Development of Advanced Simulation Methods for Solid Earth Simulations

Project Representative

Akira Kageyama — Graduate School of System Informatics, Kobe University

Authors

Akira Kageyama[*1], Mamoru Hyodo[*2], Mikito Furuichi[*2], Takehiro Miyagoshi[*2], Masanori Kameyama[*3] and Nobuaki Ohno[*4]

*1 Graduate School of System Informatics, Kobe University

*2 Institute for Research on Earth Evolution, Japan Agency for Marine-Earth Science and Technology

*3 Geodynamics Research Center, Ehime University

*4 Earth Simulator Center, Japan Agency for Marine-Earth Science and Technology

Geodynamo: We have found a new convection regime of the core that exhibits a dual structure in the low viscous frontier of numerical simulation of geodynamo; sheet-like radial plumes inside and a westward cylindrical zonal flow outside. The dual convection structure with the zonal flow is stable under a self-generated strong magnetic field. Mantle convection: We have improved our numerical code "ACuTEMan" by employing the truncated anelastic liquid approximation (TALA). This has enabled us to successfully incorporate the effects of adiabaic compression on thermal convection in Cartesian domains. Plate-mantle unified system: we have developed an iterative solution technique for solving the Stokes flow problem with a large and sharp contrast in viscosity, for example, between the upper mantle, tectonic plate and sticky air for free surface treatment. Our method with Schur complement approach with mixed precision arithmetic is very robust against large and sharp viscosity contrast, given by the one to several grid resolutions.

Keywords: Geodynamo simulation, Yin-Yang grid, Mantle convection, Anelastic Liquid Approximation, Schur complement, Double-double.

1. Geodynamo simulation

Zonal jets are omnipresent in nature. Well known examples are those in atmospheres of giant planets, alternating jet streams in the Earth's ocean, and zonal flow formation in nuclear fusion devises. A common feature of these zonal flows is that they are spontaneously generated in turbulent systems. Since the Earth's outer core is also believed to be in a turbulent state, it is an interesting question if a vigorous zonal flow exists in the liquid iron of the outer core. We have found, in the low viscous frontier of numerical simulation of geodynamo, a new convection regime of the core that exhibits a dual structure; sheet-like radial plumes inside and a westward cylindrical zonal flow outside. The dual convection structure with the zonal flow is stable under a self-generated, strong dipole magnetic field.

Convection motion of the Earth's liquid core generates the geomagnetic field through the magnetohydrodynamic (MHD) dynamo process. Computer simulation of the geodynamo has successfully reproduced the Earth's dipole magnetic field and its reversals, and various scaling lows are reduced. The convection motion in those simulations are basically composed of columnar cells aligned to the rotation axis. Recently, we have performed geodynamo simulations on the Earth Simulator with the highest spatial resolution ever achieved. The fine resolution has enabled us to investigate the numerical frontier of the low viscous simulation. One of the most important non-dimensional parameters of the core defined with the viscosity is Ekman number Ek. We have reported that when Ek is sufficiently small (of the order of 10^{-7}), the convection motion is composed of sheet-like plumes rather than columns [1].

In this fiscal year, we have found that the sheet-like plumes are surrounded by a zonal flow (Fig. 1). The simulation model and results are reported in our recent paper [2]. In the zonal flow region, the radial flow component in the cylindrical coordinate, v_s, is feeble while the azimuthal component, v_ϕ, is dominant. The zonal flow is westward. We call this convection structure---the sheet plumes inside and the zonal flow outside---as "dual convection". We have found that the dual convection state always appear when Ek is small and the Rayleigh number, Ra, is sufficiently large.

To prove the stability of the dual convection under the

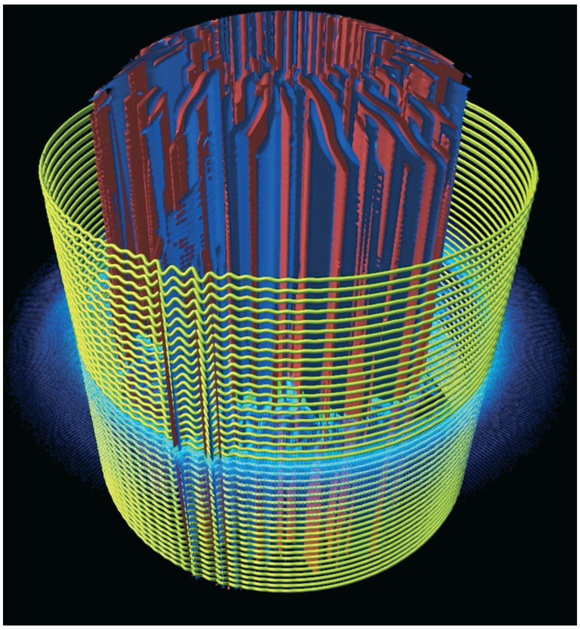

Fig. 1 Convection structure viewed from equator. Red and blue surfaces are positive (red) and negative (blue) isosurfaces of axial vorticity . The yellow lines are streamlines.

