Numerical Simulations for Understanding of the Earth's Core Convection

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Some results of numerical experiments are reported for better understanding of the Earth's core convection and the geomagnetic field generation. We find a dynamo solution that produces a high ratio of magnetic to kinetic energy densities in a relatively low Ekman-number condition. The solution exhibits a very large-scale convection pattern, which we surmise is a general tendency in a convective regime of Earth-like high magnetic energy ratio. The processes of the magnetic field generation are analyzed in detail and the importance of the field stretching due to the large-scale azimuthal flow is pointed out. Numerical experiments of a spherical-shell dynamo with a thermally stable layer at the top are performed for a rough understanding of compositional convection. The results indicate that the existence of the stable layer suppresses the large-scale flow structure and decreases the energy ratio, and this tendency becomes more prominent when the Prandtl number is greater than unity. A comparison study is made between laboratory experiments and numerical simulations of liquid-metal Bénard convection in the presence of a magnetic field and rotation. Numerical simulations for the magnetoconvection problem well reproduce four characteristic convective regimes including a regime of random reversals of roll-type circulations that have found in the laboratory experiments.

Keywords: Geodynamo, Geomagnetic field, Core convection, Rayleigh-Bénard convection, Magnetohydrodynamics

1. Introduction

The primordial internal energy of the Earth is the primary origin of a variety of dynamic natural phenomena on our planet. The mantle convection, driven by both cooling of the Earth and internal radiogenic heating, takes the form of rigid-plate motions at the surface, and sometimes leads to disastrous earthquakes. As the mantle cools, the underlying liquid metallic core becomes thermally unstable and the resulting convective motion causes generation of the geomagnetic field and its time variations. As a result of cooling of the liquid core containing some lighter elements, a solid denser inner core grows from below, causing compositional instability in the liquid outer core too.

Our group has made step-by-step efforts to understand the dynamic state of the Earth's deep interior by means of realistic numerical modeling of the core and mantle convection. Each subsystem of this global heat engine can be treated independently because of vast disparity of convective time scales. We have mainly studied physically essential processes in each convective subsystem, and here report some progress of the research on the core convection, but our ultimate goal is to assess possible interactions between the core and the mantle and to illuminate behaviors of the overall solid-Earth system in geological time scales.

2. Emergence of a large-scale flow structure with a strong magnetic field

The generation of magnetic fields by convective motions of an electrically conducting fluid in a rotating spherical shell is a fundamental problem of geophysical and astrophysical fluid dynamics. Owing to recent advances in three-dimensional numerical magnetohydrodynamic (MHD) dynamo simulations, we have obtained many insights into how convection and magnetic field generation generally occur in rotating spherical shells [1].

The Earth's core convection is considered to be in a peculiar dynamical state known as the magnetostrophic state, where viscous and inertial forces are of little importance and Coriolis and Lorentz forces dominate instead. The ratio of viscous to Coriolis forces can be represented by the Ekman number, $E = \nu/2\Omega D^2$, where Ω is the angular velocity of the spherical shell, D is the shell thickness and ν is the kinematic viscosity. A realistic value of E in the Earth's core is $O(10^{-15})$, whereas most numerical dynamo simulations are performed at $E > 10^{-6}$. Similarly, the ratio of Lorentz to inertial forces can be represented by the ratio, ε, of magnetic to kinetic energy densities. The energy ratio in the outer core is believed to be $O(10^2)$ to $O(10^3)$, but most of previous numerical models could reach a state where ε was only around 10 or less. The Ekman number is an input parameter in simulating MHD dynamos and should be set as small as possible, but the energy ratio is an output parameter that depends on other dimensionless parameters and boundary conditions even when E is same.

Keeping the remoteness from the real Earth in mind, we here focus on understanding of fundamental physical processes of an MHD dynamo in a rapidly rotating spherical shell rather than attempt to predict the magnetic fields of planets and stars quantitatively. Although a similar approach has been taken in many papers [2-4], they are restricted to relatively large values of the Ekman number, $E > 10^{-4}$, except for a few numerical experiments [5, 6]. In this study, we extend our investigation to $E = 10^{-5}$ using our new code [7, 8].

