Generation of a Strong Dipole Magnetic Field in a Geodynamo Model with Low Viscosity

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We present numerical simulations of one of the lowest-viscosity geodynamo models, in which magnetic field generation is caused by thermal convection in a rotating fluid spherical shell that mimics the Earth's metallic core. In contrast to other similar studies, in which small-scale, sheet-like convective motion generated weak or non-dipolar magnetic fields, our solution exhibits a strong dipole magnetic field with intensity similar to the geomagnetic dipole, accompanied with large-scale velocity and magnetic field structures inside the fluid core. Some geomagnetic observations such as the geomagnetic westward drift are well simulated. The difference to other studies can be attributed to thermal boundary condition at the core-mantle interface. A uniform heat-flux boundary that we assumed at the surface of the fluid core effectively drives a large-scale meridional circulation that is responsible for sustaining a strong axial dipole. On the other hand, an isothermal condition that other low-viscosity models used is geophysically unrealistic and leads to failure in simulating actual geomagnetic signatures.

Keywords: Geomagnetic field, Geodynamo, Thermal convection, Magnetohydrodynamics, Core dynamics

1. Introduction

We have attempted to understand geomagnetic field behavior by high-resolution numerical simulations of geodynamo models with low viscosity. In our group, Takahashi and his subgroup (TMH [1]) made thermally-driven dynamo simulations in a parameter regime of $E = 4 \times 10^{-7}$ and $Pm = 0.2$, where E and Pm are respectively the Ekman and magnetic Prandtl numbers, both representing viscous effects in a rotating and electrically conducting fluid (see Table 1 for definition of nondimensional parameters and notation). As

Table 1 Nondimensional input (top four lines) and diagnostic (bottom two) parameters in our model and those estimated in the Earth's core [14]. The Earth's core is modeled by a rotating fluid spherical shell, of which the radius is c, the kinematic viscosity ν, the magnetic diffusivity η, the thermal diffusivity κ, the spin angular velocity Ω, the density ρ, the magnetic permeability μ, and the thermal expansivity is α. The temperature gradient at the CMB in a hydrostatic state is β. The acceleration due to gravity is g at the CMB. *1) Calculated from mean kinetic and magnetic energies in the uniform-flux model. *2) Calculated from those in the isothermal model.

Parameters	Definition	Our model	Earth's core
Ekman number	$E = \nu/2\Omega c^2$	5×10^{-7}	10^{-15}
magnetic Prandtl number	$Pm = \nu/\eta$	0.2	5×10^{-6}
Prandtl number	$Pr = \nu/\kappa$	1	0.2
Rayleigh number	$Ra = \alpha\beta g c^4/\kappa\nu$	3.2×10^{10}	10^{30}
magnetic Reynolds number	$Rm = uc/\eta$	180^{*1} / 220^{*2}	$O(10^2)$
Elsasser number	$\Lambda = b^2/2\rho\mu\Omega\eta$	0.40^{*1} / 0.19^{*2}	$O(10)$

these numbers are believed to be very small in the Earth's liquid core, it is important to decrease them to simulate actual geomagnetic signatures. Their numerical solution exhibits a dipole magnetic field but with weaker intensity than in more viscous (high-E) simulation results. There is a tendency that the magnetic field near the equatorial plane is axial and mostly confined inside the solid inner core. Kageyama and his group (KMS [2]) recently made a geodynamo simulation in a parameter regime of $E = 2.3 \times 10^{-7}$ and $Pm = 1$. They found that a magnetic field was generated by sheet-like convection with a high azimuthal wavenumber. Surprisingly, the generated magnetic field was non-dipolar quite contrary to the geomagnetic field.

