Five-Dimensional Kinetic Simulation of Anomalous Transport in Fusion Plasma

Project Representative Takaya Hayashi

National Institute for Fusion Science, National Institutes of Natural Sciences

Author

Tomo-Hiko Watanabe National Institute for Fusion Science, National Institutes of Natural Sciences

With the aim of understanding the anomalous transport mechanism in the magnetic confinement fusion plasma and quantitatively predicting the transport level, gyrokinetic simulations of plasma turbulence have been carried out by means of the originally developed code (GKV code). The GKV simulations solving time-evolutions of particle distribution function in the five-dimensional phase-space have successfully reproduced the ion (or electron) temperature gradient turbulence in tokamak and helical devices, and have provided ones with valuable results for clarifying their saturation mechanisms.

Keywords: Nuclear Fusion, Plasma, Turbulence, Transport, Simulation

1. Introduction

Controlled nuclear fusion research has been extensively promoted in the world with expectation of an possible energy source in the future. In the long history of the magnetic confinement fusion study, there remain some important issues to be resolved. Among them, anomalous transport of particles and heat from the inside to the outside of fusion devices is one of the problems investigated long time.

In order to continuously make fusion reactions, it is necessary to confine gas of hydrogen isotope with high density and temperature in a limited volume for a certain time period. The ionized gas with high-temperature (exceeding 10 keV) is called "plasma" which consists of huge number of charged particles, ions and electrons. In the magnetic confinement fusion, many devices have a donut shape (torus), where the magnetic field is used to detach the hightemperature plasma from a wall. Because of the limited volume, fusion plasma intrinsically has inhomogeneity in profiles of the density and the temperature. The negative gradients of density and temperature profiles toward the outside of the device drive transport of particles and heat. The transport levels observed in experiments are higher in orders of magnitude than those expected from transport theories based on binary Coulomb collisions of particles (The theories are called "classical" or "neoclassical" ones in plasma physics). Then, the observed transport is called "anomalous". It is considered that the anomalous transport is mainly due to plasma turbulence driven by the density and temperature gradients.

Many efforts in the past devoted to investigation of the anomalous transport have deepened our understandings on the transport mechanism. However, quantitative predictions of the transport fluxes have not been fully confirmed yet by theoretical approaches. Main difficulties in theoretical analyses of the anomalous transport are summarized as follows: First, turbulence as origin of the anomalous transport intrinsically involves strong nonlinearity which prevents ones from approaching the problem analytically. Second, the anomalous transport phenomena are resulted from interactions of elementary processes in several spatial and time scales. For example, the turbulent transport may affect the density and temperature profiles with a typical device size (about 1 m), while the smallest scale of eddies is characterized by gyration radius of particles in the magnetic field (of the order of 1 mm). Third, a full kinetic approach, taking account of dynamic behaviors of the distribution function in the multi-dimensional phase space, is indispensable in the anomalous transport theory. This is because the fluid approximation can not be valid in the high-temperature fusion plasma with low collision frequencies where the mean-free-paths of ions are thousands times longer than the typical device size (radius of the poloidal cross-section of the torus). The last point given above demands a kinetic approach beyond the standard fluid dynamics. In order to reveal real pictures of the anomalous transport in the fusion plasma while resolving the above difficulties, numerical simulations based on the recent progress of the plasma theory should be pursued by means of a 10 Tflops-scale computer, such as the Earth Simulator, with the help of modern computer technologies.

In the last decade, the so-called gyrokinetic theories and simulations of magnetized plasmas have largely developed with the aim of studying the anomalous transport. In the gyrokinetics, the phase-space dimension is reduced to five (the three-dimensional real space and the two-dimensional velocity-space) by taking average of the gyration of particles. Although several numerical simulations of the turbulent transport have been performed, it is still a big challenge to directly and accurately simulate fluctuations of the distribution function in the five-dimensional phase-space. By using the Earth Simulator, therefore, we would reveal real pictures of kinetic turbulent transport in the magnetic fusion plasma.