At $E = 10^{-5}$, we obtain a strong dipolar solution that produces $\varepsilon = 55$ on average, which is fairly higher than in other previous models. The convection structure consists of a few large-scale retrograde flows in the azimuthal direction and some localized thin sheet-like plumes (Fig. 1). Considering our small E and large ε, we surmise that the flow structure and the dynamo processes simulated in this study well represent those in a magnetostrophic regime of a spherical-shell dynamo. A detailed term-by-term analysis of the magnetic field amplification processes shows that the magnetic field is amplified through stretching of magnetic lines, which occurs typically through four types of flow: the retrograde azimuthal flow near the outer boundary, the downwelling flow of the sheet plume, the prograde azimuthal flow near the rim of the tangent cylinder, and the cylindrical radially alternating flows of the plume cluster (Fig. 2). The current loop structure emerges as a result of stretching the magnetic lines along the magnetic field by the flow acceleration. The most remarkable effects of the generated

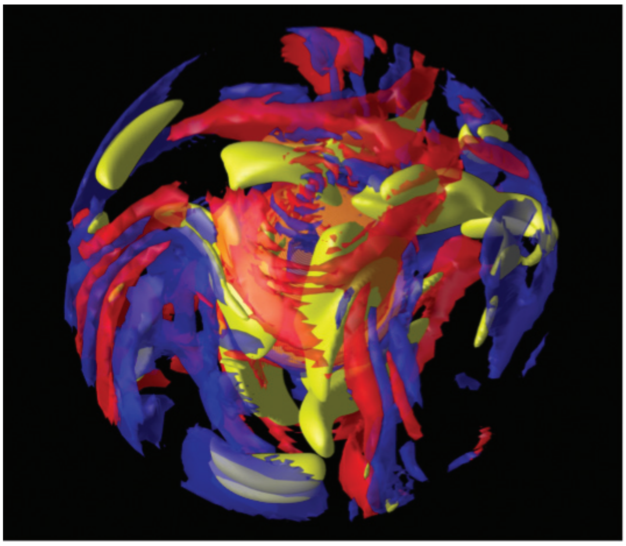

Fig. 2 Bird's-eye view of a snapshot of magnetic field and convection. The magnetic field is represented by yellow isosurfaces of $|B|$ = 2.5. The axial vorticity ω_z is also shown by isosurfaces. Positive (negative) values are denoted in red (blue).

