Disappearance of Surface Banded Structure Produced by Thermal Convection in a Rapidly Rotating Thin Spherical Shell

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In order to investigate the origin of the banded structures observed at the surface of Jupiter and Saturn, we perform a numerical simulation of Boussinesq thermal convection in a whole thin spherical shell. The Prandtl number, Rayleigh number, Ekman number and radius ratio are 0.1, 0.05, $3x10^{-6}$ and 0.85, respectively. The boundary conditions at the inner and outer boundaries are free-slip and horizontally uniform temperature. We do not assume any longitudinal symmetry adopted in a previous study.

As the time integration proceeds, there appear an equatorial prograde surface zonal jet and alternating banded zonal jets in mid and high latitudes simultaneously, which are similar to the zonal jet structures observed in Jupiter and Saturn. However, extending time integration further, mid- and high- latitudinal regions are entirely accelerated eastward, zonal banded structures disappear, and finally, one broad eastward zonal jet appears in mid- and high- latitudes of each hemisphere. This result suggests that it is difficult to explain surface zonal flow structures of Jupiter and Saturn only by deep convection in the interior of the planets.

Keywords: Jupiter, Saturn, banded structure, equatorial prograde jet, Rossby waves

1. Introduction

Surface flows of Jupiter and Saturn are characterized by the broad prograde zonal jets around the equator and the narrow alternating zonal jets in mid- and high-latitudes. It is not yet clear whether those surface jets are a result of fluid motions in the "shallow" weather layer, or they are produced by convective motions in the "deep" region.

"Shallow" models consider atmospheric motions driven by the solar differential heating and the intrinsic heat flow from the deeper region under the assumption of hydrostatic balance in the vertical direction as a result of the thin atmospheric layer compared with the radius of the planet. These models can produce narrow alternating jets in mid- and high-latitudes, while the equatorial jets are not necessarily prograde.

On the other hand, "deep" models, which describe thermal convection in rapidly rotating spherical shells whose thickness is comparable to the radius of the planet, can produce equatorial prograde flows easily, while it seems to be difficult to generate alternating jets in mid- and high-latitudes.

Heimpel and Aurnou (2007) [1] proposed thin spherical shell models and show that the equatorial prograde zonal jets and alternating zonal jets in mid- and high-latitudes can be produced simultaneously when the Rayleigh number is sufficiently large and convection becomes active even inside the tangent cylinder. However, they assumed eight-fold symmetry in the longitudinal direction and calculated fluid motion only in the one-eighth sector of the whole spherical shell. Such artificial limitation of the computational domain may influence the structure of the global flow field. For example, alternating zonal flows may not develop due to sufficient upward cascade of two-dimensional turbulence, or stability of mean zonal flows may change with the domain size in the longitudinal direction.

Therefore, in the present study, we try to perform numerical simulations of thermal convection in the whole thin spherical shell domain.

2. Model

We consider Boussinesq fluid in a spherical shell rotating with a constant angular velocity Ω. The non-dimensionalized governing equations consist of equations of continuity, motion, and temperature [2]. The non-dimensional parameters appearing in the governing equations are the Prandtl number, $Pr = \nu/\kappa$, the Ekman number, $Ek = \nu/(\Omega D^2)$, and the modified Rayleigh number, $\mathrm{Ra} = \alpha\, g_o\, \Delta T/(\Omega^2 D)$, where ν, D, κ, α, r_o, g_o, and ΔT are the kinematic viscosity, the shell thickness, the thermal diffusivity, the thermal expansion coefficient, the outer radius of the shell, the acceleration of gravity at the outer boundary, and the temperature contrast between the boundaries, respectively.

The spherical shell geometry is defined by the radius ratio, $\chi = r_i/r_o$, where r_i is the inner radius of the shell. The boundary conditions at the inner and outer boundaries are free-slip and horizontally uniform temperature.

3. Numerical method

The numerical method used in the present study is a traditional spectral method [3]. The toroidal and poloidal potentials of velocity are introduced in order to satisfy the equation of continuity. The velocity potentials and the temperature field are expanded horizontally by the spherical harmonic functions and radially by the Chebychev polynomials. The non-linear terms of the governing equations are evaluated in the physical space and are converted back into the spectral space. The time integration is performed using the Crank-Nicholson scheme for the diffusion terms and the second-order Adams-Bashforth scheme for the other terms.

4. Experimental Setup

The combinations of the non-dimensional parameters and the resolutions of the model used in this study are summarized in Table 1. The initial condition of the velocity field is state of rest and that of the temperature field is the steady conduction solution with random temperature perturbations.

In order to save computational resources, we use hyperdiffusion with the same functional form as the previous studies [1,4]: $\nu = \nu_0$ for $l \leqq l_0$ while $\nu = \nu_0\,[1+\varepsilon(l-l_0)^2]$ for $l > l_0$, where l is total horizontal wave number. We increase the parameter l_0 step by step from 21 to 42, 85, and 170.

