Development of Advanced Simulation Methods for Solid Earth Simulations

Project Representative
Mikito Furuichi Institute for Research on Earth Evolution, Japan Agency for Marine-Earth Science and Technology

Authors
Mikito Furuichi[*1], Akira Kageyama[*2], Takehiro Miyagoshi[*1], Masanori Kameyama[*3], Nobuaki Ohno[*4], Takashi Nakagawa[*1], Kensuke Yokoi[*5] and Daisuke Nishiura[*1]

∗1 Institute for Research on Earth Evolution, Japan Agency for Marine-Earth Science and Technology
∗2 Graduate School of System Informatics, Kobe University
∗3 Geodynamics Research Center, Ehime University
∗4 Graduate School of Simulation Studies, University of Hyogo
∗5 Cardiff University

Numerical planet: We have developed simulation code for a long time scale core formation under a self-gravitating field with robust stokes flow solver and sticky air treatments. In this FY, we simulated our new scenario of a formation of anomalous region above CMB during the core formation and compare the results with 1-D semi-analytical solutions. In addition, energy equation is implemented to our code to investigate an internal thermal heterogeneity of the early Earth system. We also developed a new particle-fluid coupled simulation code targeting magma dynamics with solidifying and melting process. Geodynamo: We have developed new simulation code of magnetohydrodynamic dynamo in a spherical shell which can treat rotational speed variation or length-of-day (LOD) variation from Yin-Yang dynamo code which is produced in this project. The motivation is to study the relation between geomagnetic field variation and LOD variation caused by redistribution of water mass due to climatic change with ice and non-ice ages. We found that LOD variation leads to geomagnetic field variation. The detail mechanism of geomagnetic field variation was also revealed by the detail analysis.

Keywords: Geodynamo simulation, Length-of-Day variation, Yin-Yang grid, Mantle convection, Core formation, Stokes flow, mixed precision arithmetic, Two-phase flow, Permeable flow, DEM

1. Core formation simulation: numerical planet (Furuichi)

The formation of a metallic core is widely accepted as the major differentiation event during planetary formation. In order to investigate the global core formation process of the planetary interior, we have developed a numerical code for solving the Stokes flow in 3D. In our simulation, global sinking of the dense metal-rich material over long time scale is captured. Our new solution code named "Nplat", can solve the Stokes flow motion with free surface under a self-gravitation by using the sophisticated algorisms designed for ES2 [1-4]. Expressing the free surface motion, we employ the sticky air approach. Self-gravitating force is obtained by solving the gravity potential equation. For solving the momentum and continuity equations, we developed an iterative Stokes flow solver, which is robust to problems including jumps in the viscosity contrast [3]. Our solver design consists of an inner and outer solver utilizing a strong Schur complement preconditioner and the Arnoldi type Krylov subspace method preconditioned with geometric multigrid method. We enhance the robustness of the inner solver for the velocity problem with a mixed (quad-double) precision Krylov kernel calculation. As the high precision calculation method, we employ the double-double precision algorithm which is faster than normal quad arithmetic using a vector register or cache memory. Our mixed precision method improves the convergence of Krylov method without significantly increasing the calculation time.

Last year, we proposed the new hypothetical scenario that dynamical change of the internal structure during core formation could lead to the remaining dense less differentiated material on the CMB before the giant impact event (Fig. 1 a, b) form our simulation results. Then in order to validate our simulation result, we create a 1- D semi-analytical model and compare the results of both models. Good agreements between 3-D and 1-D models are obtained when a viscous gravitational relaxation creates smoothed layered material structure during impact intervals which is essentially 1-D layered problem. We also develop the code "Nplat" to deal with energy equation for investigating not only the compositional but also thermal heterogeneity during the core formation events (Fig. 1 c).

