Simulation Study on the Generation and Distortion Process of the Geomagnetic Field in Earth-like Conditions

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Efforts have been made to numerically represent convection and dynamo action possessing Earth-like dynamical features. Emphasis is put on energy source distributions, inner core size, viscosity ratio to the Coriolis effect and viscosity ratio to thermal and magnetic diffusivities. Simulations of dynamos with varying inner core radius show that the geometry significantly influences the amount of energy input needed to sustain the dynamo action when the nondimensional radius is larger than 0.5. Thermal convection in fluids with low Prandtl number reveals a possible mechanism of oscillatory convection, which might be applicable to strong magnetohydrodynamic turbulence in the core. Simulation results of dynamo in fluids with low magnetic Prandtl number suggest that the region where a magnetostrophic balance occurs is very limited in space but influences whole dynamics in the fluid shell.

Keywords: Geomagnetic field, Dynamo, Thermal convection, Magnetohydrodynamics, Earth's core

1. Introduction

The geodynamo problem has been one of the most difficult subjects in earth science because the magnetic field and the fluid flows in the Earth's core are intrinsically coupled with strong nonlinearity and in extremely wide ranges of time and space spectra. Since the launch of the Earth Simulator, we have made attempts to construct numerical models which represent correct dynamical features of the Earth's core. One of the directions to reach the Earth-like extreme condition is to decrease the viscous effect relative to the Coriolis effect caused by rapid rotation of the Earth. Our group has computed equations of the geodynamo with the Ekman number (*Ek*, a dimensionless number representing the viscous effect) as low as 10^{-6} and obtained a magnetostrophic state in which the viscous and inertial forces are needless and the Lorentz force plays a primary role in the dynamical force balance [1]. Another one of the directions is to correctly treat the energetic sources of the geodynamo. We investigated the effect of latent heat produced at the growing inner core boundary with the total heat flux out of the core fixed and found that the ratio of the latent heat influences the stability of the magnetic dipole moment [2]. We also studied the effect of the inner core size on the geodynamo which is believed to be variable in the long history of the Earth. The results are described in Section 2 in this report.

The kinematic viscosity of the Earth's fluid core is considered to be smaller than the thermal diffusivity and much smaller than the magnetic diffusivity. Small Prandtl number (*Pr*, ratio of viscous to thermal diffusivities) enhances turbulence in thermal convection and very small magnetic Prandtl number (*Pm*, ratio of viscous to magnetic diffusivities) will cause disparity between scales of the fluid motion and the magnetic field. However, it has not been completely studied what phenomena occur in such low viscosity fluids. In particular, fully nonlinear studies have not been made thoroughly. In Sections 3 and 4, we therefore investigated thermal convection and dynamos operating in fluids with low *Pr* and *Pm*.

2. Dynamos with varying heat source distributions and inner core size

An Earth-type dynamo model is studied by calculating thermal convection in a spherical shell filled with an electrically conductive Boussinesq fluid and spun with a constant angular velocity. The model and numerical method have been explained in detail in the previous annual report [3]. Our model core is heated by uniform heat sources and an additional heat at the inner core surface, the latter representing latent heat released by the solidifying inner core. We examined the cases where the nondimensional heat source parameter $Q = Q_1/Q_0$ is 0, 0.5 and 1, where Q_1 is the latent heat production per unit time and Q_0 is the total heat production in the hydrostatic core. The temperature of the inner core surface is horizontally uniform but changes in time due to convective heat transport in the outer core. The heat flux at the outer core surface is horizontally uniform, with the horizontal average of the temperature fixed. Therefore, the total heat flux out of the convecting core remains approximately Q_0 if the convection is in a quasi-steady state. The media outside the fluid outer core (the mantle and the inner core if exits) are electrically insulating. Both of the inner and outer boundaries of the fluid shell are co-rotating, and the rigid boundary condition is applied there. The Rayleigh number *Ra* is defined in terms of Q_0 so that the same *Ra* means the same energy input to the system.

