Three Dimensional Simulations of Tsunami Generation and Propagation

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A parallel finite-difference numerical simulation program based on the Navier-Stokes (NS) equations is developed for simulating 3-D tsunami generation and propagation. We can simulate 3-D tsunami propagation over several thousand kilometers using this program, although such tsunami propagation over long distances has usually been conducted based on 2-D simulations. We apply the tsunami simulation program to simulate tsunamis caused by the 2004 off Kii Peninsula earthquake (M 7.4). The simulated results are visualized for understanding the tsunami generation and propagation processes. The dispersive tsunamis can be well simulated by the 3-D tsunami simulations while the 2-D linear long-wave equations that are often used in conventional tsunami studies cannot simulate the dispersion.

Keywords: tsunami, simulation, finite-difference method, Navier-Stokes equations

1. Introduction

Tsunami simulations have been conducted mostly in 2-D space based on the long-wave approximation, instead of solving 3-D Navier-Stokes (NS) equations. Simulations based on the 2-D linear long-wave (LLW) equations are cost effective and widely used for the estimation of the slip distribution along seismic faults or tsunami hazard assessments for past and expected future large earthquakes. Tsunami simulations have also been employed for constructing tsunami databases used for tsunami alert systems such as those operated by the Japan Meteorological Agency (JMA).

Because 2-D simulations employ long-wave approximations and so do not allow calculation of the vertical flow, the tsunami generation process due to the sea-bottom uplifting is not modeled in the 2-D simulations, but given as an initial tsunami height distribution. Numerous 2-D tsunami simulations assume that the initial tsunami-height distribution is identical to the sea-bottom deformation caused by the earthquake [e.g. Satake 1989]. This assumption may be valid for many tsunamigenic earthquakes that occur along the subducting plate and cause deformation of the sea-bottom over a large area. However, a small sea-bottom deformation area would not result in as great an uplift of the sea surface as would occur for the sea-bottom.

On the other hand, 3-D tsunami simulation can simulate the tsunami generation process caused by the sea bottom deformation and includes a dispersion effect that is neglected in the 2-D LLW equations. We report 3-D tsunami simulation for the tsunami excited by the 2004 off Kii Peninsula earthquake.

2. Navier-Stokes Equations for Tsunamis

The motion of a fluid is described by the following 3D NS equations in Cartesian coordinates (x, y, z),

$$\begin{cases} \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z} = -\dfrac{\partial}{\partial x}\left(\dfrac{p}{\rho_0}\right) + \nu\nabla^2 u \\ \dfrac{\partial v}{\partial t} + u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + w\dfrac{\partial v}{\partial z} = -\dfrac{\partial}{\partial y}\left(\dfrac{p}{\rho_0}\right) + \nu\nabla^2 v \\ \dfrac{\partial w}{\partial t} + u\dfrac{\partial w}{\partial x} + v\dfrac{\partial w}{\partial y} + w\dfrac{\partial w}{\partial z} = -\dfrac{\partial}{\partial z}\left(\dfrac{p}{\rho_0}\right) + \nu\nabla^2 w + g \end{cases} \tag{1}$$

where u, v, and w are velocity components along the x, y, and z-axes, respectively, p is pressure, g is acceleration due to gravity, ρ_0 is the density, and ν is the kinematic viscosity coefficient for water. The viscosity of water is characterized by a very small value of $\nu = 10^{-6}$ m^2/s.

Assuming incompressible fluid flow, the continuity equation is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{2}$$

When we take a free surface at rest as $z = H_0$, the bottom is given by $z = h_b = H_0 - h(x, y)$ where $h(x, y)$ is the water depth. The free surface is given by $z = h_s(x, y, t) = H_0 + \eta(x, y, t)$, where $\eta(x, y, t)$ is the fluctuation of the surface at time t. We may consider $\eta(x, y, t)$ as the tsunami. The kinematic boundary condition at the sea surface is given by

$$\frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y} = w \text{ for } z = h_s(x, y, t). \tag{3}$$

These are the equations governing the behavior of water waves, which can be used to describe tsunami generation and propagation.