2. Granular media simulation: numerical planet (Furuichi)

A fluid-particle two-phase flow has been widely studied in geodynamics, because particle-saturated fluid layer is important for understanding the dynamics of solidifying and melting process in the magma chamber or magma ocean. In order to deal with such particle-fluid systems as the geodynamical modeling in 3-D, we lunch a new sub-project for coupled simulation code of Finite Difference method (FDM) for fluid flow and Discrete Element method (DEM) for solid particles. Although this type of simulation method has been developed in the engineering field especially for high Reynolds number problems, the method for viscous granular media over long time scales has not yet been fully addressed.

We employ empirically derived a coupling term between fluid flow and particle motion providing good fit with experimental data of the creeping flow. When this coupling force is directly introduced to the normal DEM equation of particles, the normal DEM-fluid formulation requires a solution of dumped oscillation with a small time step dt $\sim 1/\eta$ for high fluid viscosity η. Thus the normal formulation is not suitable for our target problems. We therefore propose to drop off the inertial term from the equation of particle motion likewise the Stokes flow. With this approach, we can employ the large dt$\sim\eta$ for the problems with highly viscos fluid. In addition, since our original solution algorithms for both of FDM and DEM are designed for the massively vector parallel architectures, we can solve large size of problems in 3-D geometry (Fig. 2).

3. Geodynamo simulation (Miyagoshi)

Geomagnetic field protects the Earth's surface environment from harmful cosmic rays and solar winds. The barrier effect depends on the intensity of geomagnetic field. Therefore, future changes in the intensity of geomagnetic field are a matter of serious concern for life on earth. The intensity of magnetic field is deeply associated with the habitability of the planet.

The intensity of geomagnetic field is not temporally constant and has experienced temporal changes of ~50% in maximum which is discovered in recent about one million years [5]. However, causes and mechanisms of changes of geomagnetic field intensity are still mysterious. For one of the keys, the correlation between climatic changes with ice and non-ice ages and geomagnetic field intensity changes has been pointed out [6][7]. The redistribution of water mass on the Earth with ice and non-ice ages causes the change of inertial moment of the Earth. It leads to rotational speed variation or length-of-day (LOD) variation. The viscosity of fluid metal in the core is low, so the convection in the core is greatly influenced by the rotation and its change. The amplitude of the perturbation from those LOD changes will be very large, which is nearly equal to the convection velocity [6] because of the low Ekman number (strong rotation effect) in the core. This perturbation would affect the convection in the core, and thus magnetohydrodynamic (MHD) process which produces geomagnetic field. Actually, correlations between geomagnetic field variations and Milankovitch cycle which is associated with the climatic variation have been pointed out. However,

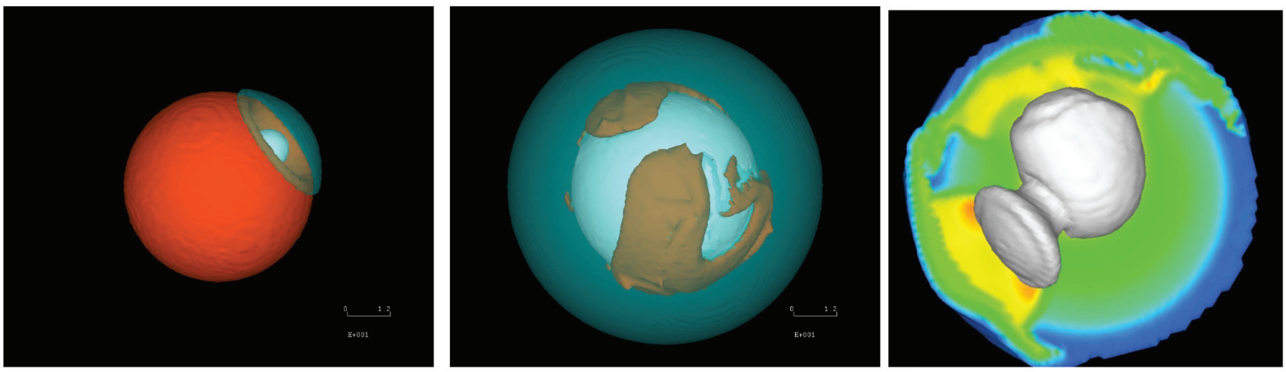

Fig. 1 Snapshot of the simulated evolution of the planet with core formation by the impact (a) initial state with first impact; (b) final state with a radius r = 0.8 Re; (c) thermal structure during the core formation. The white and orange isosurface represent metal-rich and proto planet materials respectively. The semi-transparent iso-surface indicates the silicate rich material.