In this report, we show results of similar numerical simulations but using a different boundary condition at the surface of the core; namely, a uniform heat-flux boundary rather than an isothermal boundary that TMH and KMS used [3]. In general, an isothermal boundary is a good approximation when the medium outside can transport heat more effectively by high thermal conductivity [4, 5] or vigorous convection. However, the silicate mantle is considered to be thermally less conductive than the liquid metallic core and the time scale of mantle convection is much longer, which implies that a uniform-flux condition should be geophysically more realistic in geodynamo simulations, although it is also an approximation. We expect that the thermal boundary condition causes a critical effect on convection in the presence of both rotation and a magnetic field [6].

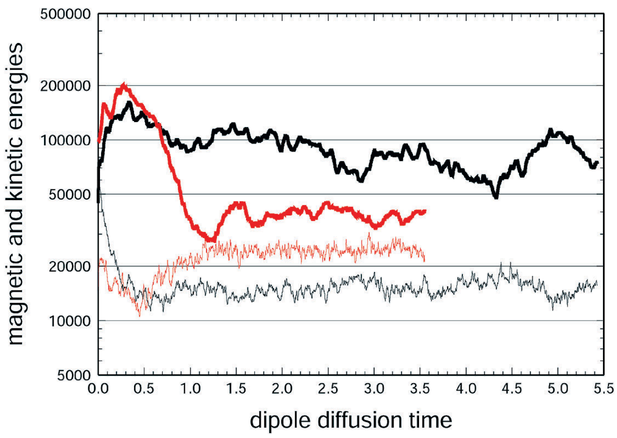

Fig. 1 Nondimensional magnetic (thick lines) and kinetic (thin lines) densities averaged in the fluid core plotted as functions of dipole diffusion time ($c^2/\pi^2\eta$ = 19400 years). Black and red colors show the uniform-flux and the isothermal models, respectively.

2. Numerical modeling

We use a spectral transform code based on spherical harmonic expansion to simulate thermal convection and magnetic field generation in a rotating Boussinesq fluid spherical shell. The radial structures of the flow and magnetic field are resolved by Chebyshev polynomials. Calculations are mostly performed by taking the spherical harmonic degrees and orders less than 256 and the Chebyshev degrees less than 160, but in some simulations the maximum degrees are 320 and 192, respectively. The inner core has the same electrical conductivity as the outer core and is free to rotate around the same axis of the outer core rotation (the z axis). Chebyshev expansion is applied to the magnetic field in the whole core, so that its spatial resolution is lower than the velocity field that is defined only in the outer core. This approximation would be justified when the magnetic Prandtl number is less than one. In addition to a uniform heat source in the whole core, a localized heat source is assumed at the surface of the inner core to mimic latent heat released through inner core crystallization. The inner core temperature varies in time because of convective heat transport. We assume that the temperature is uniform in the inner core and identical to the bottom temperature of the outer core just for economy of computation. For comparison to the isothermal model, our uniform-flux model assumes that the laterally averaged tem-