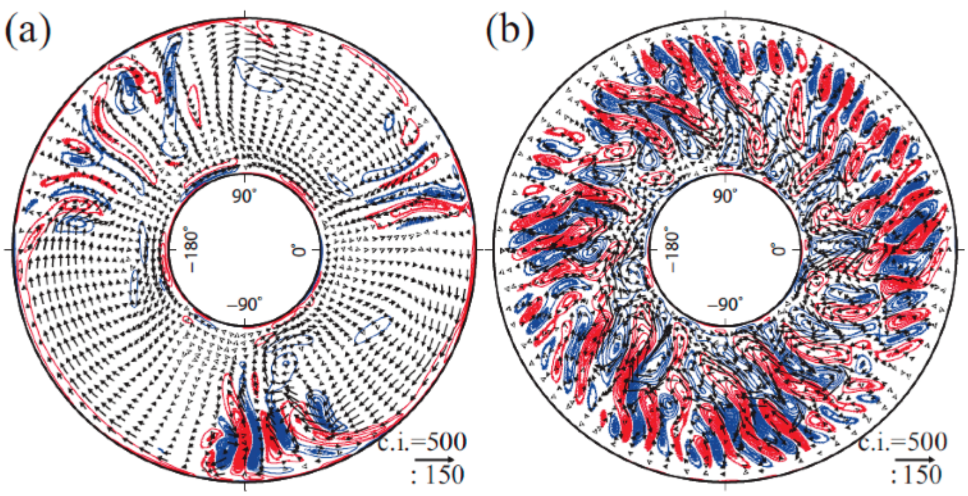

Fig. 1 Snapshot of convection structure on the plane at z = 0.2 viewed from the north, where z is the distance from the equatorial plane. Cases for dynamo (a) and purely thermal convection (b) are shown. The axial component of the vorticity (ω_z) is drawn by contour lines. Red (blue) lines represent positive (negative) values. The horizontal component of convection (u_h) is drawn by arrows.

magnetic field on the flow come from the strong azimuthal (toroidal) magnetic field. Similarities of the present model in the convection and magnetic field structures to previous studies at larger and even smaller Ekman numbers suggest universality of the dynamo mechanism in rotating spherical dynamos.

3. Conditions for a large-scale flow structure

The large-scale flow with a strong dipolar magnetic field as shown in the previous section is definitely a preferred convective state in low-E dynamos and we believe that the Earth's core convection is in such a state. However, this is not always the case of low-E dynamos. Sakuraba and Roberts [5] performed two low-E dynamo simulations with different boundary conditions for the core surface temperature and found that the uniform surface-temperature (UST) model did not produce a large-scale flow pattern contrary to the case of the uniform heat-flux (UHF) boundary condition. Here we show that a similar phenomenon can occur when there is a thermally stable layer at the top of the core. In the present model, the heat flux is uniform at the core surface like the UHF model but zero everywhere. We assume that the outer core is heated from below but a uniform volumetric heat sink in the fluid layer compensates the basal heating. A similar model has been considered to mimic compositional convection (CC) caused by release of lighter elements from the inner core surface [9].

Using dimensionless parameters comparable to the previous study [5], we find that the "CC" (zero-flux) model produces a relatively weaker magnetic field compared to the thermal-convection (TC) model that uses the UHF condition (Fig. 3). The large-scale structure, as seen in the TC model, does not develop very much. The characteristics of the CC model are rather similar to those of the previous UST models [5, 6], in which the energy state tends to be equipartition ($\varepsilon = 1$). If our zero-flux model represents Earth-type compositional convection, the Prandtl number (Pr) should be greater than unity because the diffusion coefficient of the light element is much smaller than