Table 1 Summary of the nondimensional parameters and the resolutions used in this study. "HA2007" means Heimpel and Aurnou (2007).

Parameters	Our experiment	HA2007
Radius ratio χ	0.85	0.85
Prandtl number *Pr*	0.1	0.1
Rayleigh number *Ra*	0.05	0.05
Ekman number *Ek*	3x10^{-6}	3x10^{-6}
Resolution (lon,lat,rad)	1024x512x65	128x512x65
Hyper viscosity	l_0=21-170, ε=0.01	?

5. Results

Figure 1 shows snapshots of the surface zonal flow, temperature and mean zonal flow at t=47630 (about 7500 rotation) for Ra=0.05, Pr=0.1, Ek=3x10^{-6}, and χ=0.85. An equatorial prograde surface zonal jet and alternating banded zonal jets emerge, which seem to be consistent with the result of Heimpel and Aurnou (2007).

Figure 2 shows time variation of surface zonal flows when the time integration is further extended. It can be observed that mid- and high- latitudinal regions are entirely accelerated eastward, zonal banded structures disappear, and finally one broad eastward zonal jet appears in mid- and high- latitudes of each hemisphere at t=80430 (around 12800 rotation). Note that further time integration is necessary to obtain statistically steady state since kinetic energy still increases in the final state of the present calculation.

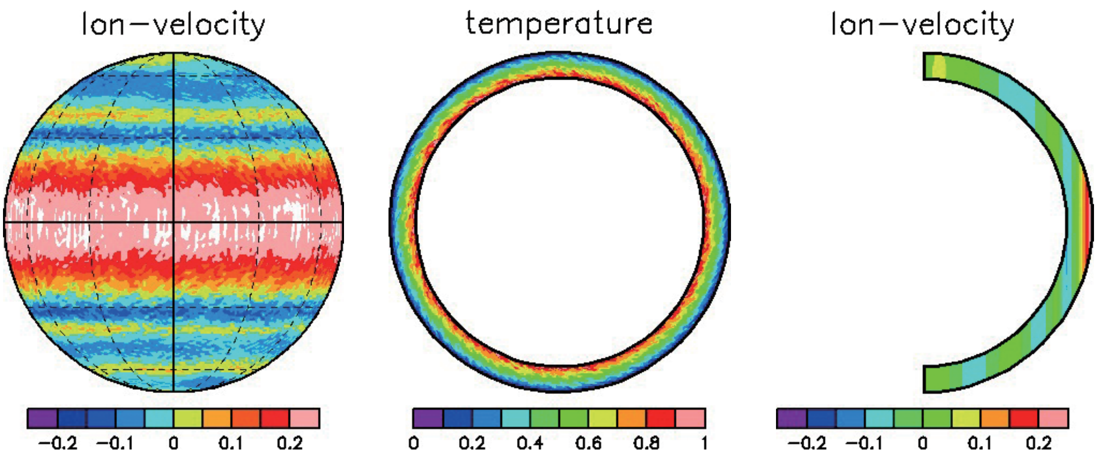

Fig. 1 Snapshots of surface zonal flow (left), temperature at the equatorial cross section (center) and mean zonal flow (right) at t=47630 for Ra=0.05, Pr=0.1, Ek=3x10^{-6}, and χ=0.85