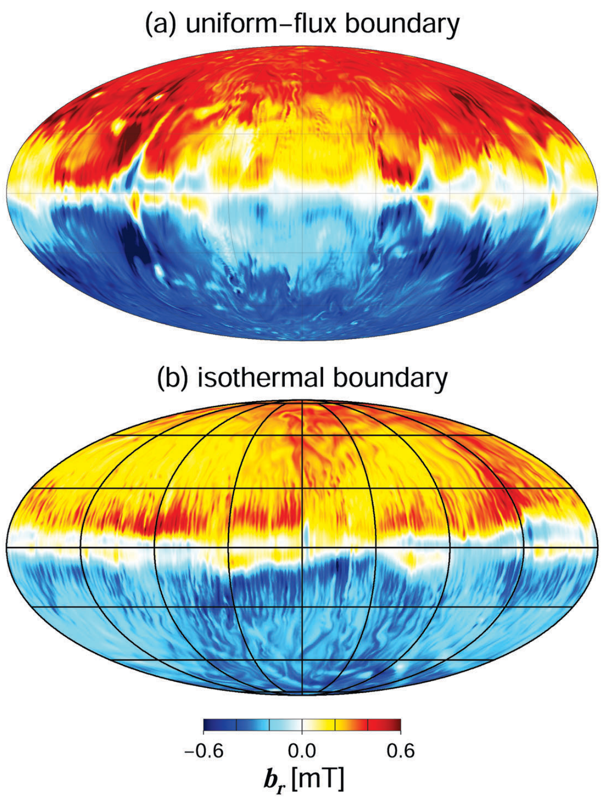

Fig. 2 Radial magnetic field at the fluid core surface in Mollweide projection. (a) A snapshot from our uniform-flux model. (b) A snapshot from our isothermal model. The color scales are the same in both cases.

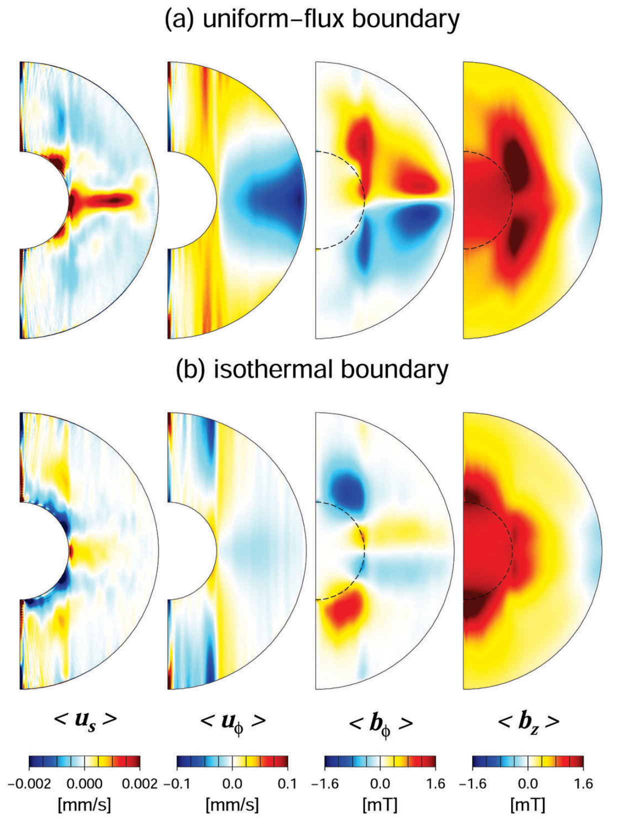

Fig. 3 Cylindrical components of velocity and magnetic field averaged in time and longitude and projected onto the meridional plane. (a) The uniform-flux model. (b) The isothermal model. From left to right, the radial velocity (u_s), the zonal velocity (u_ϕ), the zonal magnetic field (b_ϕ), and the axial field (b_z) are plotted. The dotted line shows the inner core boundary.

perature at the core-mantle boundary (CMB) is fixed so that the total heat flow across the CMB fluctuates in time due to convective heat transport in the outer core [7].

We set the Ekman number to 5×10^{-7}, the magnetic Prandtl number to 0.2 and the Prandtl number to 1. The Rayleigh number (*Ra*) is increased step-by-step. Note that our Rayleigh number is based on total heat flow across the CMB. Since viscous and Joule dissipations are neglected, the heat flow in a quasi-steady convective state is basically the same as before the onset of convection. In the following discussion, all physical quantities are converted to dimensional ones by taking the core radius as c = 3480 km, the magnetic diffusivity $\eta = 2.0$ m^2/s, the density $\rho = 1.1 \times 10^4$ kg/m^3, and the angular velocity $\Omega = 7.27 \times 10^{-5}$ rad/s. The velocity and magnetic field are respectively denoted by $\boldsymbol{u} = (u_s, u_\phi, u_z)$ and $\boldsymbol{b} = (b_s, b_\phi, b_z)$, where (s, ϕ, z) are the cylindrical coordinates defined in a frame co-rotating with the core.