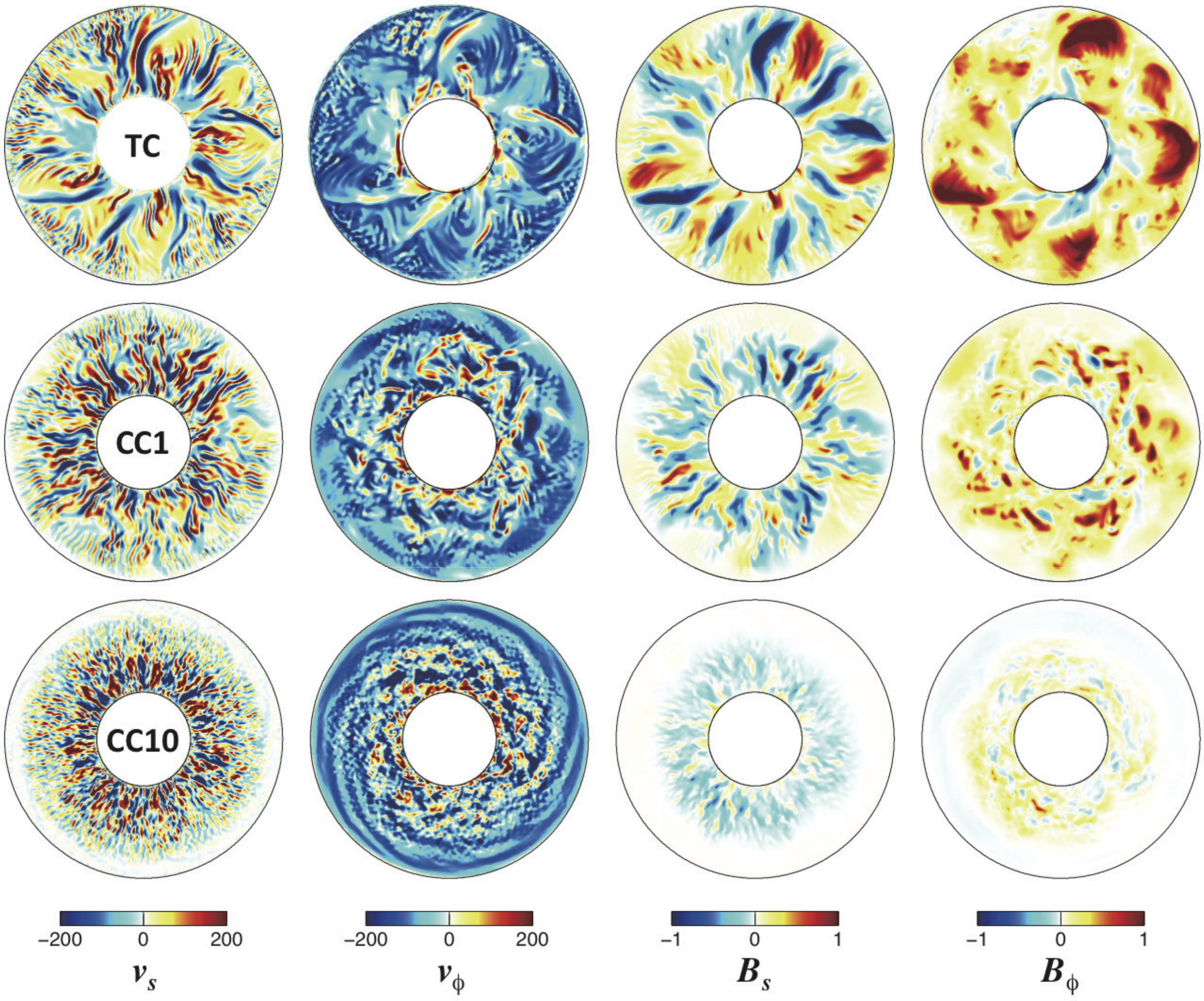

Fig. 3 The convective structures of our TC model of Pr = 1 (top), CC model of Pr = 1 (middle), and CC model of Pr = 10 (bottom). From left to right, the cylindrically radial and the azimuthal velocities, the radial and the azimuthal magnetic fields are shown at z = 0.1. The TC model is identical to our previous UHF model [5]. In the CC model, the reference temperature gradient is assumed to be proportional to $r - 1/r^2$, where r is the radius normalized by the core radius.

the thermal diffusivity. Our numerical experiment of Pr = 10 indicates that the suppression of the large-scale structure and the decrease of the magnetic energy are more evident than the case of Pr = 1.

4. Liquid-metal thermal convection in the presence of a magnetic field and rotation

The study on the nature of thermal convection of liquid metals is essential to understand the dynamics of the Earth's outer core, where Lorentz and Coriolis forces dominate, and experiments on turbulent flows in the presence of a magnetic field (Case I) and rotation (Case II) are of great importance. We performed both laboratory experiments and numerical simulations of Rayleigh-Bénard convection of highly conductive and low-Pr fluids. In many dynamo simulations, Pr is set to be around unity, but the actual value of Pr for liquid metals is $O(10^{-2})$. Here we provide fundamental dataset for realistic liquid metal convection.

Theoretical studies [10, 11] have shown that for highly conductive fluids like liquid metals the axis of the roll structure is forced to align in the direction of the applied magnetic field in Case I, and the Rayleigh number, Ra, at which transition to time-dependent flows occurs is increased. Even at such higher Ra that turbulence develops when the magnetic field is absent, suppression of turbulence and formation of anisotropic flow structure are expected under a strong magnetic filed.