2. Purpose and Plans in 2005–2006

2.1 Simulation Code

We have developed a gyrokinetic simulation code which can reproduce plasma turbulence from a microscopic level of fluctuating distribution functions f(x,v,t) in toroidal geometries. In our simulations, time-evolutions of f(x,v,t) is explicitly obtained by solving the gyrokinetic equations, such that

$$\begin{bmatrix} \frac{\partial}{\partial t} + v_{\parallel} \hat{\mathbf{b}} \cdot \nabla + \mathbf{v}_{d} \cdot \nabla - (\mu \ \hat{\mathbf{b}} \cdot \nabla \Omega) \frac{\partial}{\partial v_{\parallel}} \end{bmatrix} \delta f + \frac{c}{B_{0}} \{\psi, \delta f\}$$

= $(\mathbf{v}_{*} - \mathbf{v}_{d} - v_{\parallel} \hat{\mathbf{b}}) \cdot \frac{e \nabla \psi}{T} F_{M} - C(\delta f)$ (1)

where δf denotes a perturbed part of the total distribution function f. The Maxwellian distribution function for the equilibrium part of f is denoted by F_M . The advection term of the plasma flow caused by the gyro-averaged electrostatic potential fluctuations, ψ , is represented in the Poisson bracket form, $\{\psi, \delta f\}$, on the left-hand-side of Eq.(1). The parallel velocity, v_{\parallel} , and the magnetic moment, μ , of particles are chosen as the two-demensional velocity-space coordinates. Detailed definitions of other notations are found in Ref.[1]. The partial differential equation on δf in the five-dimensional phase space is directly dealt with in our gyrokinetic simulation code (it is called a "Vlasov" type code in the literature), which has recently been developed at Theory and Computer Simulation Center, National Institute for Fusion Science, and is named GKV code. The GKV code is highly optimized for vector and parallel operations with implementing automatic and MPI (Message Passing Interface) hybrid parallelization.

2.2 Objectives

In our project of utilizing the Earth Simulator in 2005-2006, it is aimed to clarify the kinetic picture of the anomalous transport in the magnetically-confined fusion plasma. In the kinetic plasma turbulence, the perturbed distribution function is far from the equilibrium given by F_M because of the weak collisionality. In conventional gyrokinetic simulations of plasma turbulent transport, however, kinetic aspects of turbulence characterized by velocity-space structures of δf have hardly been investigated so far except in our previous works [1–3]. Then, it is expected that the GKV code with high velocity-space resolution implemented on the Earth Simulator can be a quite powerful tool for analyzing the anomalous transport, since applicability of our previous simulations was restricted due to limited computer resources.

We have started our project in 2005–2006 with the objectives summarized as follows:

- Optimization of the five-dimensional gyrokinetic-Vlasov simulation code (GKV code) to the Earth Simulator (including modification of the code for vector and parallel operations)
- (2) Full toroidal angle simulation of the ion temperature gradient (ITG) turbulent transport in tokamak configurations
- (3) Simulation of the electron temperature gradient (ETG) turbulent transport in tokamak configurations (in comparison with the ITG case)
- (4) Extension of the GKV code for ITG turbulence simulation in helical systems
- (5) Simulation of the ITG turbulent transport in helical systems Here, the major concepts of the magnetic confinement fusion should be briefly explained, that is, tokamak and helical systems with different toroidal magnetic field configurations. The tokamak configuration has symmetry in the toroidal direction, while helical systems have three-dimensional variations in their equilibrium. Obtained results in this project are summarized in the next section.

3. Results

3.1 Code Optimization

After the code optimization, the five-dimensional gyrokinetic-Vlasov simulation code, GKV, can be successfully run on 192 computational nodes (1536 processor elements in total) of the Earth Simulator. The maximum computation speed of 4.8 TFlops is achieved by hybrid implementation of automatic and MPI parallelization. In the full toroidal angle simulation of the ITG turbulence explained below, we have used (85, 169, 128, 128, 48) grid points (or mode numbers) in the $(k_x, k_y, z, v_{\parallel}, \mu)$ -space, where k_x, k_y , and z are wave numbers perpendicular to the magnetic field and the parallel coordinate. It demands 2.6 TBytes memory even after the thorough optimization of the code. Automatic parallelization in a computational node is basically applied to the μ coordinate, while the three-dimensional domain decomposition for MPI parallelization is employed in the (z, v_{\parallel} , μ)-space. Then, the FFT operations for spectral calculations of the nonlinear advection term, $\{\psi, \delta f\}$, can be carried out in a single node with no inter-node communication, which enables us to develop the code in a short term.