3. Results

We first show results of the uniform-flux model when *Ra* is 3.2×10^{10} and then make a comparison to the isothermal model using the same parameters. The time-averaged magnetic energy density is 5.2 times as large as the kinetic energy (Fig. 1), implying a magnetostrophic balance is nearly satisfied. The magnetic Reynolds number and the Elsasser number calculated from the mean kinetic and magnetic energy densities become 180 and 0.40, respectively. The inner core rotates prograde at an angular velocity of about 0.1 degree per year. A quasi-steady, axial dipole magnetic field is dominant at the CMB (Fig. 2a). The dipole moment is about 1.4 times as strong as the present geomagnetic dipole, although the calculated higher-order multipole components are weaker. There are several magnetic flux patches near the equator that move retrograde like the geomagnetic westward drift [8]. The flux-patch motion is basically explained by advection due to retrograde mean zonal flow just beneath the equatorial part of the CMB (Fig. 3a).

The low-latitude flux patch may be interpreted as manifestation of a strong zonal (toroidal) magnetic field inside the core [9]. When averaged over the azimuthal coordinate, the zonal magnetic field has two oppositely-directed local extrema near the equatorial part of the CMB (Fig. 3a). The toroidal field actually has a wavy three-dimensional structure with an azimuthal wavenumber of approximately 6 (Fig. 4a). The local intensity exceeds 4 mT, which is about ten times as strong as the poloidal field intensity at the CMB. The large-scale wavy pattern is also seen in the velocity field, which is approximately independent of z, in agreement with the Proudman-Taylor theorem. Small-scale turbulent flows are also dominant particularly in regions where the magnetic field is weak. There exist thin sheet-like jets across which the magnetic field intensity varies almost discontinuously [10]. The mechanism of magnetic field generation is basically explained by a macroscopic α effect caused by large-scale helical columnar vortices and also by an ω effect caused by a mean zonal flow, as have been identified in previous studies [9].

The isothermal model produces a completely different result. The initial condition is taken from a solution of our uniform-flux model. Once the boundary condition is changed, the mean magnetic energy density starts decreasing and finally becomes less than twice the kinetic energy (Fig. 1). The generated magnetic field is still dipolar, but the intensity at the CMB is weaker than in the uniform flux model (Fig. 2b). The magnetic field pattern at the CMB is so broad that no localized flux patches are found. The westward drift is not clear because the mean zonal flow is weak (Fig. 3b). There is a tendency that the axial magnetic field is confined in the solid inner core. The most striking difference is the velocity and magnetic field structures inside the core (Fig. 4). The flow pattern becomes more sheet-like with a higher azimuthal wavenumber. The magnetic field also has a fine structure and the large-scale wavy zonal field disap-