strong magnetic field, we would have to continue the run for much longer time, but it is practically impossible because the simulation would take for years with our the total grid size of 511 (in radial) x 514 (in latitudinal) x 1538 (in longitudinal) x 2 (Yin and Yang).

We have therefore made a survey to find a parameter set in which the dual convection structure is formed under a coarser grid resolution 255 x 258 x 770 x 2, by which the simulation can be performed in much more swiftly. With this grid size, we were able to integrate the calculation for sufficiently long time to confirm the stability of the dual convection structure. The dual convection structure is stable even under self-generated strong magnetic field.

The zonal flow is generated and maintained by the non-linear coupling of the velocity components, or the Reynolds stress. Small scale flows ejected from the inner plumes transport the westward angular momentum at the zonal flow radius. The three-dimensional structure of the zonal flow is fairly uniform in the direction of the rotation axis. The flow appears as in a thin zone in the equatorial plane is actually a cross section of a thin cylinder of westward flows.

It is known from numerical simulations of planetary atmospheres that relatively strong azimuthal flows are formed in a rotating spherical shell convection under the stress-free boundary condition for the spherical boundaries. In contrast to those systems, the core is "capped" by the mantle. Our simulation suggests that convection in a spherical vessel may be closer than we had expected to the planetary atmospheres and oceans when viscosity or Ekman number is sufficiently small.

2. Development of mantle convection simulation code with adiabatic compression

In this FY, we developed a simulation code of mantle convection in a three-dimensional Cartesian domain which includes of the effect of adiabatic compression of mantle materials [3]. In most of numerical models of mantle convection, including our previous ones [4,5], the effects of adiabatic compression is ignored simply because of the numerical difficulty, although it has been commonly acknowledged that there occurs a significant compression of mantle materials by an extremely high pressure (exceeding 100GPa) expected in the Earth's interior. Here, by improving the thermodynamic treatment of the convecting fluid, we successfully incorporated the effects of adiabaic compression on thermal convection in Cartesian domains.

In this study, we improved our numerical code "ACuTEMan" by employing the truncated anelastic liquid approximation (TALA). The term "truncated" means that the effect of compression by dynamic pressure is ignored in the buoyancy force. This assumption is sufficiently validated by the fact that the dynamic pressure due to the fluid motion is much smaller than the static pressure by the overburden load. Compared to the well-known incompressible and/or Boussinesq approximation, the TALA enables us to include the effects of (i) the change in reference (i.e., zero-motion) density with pressure (or depth) and (ii) the conversion from mechanical to thermal energies such as an adiabatic (de)compression and viscous dissipation (or frictional heating). In particular, the first effect is taken into account through the change in the definition of thermal buoyancy and the divergence-free constraints on the mass flux (not on velocity). In the actual numerical model, the effect of adiabatic compression is expressed by a nondimensional parameter called "dissipation number", which is a measure of a ratio of the scale-height of density change to the thickness of the convecting layer. By conducting series of calculations of steady-state and time-dependent convection with systematically varying the Rayleigh and dissipations numbers, we confirmed that accurate results are successfully reproduced even for the cases with viscosity variations of several orders of magnitude. The present numerical code was chosen as one of the benchmark programs for a thermal convection of compressible fluids in two-dimensional Cartesian geometry.

3. Development of robust Stokes flow solver against viscosity jump by Schur complement approach with mixed precision arithmetic.