The effect of the heat source distribution is examined in the case where the ratio of the radii of the inner to the outer boundaries of the shell χ is 0.35. The Ekman number is 10^{-5} and the Prandtl numbers are all unity. In Fig. 1, the time-averaged kinetic and magnetic energies induced in the fluid shell are shown together with the results already reported in the previous report. It has been claimed from the previous results that there are three kinds of dynamo regimes when *Ra* is changed:

1. low-*Ra* regime: the magnetic field is dipolar, the magnetic energy increases efficiently with the increase of *Ra*, and E_{mag} (the magnetic energy density) is greater than E_{kin} (the kinetic energy density);
2. moderate-*Ra* regime: the magnetic field is dipolar, the magnetic energy does not increase efficiently with *Ra*, and E_{mag} is greater than E_{kin};
3. high-*Ra* regime: the time-averaged magnetic field is non-dipolar, polarity reversals of the dipole component occur irregularly (unstable dynamo), the magnetic energy decreases from that in the moderate-*Ra* regime, and E_{mag} is less than E_{kin}.

We confirmed that the high-*Ra* regime still exists in the cases of $Q = 0.5$ and 1, and found that manifestation of these three dynamo regimes is a general feature irrespective of the difference of the heat source distributions. The results indicate that the dynamo with $Q = 0.5$ and $Ra = 12800$ sustains a strong and stable dipolar magnetic field, though in the case of $Q = 1$ with the same *Ra* the solution exhibits an unstable dynamo in which the magnetic field is weak and non-dipolar (Fig. 2(a)). There is no notable difference between the convection patterns near the core surface (Fig. 2(b)). These results indicate that a dynamo with higher *Q* has a tendency to become unstable at a smaller threshold value of *Ra*, and the flow near the inner core, which is enhanced by the increase of *Q*, plays a primary role to make the dynamo unstable.

We further performed numerical simulations with the

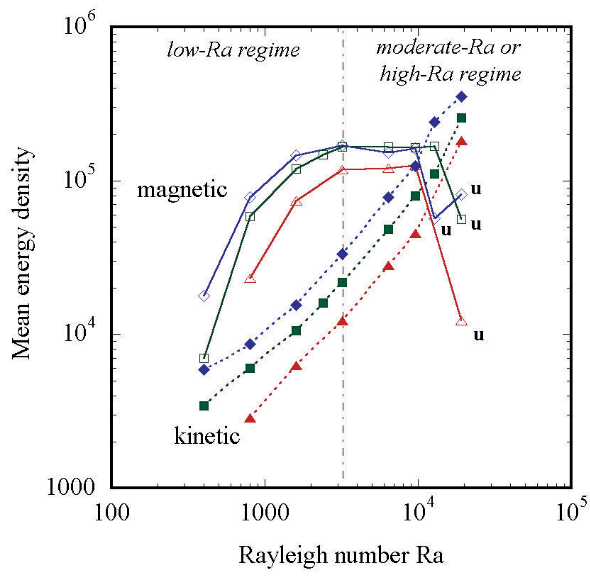

Fig. 1 Mean kinetic (filled symbols) and magnetic (open symbols) energy densities are plotted versus *Ra*. The results for $Q = 0$ (red), 0.5 (green) and 1 (blue) are shown. The symbol "u" denotes an unstable dynamo.

nondimensional inner core radius χ varied from 0 to 0.7 and *Ra* also changed. The heat source parameter *Q* is fixed to 0. The numerical results examined are summarized in Fig. 3. The results indicate that the minimum core heat flux needed to sustain a dynamo does not depend on χ when χ is less than 0.5. When χ is greater than 0.5, however, the shallower the fluid shell is, the more the system requires heat to achieve a self-sustaining dynamo. In the case of the highest χ examined, no self-sustaining dynamo is found. To demonstrate the effect of *Q*, we researched the case of $Q = 1$ in the case of the highest χ, but found no self-sustaining dynamo. These results indicate that the geometry of the fluid core is the most important constraint on existence of a dynamo when $\chi > 0.5$. Similar studies have been already performed but most of them assume $Q = 1$ [4]. We anticipate that the basal heating disturbs the geometrical effect on dynamos. Our result elucidates the purely geometrical effect and might be applicable in predicting how much energy is needed to maintain planetary dynamos.