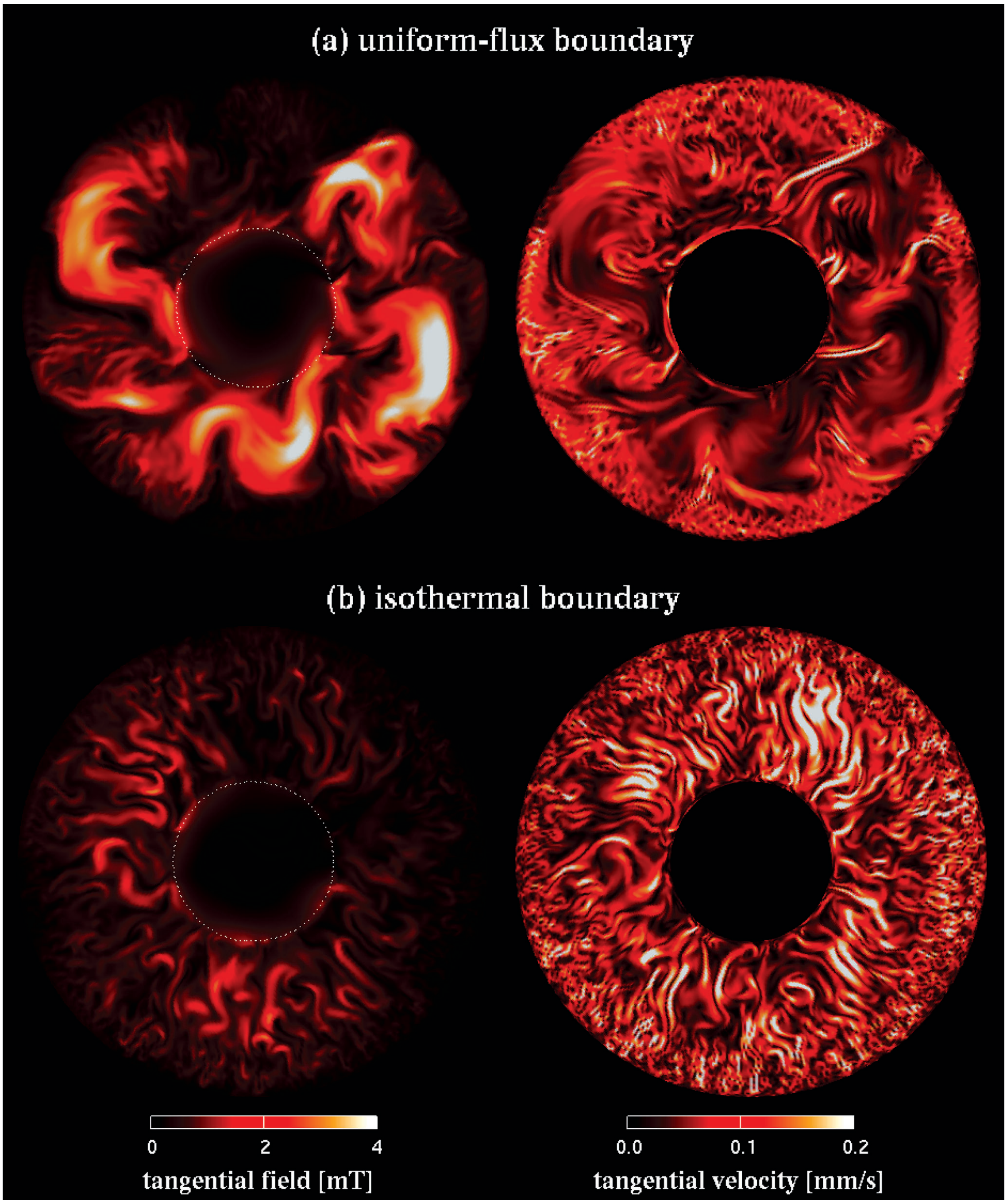

Fig. 4 Magnetic field (left) and velocity (right) structures on a plane parallel to the equatorial plane (z = 0.1c) viewed from the north. (a) A snapshot from the uniform-flux model. (b) A snapshot from the isothermal model. The magnetic field structure is represented by the tangential field intensity $(b_s^2+b_\varphi^2)^{1/2}$. The velocity structure is represented by the tangential speed $(u_s^2+u_\varphi^2)^{1/2}$.

pears. These characteristics have been commonly seen in simulation results of TMH, KMS, and other low-viscosity isothermal models [11].

When averaged in time and longitude, the azimuthal components of the Coriolis force, $-2\rho\Omega\langle u_s\rangle$, and the Lorentz force, $\langle j_z b_s\rangle - \langle j_s b_z\rangle$, are nearly in balance, where $\boldsymbol{j}$ stands for the electric current density and <.> means time and azimuthal average. At the equatorial plane (where values are denoted by E), the balance is essentially reduced to

$$-2\rho\Omega\langle u_s\rangle_E - \langle j_s\rangle_E \langle b_z\rangle_E = 0 \quad (1)$$