The deformation of the bottom is directly introduced into the simulation through the water flow at the cell at the bottom. Suppose that the bottom is rising linearly with constant velocity and with final vertical displacement distribution at the bottom after the source process time t_s being given by $d(x, y)$; the corresponding boundary condition at the bottom is then given by,

$$w(x, y, h_b(x, y)) = \begin{cases} d(x, y)/t_s & \text{for } 0 < t_s \le t \\ 0 & \text{for } t > t_s. \end{cases} \tag{4}$$

The vertical flow caused by the bottom deformation results in elevation of the surface. The elevation, or the height distribution at the surface, is propagated as a tsunami.

We use the SOLA technique originally developed by Hirt et al. [1975] to solve above equations and boundary conditions. The NS equations are solved numerically using the FDM with a staggered-grid model. We developed a parallel computation method for the SOLA technique and reported its calculation efficiency. This study uses the code to simulate tsunami generation and propagation processes.

3. Tsunami Simulations of the 2004 off Kii Peninsula Earthquake

3.1 Tsunami of the 2004 off the Kii peninsula earthquake

The Philippine Sea Plate is subducting underneath the Eurasian Plate in southwestern Japan [Fig. 1], where M8 class earthquakes have repeatedly occurred between the two plates. The last two M8-class events were the 1944 Tonankai (M7.9) and the 1946 Nankai (M8.0) earthquakes. The both events created tsunamis, which caused serious damage along the Pacific coast of Japan. Neighboring the Tonankai- and Nankai- earthquake regions, a large M7.4 earthquake occurred off the Kii peninsula on 5 September 2004. The event was occurred in the subducting Philippine Sea Plate and characterized by thrust fault event with large dip-angle. We simulate the tsunamis of the 2004 off Kii peninsula event comparing 3-D and 2-D simulations in the following.

3.2 Simulation model, the 2004 off the Kii peninsula event

The area of the simulation model was 580 km by 896 km horizontally, and extended to a depth of 10 km, which is discretized with a uniform mesh size of 1 km in horizontal directions and 0.2 km in depth [Figure 1]. Digital bathymetric data of J-EGG500 provide by Japan Oceanographic Data Center was employed in the simulation.

The fault model for the Kii event was derived from the analysis of far-field seismograms by Yamanaka [2004]. Referring to this model, we set parameters for a constant slip model on a flat fault plane. The top depth of the fault was 2 km and the fault sizes were 60 km and 32 km in the strike and dip directions, respectively. The focal parameters were given by dip = 40degree, strike = 135degree, rake = 120degree. The source process time was 30s.

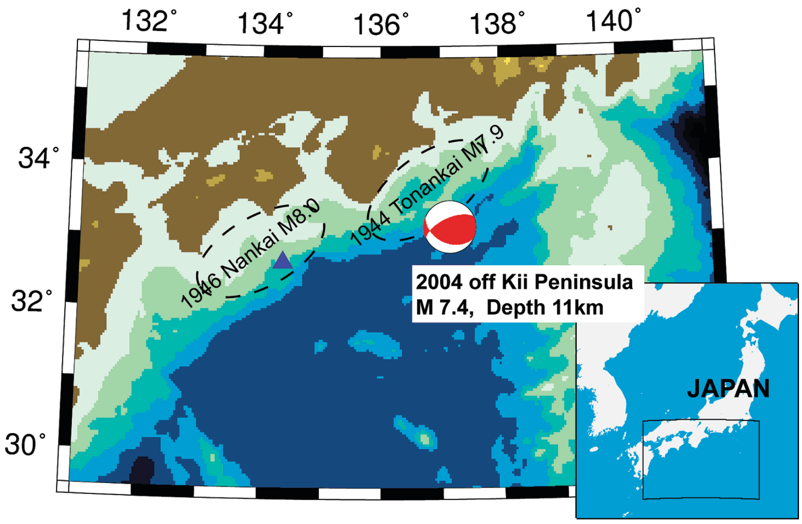

Fig. 1 The area and bathymetry for the tsunami simulation of the 2004 M 7.4 off Kii Peninsula earthquake (red). Source areas of the 1944 Tonankai (M 7.9) and the 1946 Nankai (M 8.0) earthquakes are also shown by dashed curves. The point for the tsunami records in Fig. 3 is shown by a triangle.