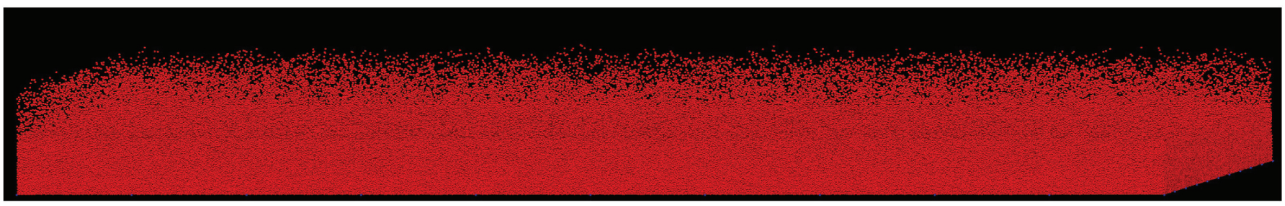

Fig. 2 Snapshot of the simulated falling 5 million particles in very viscous fluid (η = 8.5e3[Pas])

there have been no MHD numerical geodynamo model with LOD variation. In this project, we have developed numerical geodynamo simulation code in a spherical shell with LOD variation from Yin-Yang dynamo code which is produced in this project [8][9].

Until the last fiscal year, we have succeeded numerical geodynamo simulations with LOD variation by the newly developed code. In this fiscal year, we have analyzed details of the geomagnetic field variation. We especially focused on the investigations for variations of physical values except for geomagnetic field intensity, and understanding the mechanism of geomagnetic field change. The employed parameters are, the Ekman number $O(10^{-5})$, Rayreigh number $O(10^{8})$, and both Prandtl and magnetic Prandtl numbers are unity. To the magnetic dipole moment dominant solution like the Earth, LOD variation is inputted. We found that geomagnetic field variation arises and its amplitude depends on the Ekman number. To understand the mechanism of geomagnetic field variation, we performed calculations for high time cadence output with calculations of other important physical values. We found that LOD variation firstly affects the convection of fluid metal, and confirmed the detail for the change of convection activity. The relation between convection activity and geomagnetic field variation was also found. There are changing physical values except for geomagnetic field intensity, for example, heat flux at the boundary. And we found that the heat flux and other variations are deeply associated with geomagnetic field change. The first report for details will be published as a Letter in the next fiscal year.

4. Mantle convection simulation Miyagoshi & Kameyama

For mantle convection in this project, in this fiscal year we especially focus on the mantle dynamics in super-Earths. Since the first discovery of the extra-solar planet at 1995, many super-Earths have been found. Super-Earths have a high density like the Earth, not a gas planet as Jupiter, and the mass of the super-Earths is up to about ten times the Earth's mass. It is an interesting question whether those super-Earths are habitable or not. There are many factors for habitability on super-Earths, and it is no doubt that mantle convection in super-Earths is one of the most important factors to know each super-Earths are habitable or not. Because it determines the surface environment and cooling rate of the fluid metal core in super-Earths. For cooling rate of the core, it is associated with the intensity of super-Earth's magnetic field which protects surface of the planet from harmful cosmic rays or star winds.