3.2 Full Toroidal Angle Simulation of ITG Turbulence

The ion temperature gradient (ITG) turbulence is considered as a main cause of the anomalous ion heat transport in a core region of tokamak and helical fusion devices. The source of the turbulence drive is a gradient of the ion temperature in the equilibrium profile. In the simulation, we set constant gradients of equilibrium density and temperature, where the ITG instability is linearly unstable and begins exponential growth. Saturation of the instability growth is given by a nonlinearly generated mean flow with sheared profile in the direction of the minor radius of the torus. The sheared flow is called "zonal flow" in analogy to the sheared longitudinal jet flows found in the Jupiter's atmosphere [4], and plays the most important role in determining the transport level of the ITG turbulence. In Figure 1, one can see the elongated eddies found in the linear growth phase of the instability (left panel) are suddenly broken by the nonlinearly excited zonal flows (middle). Then, one finds the statistically steady state of the ITG turbulence (shown in the right panel of Fig. 1) with nearly constant transport flux (see also Fig. 2).

During the saturation of the ITG instability, we have found a new excitation process of the zonal flow with higher-order nonlinearity. Radial harmonics of the zonal flow components with shorter scale lengths are successively generated with higher growth rates, and effectively destroy the elongated eddy structures. In the statistically steady state after saturation, however, the zonal flow with longest radial wavelength dominates and co-exist with turbulent eddies (see right panel of Fig. 1). This is related to behaviors of the geodesic acoustic mode (GAM) oscillations. The GAM is an oscillatory component of the zonal flow, and has a larger collisionless damping rate for a shorter radial wavelength because of the finite-orbit-width effects [5]. It is also noteworthy that resolution in the wave number (k_{y}) space in the present full toroidal angle simulation is largely improved in comparison to the conventional GKV simulations [1]. Thus, a fine structure of turbulent eddies can be clearly reproduced



Fig. 2 Entropy balance in the full toroidal angle simulation of the ITG turbulence, where δS , W, Q_i , and D_i denote the entropy variable, the potential energy, the ion heat transport flux, and the collisional dissipation, respectively.

as seen in Fig. 1.

Entropy balance in the ITG turbulent transport is also accurately confirmed in the present simulation. Figure 2 shows time evolutions of the entropy variable, δS , the potential energy, W, the ion heat transport flux, Q_i , and the collisional dissipation D_i , where δS is defined by a square-integral of δf in the phase space, and reflects fine-scale structures of δf enhanced in the kinetic turbulence. Ratio of radial scale-lengths of the ion density and temperature gradients are represented by η_i . In the statistically steady state, mean values of $\eta_i Q_i$, and $-D_i$ should balance if the numerical error Δ is negligible. The obtained result confirms the accuracy of the present GKV simulation where the phase-space structures of δf including fine-scale fluctuations in the velocity space are successfully reproduced. It should also be mentioned that a small fluctuation level of the transport flux observed in the statistically steady state enables us to accurately evaluate the anomalous transport coefficient ($\chi = Q_i/\eta_i$) from the numerical simulation result.



Fig. 1 Results of gyrokinetic-Vlasov simulations of the ion temperature gradient turbulence in a tokamak configuration of the confinement magnetic field. Color contour maps represent electrostatic potential perturbations, ϕ , accompanying turbulence, where only a portion of torus in the three-dimensional real space is plotted for clarity. Snapshots of ϕ in the growth, saturation, and steady phases are shown in left, middle, and right panels, respectively. Red and blue colors in figures also mean directions of vorticity. It is clearly found that elongated eddies are destroyed by the zonal flow (a sheared poloidal mean flow), and that complicated structures of turbulent eddies co-exist with a large-scale zonal flow in the statistically steady state after saturation.