3. Thermal convection in fluids with low Prandtl number

Thermal convection in a three-dimensional rectangular Boussinesq fluid is studied numerically. The dimensions of the fluid container are 1 × 1 × 4, with the longest axis (*x*-axis) being horizontal, in order to make a comparison to a laboratory experiment in which some of the authors participate. The Prandtl number *Pr* is 0.03 to mimic the liquid gallium used in the experiment. The side walls of the container are thermally insulating and the temperatures of the top and bottom boundaries are constant to maintain a temperature difference, which is controlled by the Rayleigh number in

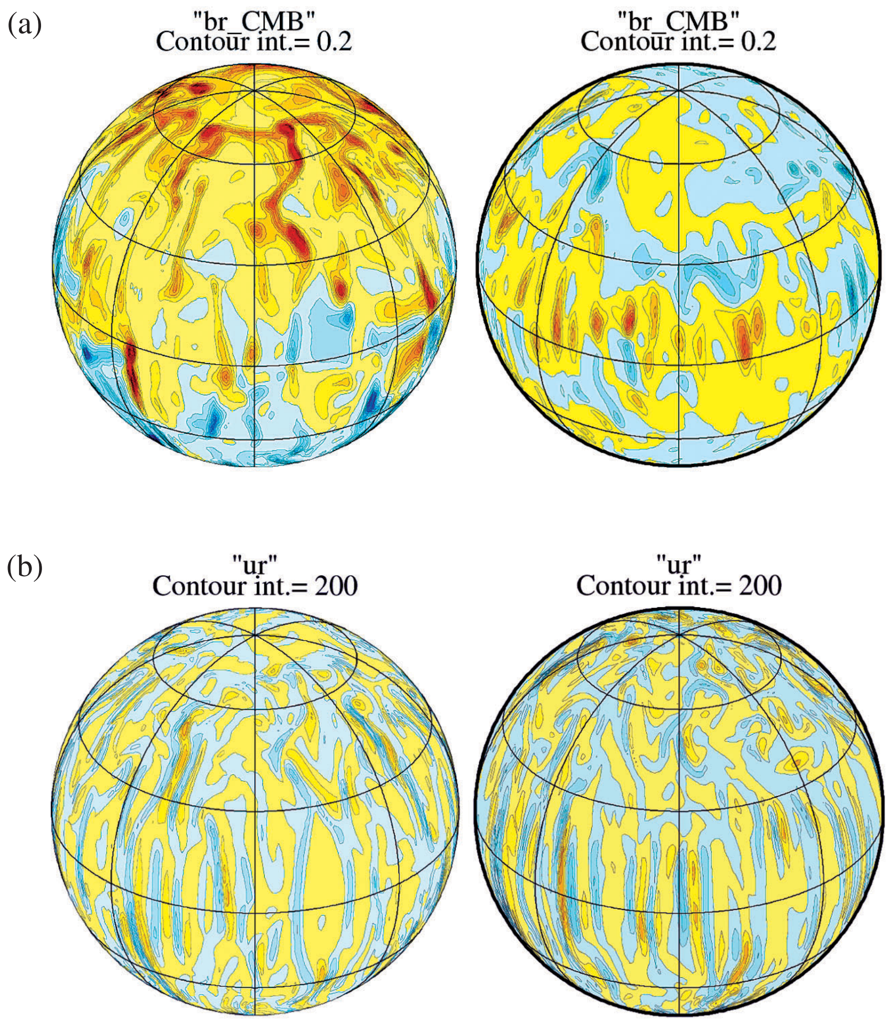

Fig. 2 The radial components of (a) the magnetic field at $r = 1$ and (b) the velocity at $r = 0.96$ are visualized. The results for (Q, Ra) = (0.5, 12800) are displayed in the left and for (Q, Ra) = (1, 12800) are in the right.

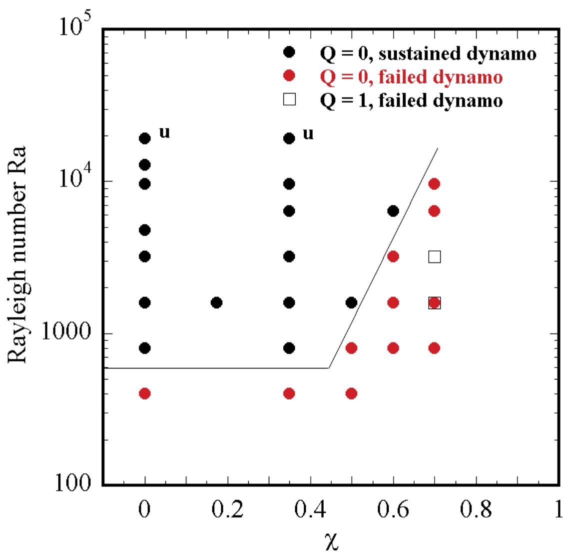

Fig. 3 The nondimensional parameters researched are summarized in the χ-Ra space. The symbol "u" denotes an unstable dynamo.

the numerical model.