The static deformation of the sea bottom caused by the earthquake was calculated by using a program of Okada [1992] where an analytical expression of the static deformation of an elastic half space is given. The elastic parameters were taken as V_p = 5.8 km/s, V_s = 3.2 km/s, and ρ = 2.6 g/cc for our simulation, corresponding to the *P* and *S*-wave velocities of the upper crust of the PREM. The source process time is set to be t_s = 30 s. The time step was chosen as Δt = 0.1 s during the source process time ($t < t_s$) and a larger time step of Δt = 1 s was used after the source process time.

3.3 Comparison of 3D Navier-Stokes and 2-D Linear Long-Wave Simulations

The results of tsunami simulation derived from the 3D simulation are compared with those calculated by the LLW simulation. Both simulations use the same bathymetry grid and seismic fault model. The results of tsunami derived from the NS and LLW simulations are compared in Fig. 2 as snapshots of tsunami propagation for elapsed time of t = 10, 20, 30, and 40 min. There is a significant difference between the two results; the dispersion of tsunami propagation can be

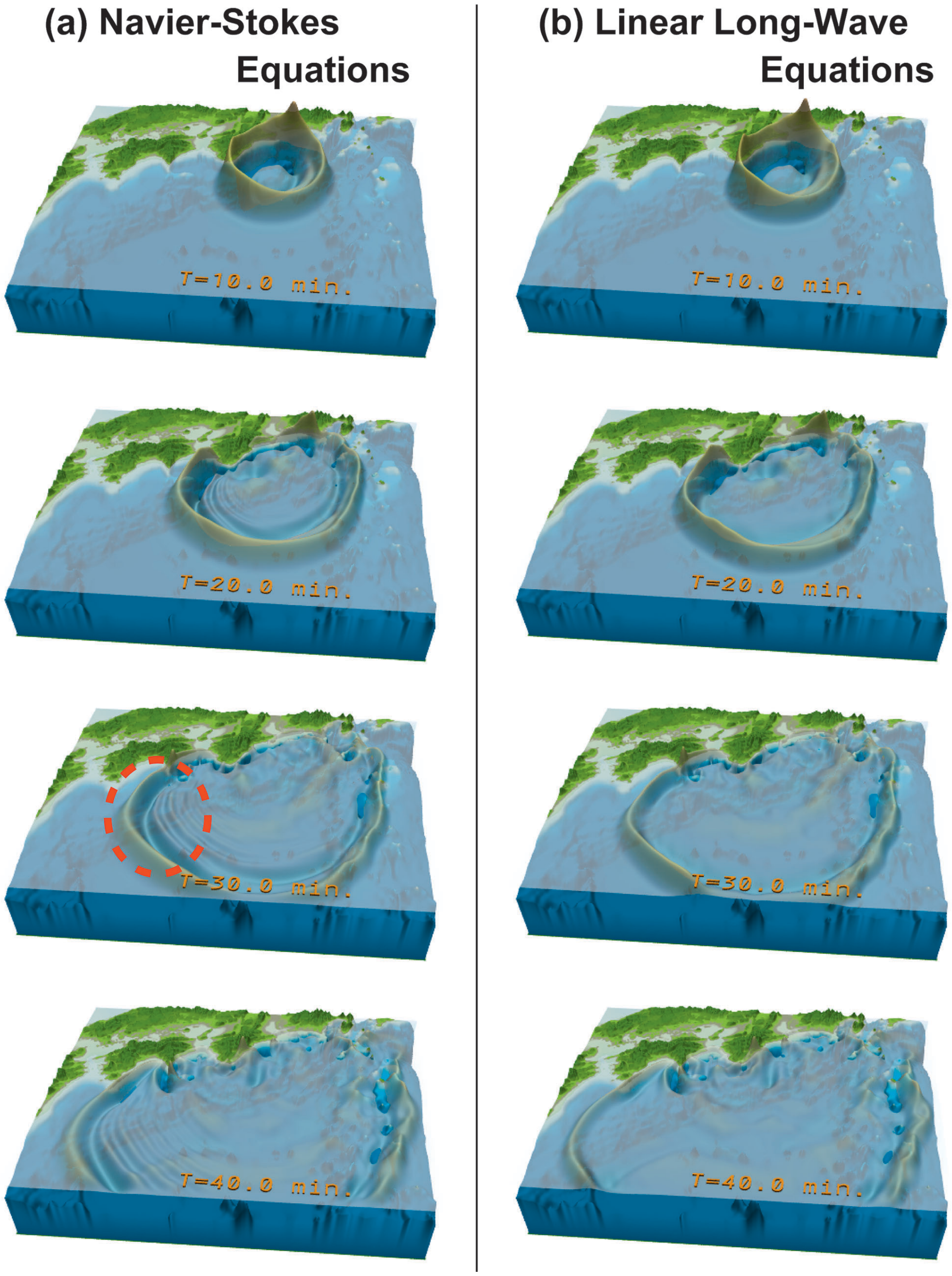

Fig. 2 Snapshots of the tsunami propagation for the 2004 off Kii Peninsula earthquake, at elapsed times of 10, 20, 30 and 40 min from the earthquake origin time calculated by 3-D Navier-Stokes equations and 2-D linear long-wave equations. Tsunami dispersion is recognized in the results of the 3-D Navier-Stokes simulations [dashed circle].

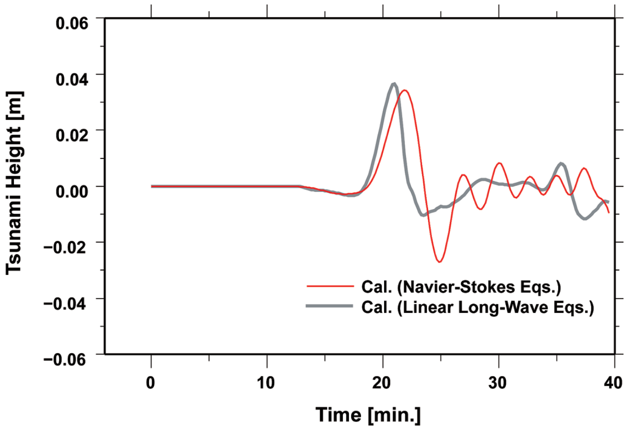

Fig. 3 The tsunami record calculated from the simulation using 3-D NS equations (red) and 2-D linear long-wave equations (black) off Muroto (a triangle in Fig. 1). The 3-D NS equations can simulate dispersive tsunami.

clearly seen from the NS simulation, while the LLW simulation shows simple impulsive tsunami.