Fig. 3 Contour plot of real part of the perturbed distribution function, δf , with $k_x \rho_i = 0.0858$, $k_y \rho_i = 0.7$, and z = 0, where the positive and negative values are represented by red and green lines, respectively.

Fine structures of δf in the velocity space are clearly shown in Fig. 3 for the linearly stable mode ($k_y \rho_i = 0.7$). The elongated patterns of δf along the μ -axis is generated by the ballistic motion of passing particles, while the weak but finite collisionality acts to dissipate the small-scale structures. Then, the entropy balance is sustained as shown in Fig. 2. It should also be noted that the perturbed distribution function can not be approximated to F_M , and that the kinetic approach beyond the conventional fluid descriptions is indispensable for studying the anomalous turbulent transport in fusion plasma.

3.3 Simulation of ETG Turbulence

The electron temperature gradient (ETG) instability is described in an isomorphic form to the ITG mode. In experiments, however, the normalized electron heat transport shows stronger anomaly than ions. It is expected that the large electron heat transport is generated by the ETG turbulence with much higher saturation level than ITG. This is because the zonal flow in the ETG turbulence is hardly excited, while it strongly suppresses the ITG turbulent transport. The different zonal flow generation stems from the disparate adiabatic responses of electrons and ions, because ions can move to shield the zonal flow potential unlike electrons strongly magnetized.

Conventional gyrokinetic simulations of ETG turbulence by different codes have shown an unignorable difference in the transport coefficient. Furthermore, it is also reported that no saturation is observed in a gyrokinetic-Vlasov simulation of the toroidal ETG turbulence. In order to clarify a possible saturation mechanism, we have performed the GKV simulation of the toroidal ETG turbulence. Two-dimensional snapshots of the electrostatic potentials observed in our simulation are presented in Fig. 4. One can see that the longer wavelength modes in the y-direction dominate after the saturation of the linear ETG instability, and that the radially elongated eddies are strongly modulated in the late phase of the simulation where nearly constant transport flux is successfully observed. More detailed analysis of the saturation mechanism of the ETG turbulence is still in progress, and will be reported elsewhere.

3.4 GKV Simulation for Helical Systems

Our recent analyses [6,7] on the zonal flow in helical systems suggest that a high-level zonal-flow response can be maintained for a long time by reducing the radial drift velocity of the helical-ripple-trapped particles. It is, thus, expected that optimization of the three-dimensional magnetic configuration for reducing the neoclassical ripple transport can simultaneously enhance the residual zonal flows which lower the anomalous transport. In fact, it is observed in the Large Helical Device that not only neoclassical but also anomalous transport is reduced by the inward shift of the magnetic axis which decreases the radial ripple transport while more destabilizing magnetic-curvature-driven instabilities such as the toroidal ITG mode [8]. This encourages us to conduct the GKV simulation of turbulence in helical systems for further investigation into the zonal-flow regulation of the anomalous transport.

In order to investigate the ITG turbulent transport in helical systems with the aim of confirming the above hypothesis, we have extended the GKV code so as to take account of helical field variations along field lines. For this purpose, we have first carried out the GKV simulation of the ITG instability. The obtained eigenfunction of the ITG mode in a helical configuration with L = 2 and M = 10 (where L and M denote the period numbers of the helical field in the poloidal and toroidal directions, respectively) has a typical oscillating



Fig. 4 Color contour plots of electrostatic potential at z = 0 obtained from the gyrokinetic-Vlasov simulation of the electron temperature gradient turbulence. The horizontal and vertical axes represent the *x* and *y* coordinates, respectively.



Fig. 5 Field-aligned profiles of real (red) and imaginary (green) parts of the eigenfunction for the ion temperature gradient (ITG) mode obtained by the GKV simulation for a helical system with L = 2 and M = 10.

profile along the field line with helical ripples (see Fig. 5) as predicted by the previous linear eigenmode analysis [9]. Then, we are ready for starting nonlinear GKV simulations of the ITG turbulent transport in helical systems.