(Fig. 5). The axial magnetic field $\langle b_z\rangle_E$ is responsible for sustaining an axial dipole, because the dipole moment is proportional to the surface integral of magnetic flux over the CMB in the northern hemisphere if higher-order multipoles are neglected, and hence proportional to the integral of b_z over the equatorial plane within the core. The radial electric current $\langle j_s\rangle_E$ sustains low-latitude toroidal fields which are

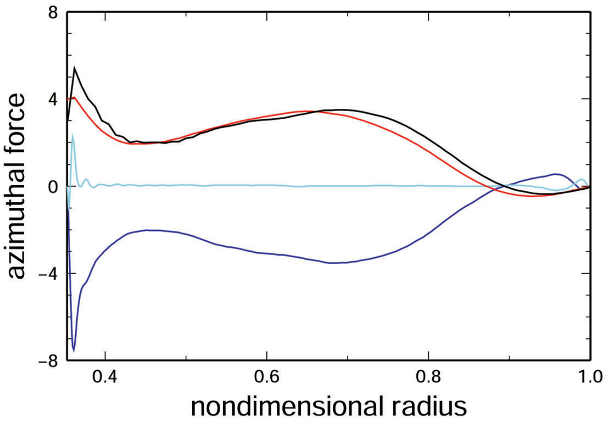

Fig. 5 The azimuthal Lorentz (black line), Coriolis (blue line) and advective (light-blue line) forces at the equatorial plane averaged in time and longitude are plotted as functions of the radius s. Red line represents $-\langle j_s \rangle_E \langle b_z \rangle_E$, which well approximates the actual Lorentz force (black line).

oppositely directed on each side of the equatorial plane. Therefore, equation (1) implies that the radial flow $\langle u_s \rangle_E$ is essential for maintaining not only a strong axial dipole, but also toroidal fields responsible for low-latitude flux patches at the CMB. Figure 3 clearly shows that the uniform-flux model produces a weak but significant radial flow along the equatorial plane, whereas $\langle u_s \rangle_E$ is largely suppressed in the isothermal model. A strong axial dipole can be sustained without the radial flow if b_z is confined inside the solid inner core, which seems to occur in our isothermal model and also in TMH. However, sustaining $\langle j_s \rangle_E$ seems to be difficult without the radial flow.

The radial flow $\langle u_s \rangle_E$ and its counter flow $\langle u_s \rangle$ toward the z axis at other latitudes form a meridional circulation pattern, which has been regarded as one of the important components in kinematic dynamos. The reason why an isothermal boundary weakens the meridional circulation is still uncertain. It should be noted, however, that laterally large-scale convection cells emerge in Rayleigh-Benard convection with uniform-flux boundaries [4, 5], implying that uniform-flux condition, which allows a lateral variation of the surface temperature, enhances flows along the boundary so that large-scale meridional circulation ensues. Previous geodynamo simulations that used higher Ekman numbers suggest that the boundary condition of temperature is not so critical [7]. In such a case, the theoretically predicted azimuthal wavenumber in non-magnetic convection, which increases as $E^{-1/3}$ [12], is not much greater than unity and a large-scale flow is intrinsically permitted. We speculate that an isothermal boundary suppresses meridional circulation but only if the Ekman number is small enough (probably less than 10^{-6}).

4. Concluding remarks

A strong axial magnetic field accompanied with large-scale fluid motion is obtained from a numerical geodynamo model with significantly low viscosity. The large-scale fluid motion of which the azimuthal wavenumber is about 6 is consistent to our prediction deduced from the observed time spectrum of the geomagnetic dipole moment [13]. Our result implies that the large-scale structure is a robust characteristic even in much less viscous conditions and that previous simulations with higher viscosity may be essentially applicable to the actual geomagnetic phenomena. Our numerical solution has several characteristics similarly observed in the actual geomagnetic field. In particular, geomagnetic phenomena of short timescales such as geomagnetic jerks and torsional oscillations may be elucidated by more detailed analyses owing to low viscosity realized in our model. The difference from other low-viscosity geodynamo models, in which small-scale, sheet-like fluid motion generated weak or non-dipolar magnetic fields, can be attributed to thermal boundary condition at the CMB. Importance of meridional circulation for sustaining an axial dipole and toroidal magnetic fields is also pointed out. In order to simulate the actual situation in the deep Earth, it would be necessary to impose more realistic, laterally heterogeneous thermal and electromagnetic boundary conditions.