In this fiscal year we especially focus on the large adiabatic compressibility effect in super-Earth's mantle. The radius of super-Earths is larger than the Earth, which indicates that the thickness of their mantle far exceeds the thermal scale height. This suggests that the adiabatic (de)compression effect is important for the dynamics in super-Earth's mantle compared with the Earth's mantle. The intensity of the adiabatic (de) compression effect is estimated by the dissipation number $D_i=\alpha gd/C_p$, which is the non-dimensional parameter. Here α is the thermal expansivity, g is the gravity, d is the thickness of the mantle, and C_p is the specific heat in constant pressure, respectively. In the Bousinnesq approximation, which is usually used in mantle convection simulation, adiabatic (de) compression effect is neglected and D_i is regarded as zero. In the Earth D_i~0.5, so the Boussinesq approximation is well method. On the other hand, in super-Earths whose mass is ten times the Earth's mass, D_i reaches 5, so adiabatic (de)compression effect is extremely larger than the Earth and the effect cannot be neglected to understand the mantle dynamics. Almost all previous studies for mantle convection on super-Earths use Boussinesq approximation. Only Tackley et al. (2013) [10] uses annelastic approximation which focus on the post-perovskite rheology and plate tectonics in super-Earths. However, the adiabatic (de)compression in their model is still weak (about twice the effect in the Earth), and they do not focus on and not investigated the effect of adiabatic (de)compression. We have performed numerical simulations on mantle convection with high adiabatic compressibility, high Rayleigh number, and strong temperature-dependent viscosity due to the large size of super-Earths. We found that the thermal structure and mantle dynamics are totally different from those which are previously known, including Tackley et al. (2013) [10].

The model is as follows. Thermal convection of compressible infinite Prandtl number fluid is solved in a rectangular box whose aspect ratio is 4:1, by the ACuTEMAN code which is produced in this project [11][12]. One of the strong points of the ACuTEMAN code is that it can calculate high Rayleigh number and strong temperature-dependent viscosity model with good precision. In super-Earths whose mass is ten times the Earth's mass, the radius becomes about twice than the Earth. The temperature difference between the surface and bottom is thought to become also large by its large radius. For these contributions, the Rayleigh number in super-Earths will become considerably larger than the Earth. The Rayleigh number in our model is 10^9, 10^{10}, and 6×10^6 (which is the Earth's Rayleigh number, for comparing with other cases). The temperature-dependent viscosity contrast between the surface and the bottom of the mantle is up to 10^7. We assume that the mass is ten times the Earth's mass. The D_i is 5. The depth-dependent thermal expansivity is taken into account. The density profile we use is suggested by Valencia et al. (2006) [13]. The ratio of thermal expansivity between the surface and the bottom is about 10, and the ratio of density is about 3. The temperature is fixed at the bottom and the surface. The employed grid number is 1024 (horizontal) and 256 (vertical).

We have calculated about 15 cases in various R_a and temperature-dependent viscosity r. The needed calculation time depends on R_a and r. For example, in R_a=10^{10} and r=10^4, ~330 hours times 2 nodes equal ~660 node hours is needed. In the most heavy parameter case, more than ~450 hours times 2 nodes equal ~900 node hours is needed (not yet finished). If our model is extended in three dimensional box with the grid number 1024 times 1024 times 256, the needed node hours is estimated as 660 node hours times 1024 is equal to ~ 680000 node hours even for one case. This is enormously larger than the total node hour we can use in this project per year (39000 node hours). So, larger scale simulations in three-dimensional model are a next challenge with the new Earth Simulator. In this fiscal year, we analyzed detail for some parameter cases and have found that the thermal structure and the mantle dynamics in super-Earths is totally different from that in the Earth by strong adiabatic (de)compression effect. For example, the efficiency of heat transport by mantle convection is considerably different from the Earth's one. This will greatly affect the thermal evolution history and habitability on super-Earths. We also found that the temperature-dependent viscosity is also important for mantle dynamics. Compared with no temperature-dependent viscosity model (Tachinami et al. 2013, submitted) [14], the thermal structure and the state of mantle dynamics is considerably different. The preliminary first Letter and the full paper will be published in the next fiscal year.

References

[1] M. Furuichi, M. Kameyama, and A. Kageyama, J. Comput. Phys. 227, 4977–4997, 2008.

[2] M. Furuichi, M. Kameyama, and A. Kageyama, Phys, Earth Planet. Inter. 176, 44-53, 2009.