We solve the Navier-Stokes equation, the equation of thermal conduction in the convecting fluid and the equation of continuity, but the latter equation is modified to

$$\varepsilon^2 \partial p/\partial t = -\vec{\nabla} \cdot \vec{u}, \tag{1}$$

where p is the pressure, $\vec{u}$ is the velocity and the speed of sound is 1/ε in the nondimensional unit. The parameter ε must be small enough to approximately represent the fluid incompressibility. We typically take ε/h = 0.05 – 0.1 so that the velocity is much smaller than the speed of sound, where h is the size of the computational grid cell. We use the second-order finite difference method with the staggered grid. Therefore, the fact that ε = O(h) is consistent to the degree of total accuracy.

Convective instability occurs in the form of rolls whose axis is parallel to the shorter horizontal axis (y-axis) of the container. The results suggest that appearance of four rolls is preferred, though detailed linear calculations are not per-

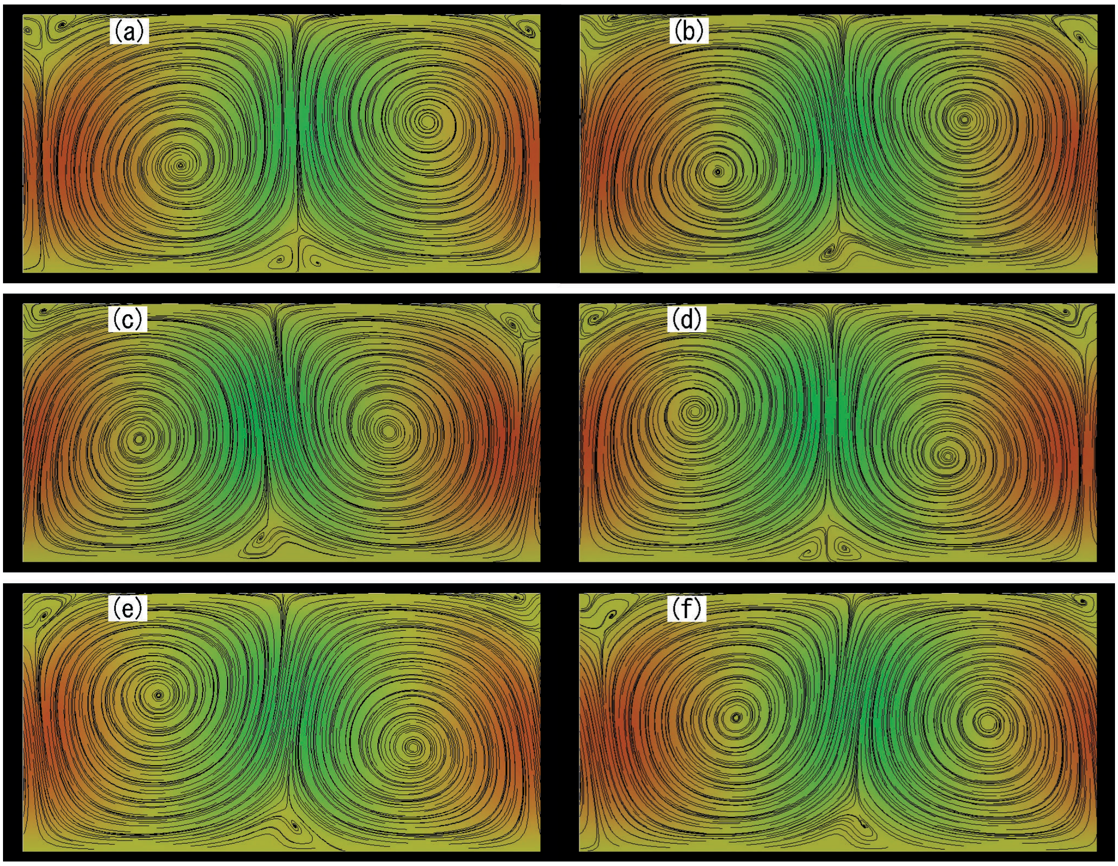

Fig. 4 The streamlines and the vertical velocity (blue and red respectively mean upward and downward flows) are shown on the xz-plane (y = 1/4 and 1 < x < 3). Time advances from (a) to (f) with a constant time interval. The Rayleigh number is 8000.

formed. As the Rayleigh number is increased, a secondary instability occurs and the convection rolls come to oscillate. The instability seems to be the so-called oscillatory instability of convection rolls [5]. The oscillations appear to be a standing wave propagating along the y-axis.