In the laboratory experiments of Case I at higher Ra, we observed four flow regimes depending on Ra and the Chandrasekhar number, Q, which measures the square of the imposed field intensity; that is, a steady two-dimensional roll regime, an oscillatory roll regime, a regime of random reversals of roll-type flow circulations, and a turbulent regime with large-scale structures [12]. In the regime of random reversals, a roll-type structure is dominant in most of the time and the flow keeps its two-dimensionality. However, a new circulation suddenly emerges near the sidewall, triggering a global reorganization that causes reversal of the overall flow direction. The process of the reversal is over in a relatively short time, which is comparable to the circulation time of a convection roll. The reversals of the flow direction occur randomly with a typical time interval between reversals much longer than the circulation time. We performed numerical simulations of magnetoconvection using parameters and boundary conditions comparable to the laboratory experiments and successfully reproduced these four flow regimes (Fig. 4). We analyzed details of the three-dimensional structure in each regime, and clarified the cause of these variations.

In Case II, linear studies on the onset of thermal instability

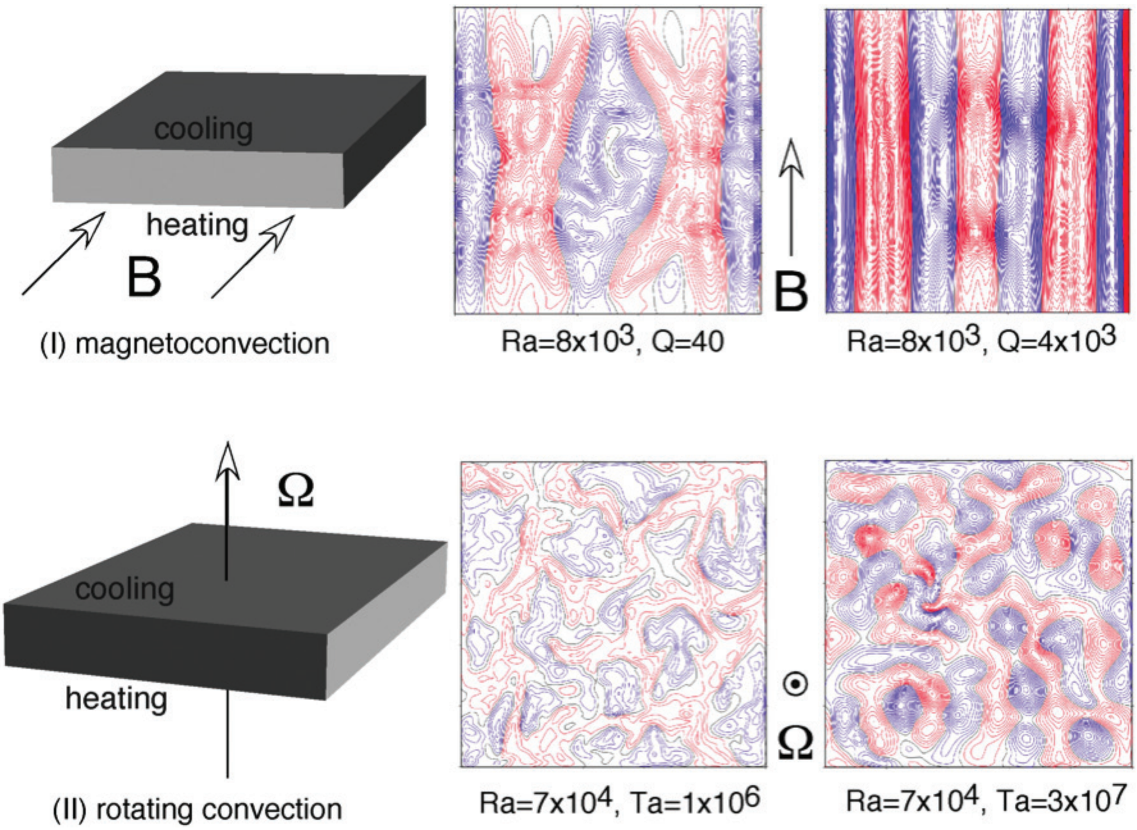

Fig. 4 Setting and result of numerical simulations. (I) Convection under uniform horizontal magnetic field. (II) Convection on a rotating system. Pr of the fluid is 0.025. The contours are flow velocity in vertical direction; red indicates the upward flow, and blue indicates downward flow. In (I), the flow pattern is restricted to be rolls in the direction of magnetic field for larger Q. In (II), the pattern is composed of vortices and gets shorter for larger Ta.