We develop an iterative solution technique for solving the Stokes flow problem with a large and sharp contrast in viscosity, for finite volume discretization in three dimensions. Robustness and scalability of the solver against viscosity contrast is important to treat the problems of the realistic geodynamical modeling. The large viscosity jump of the property on the sharp

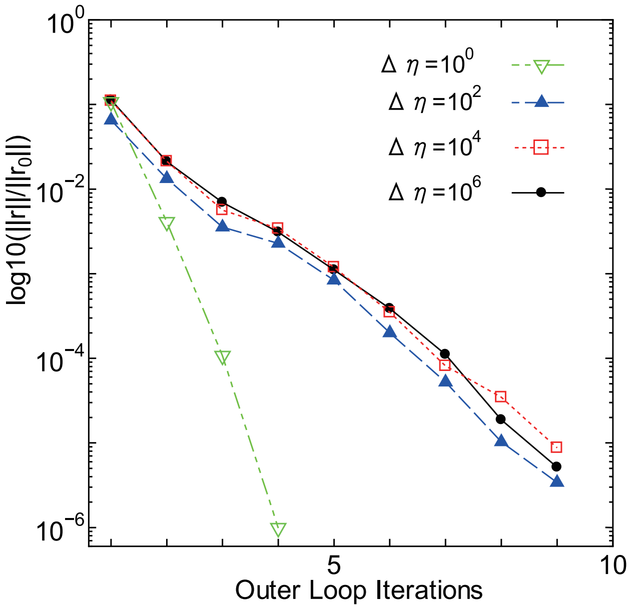

Fig. 2 Convergence history of the outer iterations by BFBt pre-conditioner for the falling block problems with different viscosity contrast.

interface is expected at the boundary interface, for example, between the upper mantle and the tectonic plate, or the treatment of free surface by the sticky air [6,7]. We use the Schur complement reduction with the Krylov subspace method. For the pre-conditioning of Schur complement, we employ scaled BFBt pre-conditioner as a scalable approach against the strong variation of viscosity profile. In addition, in order to improve the convergence of Krylov subspace method for momentum equation required for BFBt pre-conditioner, we propose to use a mixed precision technique. We implement quad precision arithmetic on Earth Simulator 2 by using the double-double precision method.

In order to see the performance of our iterative method, we solve the falling block benchmark test with a high viscosity jump $\Delta\eta$ within one discretized grid length. Figure 2 shows a history of outer solver residual. This plot presents that the number of iteration with the scaled BFBt pre-conditioner shows good scalability against the viscosity jump in the finite volume discretization.

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先端的固体地球科学シミュレーションコードの開発

プロジェクト責任者
陰山　　聡　　神戸大学大学院　システム情報学研究科

著者
陰山　　聡 *1，兵藤　　守 *2，古市　幹人 *2，宮腰　剛広 *2，亀山　真典 *3，大野　暢亮 *4
*1　神戸大学大学院　システム情報学研究科
*2　海洋研究開発機構　地球内部ダイナミクス領域
*3　愛媛大学　地球深部ダイナミクス研究センター
*4　海洋研究開発機構　地球シミュレータセンター

我々の最終的な目標は、地球シミュレータを駆使した大規模計算機シミュレーションを通じて、地球ダイナモとマントル対流をはじめとする地球内部全体の構造とダイナミクスを理解することである。そのために必要となる大規模並列計算手法や基本数値アルゴリズムの独自開発にも積極的に取り組んでいる。

地球ダイナモシミュレーション：本年度は前年度に引き続き、低エクマン数領域の地球ダイナモシミュレーションを追求した。その結果、前年度に発見したシート状のプルーム対流の周囲に西向きに流れる帯状流が形成されることを見出した。帯状流を伴うこの 2 重対流構造は、MHD ダイナモ作用で生成された磁場の下でも安定に存在することを確認した。エクマン数とレイリー数をさまざまに変えた計算を行った結果、この構造はエクマン数が低くレイリー数が高い場合にのみ現れる事が分かった。

マントル対流シミュレーション : ACuTE 法の改良を行い、地球深部の高圧力条件下におけるマントル物質の (静的な) 断熱圧縮及び粘性散逸の効果を取り入れる事を可能にした。ここで開発・改良したプログラムは、非弾性流体近似に基づいたマントル対流シミュレーションのベンチマークプログラムとして採用された。

プレート・マントル統合シミュレーション：プレートとマントルの間にある鋭い粘性差、ならびに数値惑星に向けた自由境界表面を動的に取り扱う事を目的として、鋭い粘性差（ジャンプ）に対して有効なストークス流れソルバーの開発を行った。具体的にはシュアー補行列を用いて、ストークス流れの行列問題を速度と圧力に分離し、対角項で規格化された BFBt 前処理行列を用いることにより、速度と圧力の結合する反復法の収束性を粘性差に対してスケーラブルなものにした。また局所的に double-double method による 4 倍精度演算を取り入れることで、計算時間をあまり増やすことなく効率的に粘性差に対してロバストな反復法を構築することに成功した。

キーワード : 地球地球ダイナモ , インヤン格子 , マントル対流 , 非弾性流体近似 , シュアー補行列 , 混合4倍精度