The streamlines of the convection cell is nearly circular, which is the most obvious characteristic of low-Pr thermal convection. As a result, small twin vortices form at the triangular quiet space surrounded by the top (or bottom) wall and the circular main flows away from the wall (Fig. 4). The mechanism of the formation of the twin vortices is very similar to the case of a flow behind a moving cylinder. What is expected when the Reynolds number becomes large is also the same in both examples. We consider that the oscillations of the roll axis originate from the instability of the twin vortices. In the case of a flow behind a cylinder, small vortices are periodically shed from the cylindrical surface to form a Kármán street. Similarly, the twin vortices near the boundary are shed one after another and absorbed into the main circular flows. We observed that the phase of the periodic vortex shedding slightly precedes that of the roll oscillations, indicating that the instability of the twin vortices plays an important role in oscillatory convection.

Comparison to the laboratory experiment is satisfactory. The oscillation pattern is consistent to the velocity measurement results in the experiment. The oscillation frequency is also correctly simulated. It is confirmed that the Strouhal number is around 0.2. Paleomagnetic data show that there is a characteristic frequency where the slope of the power spectrum of the geomagnetic dipole moment changes [6]. We find that the Strouhal number calculated by this characteristic frequency and some appropriate velocity and length scales in the Earth's core also gives 0.2, suggesting that the turbulent spectrum is affected by the same mechanism that we have seen in the present non-magnetic and non-rotating thermal convection.

4. Dynamos in fluids with low magnetic Prandtl number

The numerical code is the same as used in Section 2 but modified to be able to treat an electrically conductive inner core. The magnetic field is defined in both the fluid outer core and the solid inner core, while the velocity and temperature are defined only in the fluid domain. For each spherical harmonic mode representing a dependent variable, we employ the Chebyshev expansion with the same number of spectral modes irrespective of the extent of the domain in which the variable is defined. Therefore, the resolution of the magnetic field is worse than the velocity and temperature. This treatment is not adequate in the case of moderate and high magnetic Prandtl numbers, but could be a good approximation in the present situation. We employ the spectral transform method to calculate nonlinear terms in model equations. We use a second-order interpolation in the radial direction when the magnetic field is needed at the grid point associated with the velocity and vice versa.

A dynamo solution is obtained for $Ek = 5 \times 10^{-7}$, $Pm = 0.2$, $Pr = 1$, $Q = 0.5$ and $\chi = 0.35$. The maximum degree taken in the spherical harmonic expansion is 255. A dipolar magnetic field is generated with the mean magnetic energy density more than twice as large as the kinetic one. The most obvious difference from previous moderate-Pm dynamos is that the region where the magnetic field is strong is spatially limited. Figure 5 illustrates that the magnetic energy is much greater than the kinetic one and the Elsasser number is nearly unity at the mid-depth region on $z = 0.4$, while the magnetic field is rather weak on the equatorial plane. Disparity of the characteristic length scales of the velocity field is conspicuous between the places where the magnetic field is strong and not. The result suggests that a magnetostrophic balance is kept at a very limited region in the fluid shell but the very balance determines the whole velocity structure with the aid of inertial waves propagating along the spin axis.