[3] M. Furuichi, D.A. May, and P. J. Tackley, J. Comput. Phys, 230, 8835-8851, 2011.

[4] M. Furuichi, proceedings of International Conference on Computational Science, ICCS 2011, 2011.

[5] Toshitsugu Yamazaki, (In Japanese) Journal of Geography, vol. 114, pp.151-160, 2005.

[6] Yozo Hamano, (In Japanese) Kagaku, vol.62, No.1, pp.14-18, 1992.

[7] C. S. M. Doake, Nature, vol. 267, pp.415, 1997.

[8] Akira Kageyama, Takehiro Miyagoshi, and Tetsuya Sato, Nature, vol. 454 pp. 1106-1109, 2008.

[9] Takehiro Miyagoshi, Akira Kageyama, and Tetsuya Sato, Nature, vol. 463, pp. 793-796, 2010.

[10] Tackley, P. J., Ammann, M., Brodholt, J. P., Dobson, D. P., and Valencia, D., Icarus, 225, 50, 2013.

[11] M. Kameyama, A. Kageyama, and T. Sato, J. Comput. Phys. 206, 162-181, 2005.

[12] M. Kameyama, J. Earth Simulator 4, 2-10, 2005.

[13] Valencia, D., O'Connell, R. J., and Sasselov, D. D., Icarus, 181, 545, 2006.

[14] Tachinami, C., Ogawa, M., and Kameyama, M., Icarus, submitted, 2013.

先端的固体地球科学シミュレーションコードの開発

プロジェクト責任者
古市　幹人　　海洋研究開発機構　地球内部ダイナミクス領域
著者
古市　幹人 *1，陰山　　聡 *2，宮腰　剛広 *1，亀山　真典 *3，大野　暢亮 *4，中川　貴司 *1，
横井　研介 *5，西浦　泰介 *1
*1　海洋研究開発機構　地球内部ダイナミクス領域
*2　神戸大学　大学院システム情報学研究科
*3　愛媛大学　地球深部ダイナミクス研究センター
*4　兵庫県立大学　大学院シミュレーション学研究科
*5　カーディフ大学

我々は、地球シミュレータ２の特徴を生かした先端的なシミュレーションコードを開発することで惑星の形成過程から地球ダイナモの生成並びにマントル対流の駆動といった固体地球科学ダイナミクスの諸問題に包括的に取り組んでおり、またコード開発で得られた知見を積極的に他分野で応用することを目指している。

数値惑星：コア形成時のグローバルスケールの金属層の落ち込みを再現する 3 次元シミュレーションコードを開発している。本年度は、前年度までに開発したコードの検証を擬一次元モデルを用いて行ったほか、新しくエネルギー方程式が扱えるように拡張した。さらに、新たにマグマの溶融･結晶化を扱うことを目的とした混相流コードの開発を行った。

ダイナモ：気候変動による氷床消長に伴う自転速度変動が地球磁場変動を引き起こしているというアイデアに基づき、Yin-Yang ダイナモモデルを発展させたコードで計算を行っている。本年度は地球磁場変動のメカニズムについて詳細に調べるための計算を行い、磁場強度以外にも変動する複数の重要な物理量が存在する事、またそれらが磁場変動のメカニズムと深く関係している事が分かり、さらに自転速度変動がどのようにして磁場変動を引き起こすのかの過程をほぼ明らかにする事が出来た。

マントル：続々と発見されている系外惑星スーパーアース（地球と同程度の密度を持ち、質量が地球の約 10 倍までの地球型惑星）のマントル対流シミュレーションを行った。特に、サイズの大きさから期待される大きな断熱圧縮加熱、地球よりもかなり高いレイリー数、および強い粘性温度依存性に着目して計算を行った。その結果、強い圧縮性の効果により、スーパーアースのマントル対流の様式および対流による熱輸送効率は、地球のマントルとかなり異なる事が分かった。

キーワード: 地球ダイナモ, インヤン格子, 地球回転変動, マントル対流, コア形成, ストークス流れ, 2 相流, 浸透流, DEM