[10] indicate that the critical Rayleigh number is proportional to $Ta^{2/3}$ in an asymptotic form, where $Ta = 4E^{-2}$ is the Taylor number, and overstability occurs for $Pr < 0.6$. The horizontal scale of convection pattern becomes smaller with the increase of *Ta*. Our numerical results well reproduced the relation of the critical *Ra* on *Ta*, depending on *Pr* (Fig. 4). Convection patterns above the critical *Ra* are consistent with that observed in the laboratory experiments and theoretical studies.

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地球コア対流の理解のための数値シミュレーション

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地球の冷却に起因するマントル対流とコア対流は、固体地球上に生起するさまざまな自然現象の原因をつかさどる、根本的な物理プロセスであると考えられる。われわれはこれまで、マントルとコアという２つの対流系を、より実際の地球に近い条件のもとで、数値的にモデリングする研究をおこなってきた。本年度は、そのうちとくにコアの対流に焦点をあてて研究をおこなったので、その成果を報告する。

地球のコア対流は、運動方程式における粘性項と慣性項とが無視できる、いわゆる磁気地衡流状態にあると考えられる。コリオリ力に対する粘性力の比はエクマン数という無次元数であらわされ、これが小さいことがまず重要である。さらに慣性力に対するローレンツ力の比は、端的には対流の運動エネルギー密度に対する磁気エネルギー密度の比としてあらわすことができ、この比が大きいことも磁気地衡流の条件である。われわれは、比較的小さいエクマン数のもとでダイナモの数値シミュレーションをおこない、これまでの同様の研究に比べて、著しく大きい磁気エネルギー密度比をもつ解を見いだした。そこでは、東西方向の波数が小さい、大規模対流構造が顕著であり、これが磁気地衡流の一般的性質であると推察する。磁場の生成機構を詳細に調べたところ、とくに西向きの大規模流による磁力線の引き延ばし効果が重要であることがわかった。

地球のコアでは、内核成長にともない軽元素が液相に濃集する結果、組成対流もおこっていると考えられる。これをモデル化するために、コア表層に熱的に安定な層がある場合のダイナモシミュレーションをおこなった。その結果、通常の熱対流モデルでみられる大規模対流構造が顕著でなくなり、磁場も弱まることがわかった。これは、コア表面の温度を水平方向に一様とする境界条件のもとでの熱対流が駆動するダイナモの解と、特徴がよく似ており、磁気エネルギーと運動エネルギーとが等分配的になる傾向を示唆している。さらにこの傾向は、プラントル数を大きくするとより顕著になる。

コアの乱流的なふるまいをよりよく理解するために、液体金属のベナール対流の室内実験をおこない、それをさらに数値的にモデリングし、解析した。一様磁場を印加した磁気対流の場合について、レイリー数と印加磁場強度を系統的に変えてシミュレーションをおこなったところ、室内実験でみられた４つの特徴的な対流状態がよく再現された。そのうちあるパラメータ領域では、印加磁場方向にそろったロール状対流セルが、比較的短時間のうちに、循環する方向が逆転するという興味深い現象が再現された。

キーワード : 地球ダイナモ , 地磁気 , コア対流 , レイリー・ベナール対流 , 磁気流体力学