The tsunami records calculated by NS and LLW simulations are compared at a point off Muroto, located about 200 km from the source region [a triangle in Fig. 1]. Solid and dashed lines shown in Fig. 3 are the calculated tsunami derived from the NS and LLW simulations, respectively. The calculation of the NS simulation shows the later tsunami phases following the leading waves. On the other hand, the LLW simulation fails to simulate the later phase excitation.

4. Conclusions

3D tsunami simulation has been conducted for the tsunami excited by the 2004 off Kii Peninsula (M7.4). Since this event is intraplate event with steep dip angle, the tsunami wavelength of this event is relatively short compared to usual large interplate earthquakes. In that case, we need to carefully examine the long wave approximation that is often used in conventional tsunami simulations. By comparing the 3-D tsunami simulation and the 2-D simulation based on the long wave approximation, we have confirmed that the 2-D LLW simulation did not serve as a good approximation to the 3-D NS simulation for the 2004 off Kii Peninsula event. Although the 2-D LLW equations cannot simulate the dispersive tsunamis, the 3-D NS simulation is well able to model those characteristics.

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津波生成と伝播の3次元数値シミュレーション

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流体運動は3次元ナヴィエ・ストークス式（NS式）で記述されるが、津波発生および伝播過程は、津波波長が十分長いと仮定し、2次元線形長波方程式に基づきシミュレートされる場合が多い。これに対し、本研究では3次元NS式に基づき、長波近似を用いることなく津波発生・伝播過程をシミュレートする並列計算プログラムを開発した。並列計算を行うことで、長距離伝播する津波をシミュレートすることが可能となり、実際に観測される津波記録への高い適用性をもつ。これを利用し、2004年9月5日に紀伊半島沖で発生した地震による津波をシミュレートし、その発生と伝播過程の可視化を行った。これまでの代表的な津波シミュレーション法である2次元線形長波方程式によるシミュレーションでは、波の分散現象を再現することができないが、3次元NSシミュレーションによって分散現象を正確に再現することに成功した。

キーワード：津波, シミュレーション, 差分法, Navier-Stokes方程式