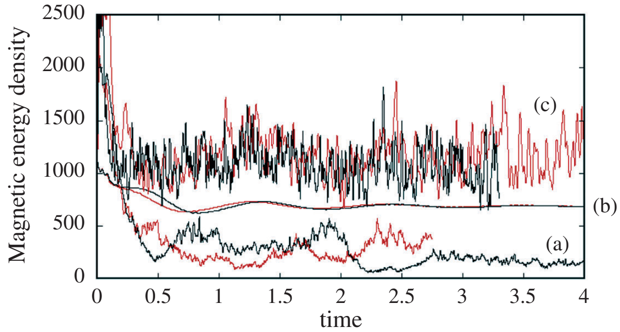

Fig. 8 Time development of the magnetic energy density for cases of (a) $E = 2.1 \times 10^{-4}$, $Ra = 2.5 \times 10^{5}$, (b) $E = 6.3 \times 10^{-5}$, $Ra = 1.3 \times 10^{6}$, and (c) $E = 2.1 \times 10^{-5}$, $Ra = 5.5 \times 10^{6}$. Time is scaled by magnetic diffusion time. Red lines: results of the FTM; black lines: results of the STM.

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実地球環境での地球ダイナモの数値シミュレーション

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本プロジェクトでは、地球磁場とその変動を手がかりとして地球内部のコアとマントルのダイナミクスを明らかにするため、コアでの磁場生成に寄与するダイナモ過程の高精度シミュレーションを実施している。本年度は、スペクトル変換法に基づく2種類のシミュレーションコードを用いて、地球ダイナモの数値シミュレーションを前年度に引き続いておこなった。レーリー数(Ra）とエクマン数(Ek）の広いパラメーター領域でシミュレーションを実施し、ダイナモ過程のレーリー数、エクマン数依存性を調べた．特に$Ek = 10^{-6}$, $Ra = 5 \times 10^{8}$のダイナモモデルにおいて粘性力が無視できる準テイラー状態が達成されたことは、低粘性の実地球のダイナモの状態を推定する上で重要な成果である。また、高精度ダイナモシミュレーション結果に基づいて、小スケールでの流れ場の実態が捕らえられ、ダイナモ過程においてMHD乱流が重要な役割を果たしていることが示された。ダイナモモデルで流体核中の運動エネルギー及び磁場エネルギーの波数スペクトルは、低波数領域での活発なダイナモ作用、中間波数領域でのMHD乱流の慣性領域の存在を示し、また高波数側では運動エネルギーと磁場エネルギーのエネルギー等分配が成り立っていることを示す。この等分配領域の存在は、小スケールの流れが粘性拡散ではなく、磁気拡散によって抑制されていることを示し、ダイナモシミュレーションの結果から、地球コアでのダイナモ作用を推定するために重要な示唆を与えるものである。
開発中のフーリエスペクトル変換法に基づくダイナモシミュレーションでは、本年度はシミュレーションコードの最適化を行い、$Ek = 10^{-4}$でのダイナモシミュレーションを行えるようになった。

キーワード：地球ダイナモ，地球磁場，地球流体核