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低粘性の地球ダイナモモデルで得られた強い双極子磁場

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われわれはこれまで、地球磁場の変動メカニズムの解明を目的として、高解像度の地球ダイナモシミュレーションを、できる限り地球に近い、低粘性のパラメータ領域でおこなってきた。高橋およびそのサブグループは、回転磁気流体の粘性効果をあらわす無次元パラメータであるエクマン数(E)と磁気プラントル数(Pm)を、それぞれ4×10^{-7}および0.2に下げて、ダイナモシミュレーションをおこなうことにすでに成功している。得られた数値解は双極子型の磁場を示したが、よりEやPmが高い場合の結果に比べて、生成される磁場強度は弱かった。また他のプロジェクトで最近おこなわれた$E = 2.3 \times 10^{-7}$という低エクマン数のシミュレーションでは、双極子型の磁場は卓越せず、むしろ地球磁場とは異なる特徴を呈している。すなわちこれまでの結果には、無次元パラメータを地球のそれに近づけると、生成される磁場がむしろ地球とは異なる性質をもつという、矛盾がみられるように思われる。

本年度の研究では、この矛盾が、設定された温度境界条件によるものであるかもしれないことを示した。これまでの多くのモデルでは、回転する流体コア内に熱的な対流不安定を起こし、それによって磁場が生成する。そしてその際、コア・マントル境界の温度を一定とする仮定をおいている。このような等温境界条件は、マントルがコアよりも高い熱伝導度をもつか、あるいはマントルがコアよりも激しく対流するような場合には適当であるが、現実のコア・マントル系では、むしろそれと対極に位置する、熱流束一様の境界条件のほうがより適当である(実際は熱流束は水平不均質をもつが、もっとも単純で理想的な条件をここでは考える)。実際、$E = 5 \times 10^{-7}$および$Pm = 0.2$という、これまでの低粘性ダイナモモデルと同程度の粘性効果を仮定した数値計算を、等温および一様熱流束の2つの境界条件のもとでおこなったところ、驚くほど異なる流れと磁場の構造がみられた。等温境界の場合、これまでの結果と同様に、経度方向の波数が大きい、シート状の細かい対流構造が見られ、磁場も同様に高波数であった。いっぽう一様熱流束境界の場合、磁場強度が増し、強い双極子磁場が生成された。コア内部の流れと磁場には、波数6程度の大規模構造がみられ、とくに赤道反対称の強いトロイダル磁場が卓越した。境界条件に起因するこれらの相違は、赤道面で内核表面からマントルに向かい、より高緯度で自転軸方向に返るような、子午面循環の強弱によって物理的に説明することができた。すなわち一様熱流束条件では、コア・マントル境界に沿って温度不均質が許され、それによって大規模な子午面循環が駆動されうるのである。この子午面循環は、強い双極子磁場やトロイダル磁場の維持にきわめて重要な役割を果たす。

本研究は、今後の、より地球に近い条件下でのダイナモシミュレーションにおいて、コア表面の温度境界条件が重要であることを示唆する。とくに等温境界条件をもちいたシミュレーションは、単に現実のコア・マントル境界にはそぐわないだけでなく、地磁気の特徴を本質的にあらわしえないかもしれない。今回得られた数値解は、赤道域に卓越する地磁気の西方移動など、現実の地磁気変動とよく似た性質を有している。今後、数値解の解析をすすめて、地磁気の短周期変動の観測結果などと比較することにより、地球磁場変動のより深い理解が得られることが期待される。

キーワード：地磁気, 地球ダイナモ, 熱対流, 電磁流体力学, コアダイナミクス