4. Summary

Gyrokinetic simulations of the ion (or electron) temperature gradient (ITG or ETG) turbulence have been carried out for tokamak and helical configurations of the magnetic confinement fusion device. Our GKV simulation code, solving time-evolutions of the one-body velocity distribution function in the five-dimensional phase-space, is optimized for the Earth Simulator. In the full toroidal angle simulation of the ITG turbulence in a tokamak, we found an excitation process of the zonal flow related to high-order nonlinearity. The simulation of the ETG turbulence has shown a nonlinear saturation with the normalized heat flux in an order of magnitude higher than that in the ITG case. The GKV simulation is also extended to the ITG instability in helical systems. The preliminary results show a good agreement with the gyrokinetic theory. Simulations of ITG turbulence in helical systems are in progress, and will be reported elsewhere.

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核融合プラズマにおける異常輸送の5次元運動論的シミュレーション

プロジェクト責任者

林 隆也 自然科学研究機構 核融合科学研究所

著者

渡邊 智彦 自然科学研究機構 核融合科学研究所

1. 目的と計画

磁気核融合プラズマ閉じ込め研究における長年の中心的課題として、装置中心部から外部への熱及び粒子の異常輸送が ある。これは、閉じ込めプラズマに付随する密度・温度勾配によりプラズマ乱流渦が発生し、それに伴う電磁場揺動を介して、 粒子間クーロン衝突に起因する拡散よりも桁違いに大きな輸送が引き起こされるという現象である。異常輸送機構の理解と予 測、さらにその制御は、核融合条件を達成・維持し、高い炉心プラズマ性能を実現するための重要な要素となっている。

高温プラズマ乱流を多次元位相空間内の分布関数変動のレベルから直接扱うための理論モデルとして、ジャイロ運動論的 方程式が知られている。この方程式系は、磁力線を旋回するジャイロ運動について平均化することで粒子運動を表すための 速度空間次元を低減し、実空間3次元および速度空間2次元の計5次元の位相空間中で、粒子の一体分布関数の時間発展を 記述するものであり、運動論的プラズマ乱流に関する基本方程式と位置づけられる。プラズマ乱流を5次元位相空間中の分布 関数揺動のレベルからシミュレーションすることによってはじめて、核融合プラズマの異常輸送現象の実相を明らかにすること ができると期待される。核融合プラズマ乱流輸送機構の解明、輸送レベルの予測とその低減に関わる研究に寄与することを目 指して、平成17年度は以下の計画で地球シミュレータ研究プロジェクトを進めた。

(1)5次元ジャイロ運動論的ヴラソフ・シミュレーション・コード(GKVコード)の地球シミュレータへの最適化(ベクトル化・並列 化手法についてのコードの改良を含む)

- (2)トカマク配位でのイオン温度勾配(ITG)乱流輸送の全トロイダル角シミュレーション
- (3)トカマク配位での電子温度勾配(ETG)乱流輸送のシミュレーション(ITGとの比較)
- (4)ヘリカル配位でのITG乱流輸送解析のためのシミュレーション・コードの拡張
- (5)ヘリカル配位でのITG乱流輸送のシミュレーション

2. 今年度得られた成果

今年度の地球シミュレータ利用により、以下にまとめた成果が得られた。

- (1)トカマク配位におけるプラズマ乱流輸送を解析するために、5次元GKVコードを地球シミュレータ用に最適化し、実効性能約4.8Tflops (192ノード利用)を達成した。
- (2)トカマク型装置の全トロイダル角を扱う大規模なITG乱流シミュレーションを行った。従来のシミュレーションと比して揺らぎ による影響が小さく確度の高い乱流輸送係数の予測が可能となり、高次の非線形性が関わる帯状流(平均流)による輸送抑 制機構が新たに見出された。
- (3)トカマク型装置におけるETG乱流による電子熱輸送のシミュレーションを行い、その飽和状態を再現することに成功した。
- (4)GKVコードをヘリカル型配位に適用できるようにコードの拡張を行い、ITG不安定性のシミュレーションを行った。これに より、次年度以降、ヘリカル系におけるITG乱流輸送シミュレーションを進める準備が整った。

キーワード:核融合,プラズマ,乱流,輸送,シミュレーション