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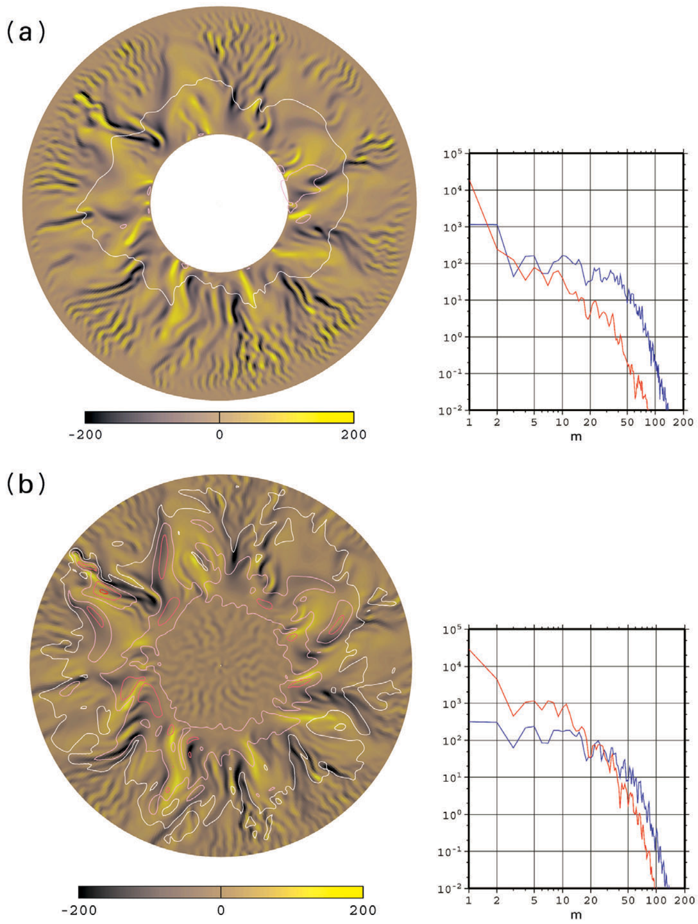

Fig. 5 The radial velocity (shades) and the magnetic field intensity (contour lines) at (a) $z = 0$ and (b) $z = 0.4$ are shown together with the azimuthal kinetic (blue) and magnetic (red) energy spectra at $s = 0.4$ on the same plane, where s is the distance from the z-axis. The magnetic contour line interval is 0.2.

実地球環境での地球磁場・変動シミュレーション

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われわれはこれまで、地球ダイナモ問題の基礎的な理解を目指し、コアの状態をよりよく数値的に再現する努力をおこなってきた。数値モデルを地球のコアに近づけるということは、基本的にはモデル流体の粘性を下げるということにほかならない。われわれはすでに、コリオリ力に対する粘性の比であるエクマン数を10^{-6}程度まで下げて計算をおこない、磁気地衡流がなりたつようなダイナモの解を得ることに成功した。しかしエクマン数を下げるだけでなく、磁気エネルギー／運動エネルギー比や、粘性拡散と熱拡散、または磁気拡散との比を地球のコアのそれと同程度にすることも、ダイナミクスを正しく表現するという意味において重要である。また地球の進化や他の惑星ダイナモとの比較という観点から、内核半径の違いがダイナモにどのような影響を与えるかも重要な興味のひとつである。

そこで本年度では、まずさまざまな内核半径比をもつ球殻流体に対して、レイリー数を変えたときにダイナモ作用が起こるかどうか、また起きたとしてどのような磁場が生成されるかを系統的に調べた。その結果、半径比が0.5以上では、内核の幾何学的な影響がダイナモ作用の発現に大いに影響を与え、薄い球殻になるにしたがって、より多くのエネルギー入力（ここではコアで発生する熱）を系が必要とすることがわかった。つぎに液体金属の一般的な特徴である、粘性拡散が熱拡散に比べて弱い流体（低プラントル数流体）の熱対流のシミュレーションをおこない、われわれのグループが一部関与しているガリウム対流の実験結果と比較した。数値シミュレーションでも、実験と同様、対流ロールの振動が見られた。そのメカニズムとして、境界付近の双子渦が境界から規則的に剥がれることが全体の振動に影響を及ぼしている可能性を指摘した。われわれがこれまでにおこなったダイナモシミュレーションで見られた双極子強度の時間スペクトルの特徴との比較から、この振動現象が、地球コア内の乱流における特徴的な周期の発生と関連性があるかもしれないことが示唆された。また磁気プラントル数（粘性拡散と磁気拡散の比）が1よりも小さい場合について、地球型ダイナモの大規模シミュレーションをおこなった。低磁気プラントル数のダイナモの特徴として、波数の小さい磁気地衡流的な速度構造をもつ部分と、そうでない波数の大きな流れの領域とが大きく二分されるのが観察された。解析の結果、磁気エネルギーが運動エネルギーに比べて大きい、磁気地衡流の領域は、球殻中の限られた領域でしか実現しないが、それでも全体のダイナミクスを決定づけるという、重要な役割を演じていることがわかった。

キーワード：地磁気, ダイナモ, 熱対流, 電磁流体力学, 地球中心核