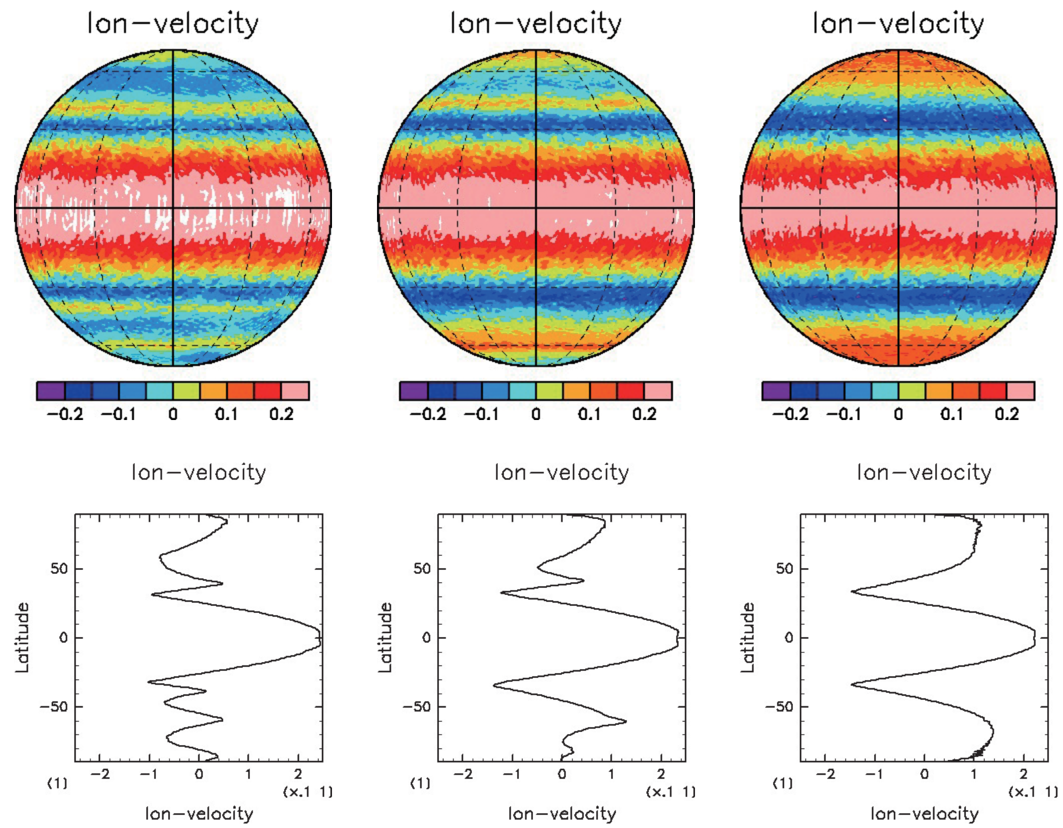

Fig. 2 Snapshots of surface zonal flows (upper panels) and mean zonal flows at the surface (lower panels). From left to right, t=47630, 64030, and 80430 for Ra=0.05, Pr=0.1, Ek=3x10^{-6}, and χ=0.85.

6. Concluding Remark

We have tried to perform numerical simulations of thermal convection in the whole thin spherical shell domain. It is found that the banded structures in mid- and high-latitudes are prominent in a transient state, however, the banded structures disappears when the time integration is further extended. This result suggest that it is difficult to explain surface zonal flow structures of Jupiter and Saturn only by deep convection in the interior of the planets.

Formation of these broad zonal jets may be attributed to the angular momentum transport in the cylindrically outward direction by topographic Rossby waves, which are excited by thermal convection inside the tangent cylinder.

Acknowledgement

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高速回転する薄い球殻内の熱対流により引き起こされる表層縞状構造の消滅

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木星･土星表面に観測される縞状構造の成因を探るために、薄い回転球殻ブシネスク熱対流の全球数値シミュレーションを行った。時間積分を行ったところ、木星・土星に見られるような赤道ジェットと中高緯度の縞状構造が同時に出現した。しかしながら、さらに時間積分を進めると、中高緯度が全体に西風加速され縞状構造が消滅していき、最終的に南北中高緯度の両半球にそれぞれ幅広の 1 本の帯状流が形成された。中高緯度の幅広な帯状流の形成は、対流運動で駆動された回転軸方向に一様な 2 次元渦が地形性ロスビー波として外側へと伝播し、東風角運動量を抜き去ることで西風加速が生じていると考えられる。

1. はじめに

木星と土星の表層の流れは、赤道周辺の幅の広い順行ジェットと中高緯度で交互に現われる互いに逆向きの幅の狭いジェットが特徴的である。この表層のジェットが深部領域の対流によって生成されているのか、表層の流体運動の結果なのかは未だに明らかになっていない。流体層の厚さが惑星半径に比して十分小さい「浅い」モデル、すなわち、鉛直方向の静水圧近似の仮定の下で深部からの熱流と太陽加熱によって大気の運動が駆動されるモデルでは、中高緯度の交互に表われる幅の狭いジェットは再現されるものの、赤道域のジェットは必ずしも順行方向とはならない。一方で、流体層の厚さが惑星半径に匹敵する「深いモデル」、すなわち高速回転する球殻中の熱対流モデルでは、赤道域の順行するジェットは容易に生成されるものの、中高緯度の交互に表われるジェットの生成が困難である。

このような問題に対して Heimpel and Aurnou (2007) [1] は、これまでに考えられていた深いモデルよりも薄い球殻領域内の深部対流運動を考え、レイリー数が十分大きく内球接円筒での対流が活発な場合に、赤道域の順行流と中高緯度の交互に現われる狭いジェットが共存する状態を数値的に再現した。しかしながら、彼らの研究では経度方向に 8 回対称性を仮定しており、全球の 1/8 の領域の運動しか解いていない。このような領域の制限は流れ場全体の構造に影響を与えている可能性がある。例えば、2 次元乱流的なエネルギーの upward cascade が十分に作用し、互い違いの縞状ジェットが生成されないかもしれない。また、生成される帯状流が不安定となって縞状ジェットが壊されてしまうかもしれない。そこで本研究では、薄い球殻対流の数値計算を全球で行うことで、赤道域および中高緯度領域の帯状流が形成されるか否かを吟味した。

2. モデルと結果

モデルは回転する球殻中のブシネスク流体の方程式系で構成されている。方程式系に現われる無次元数であるプランドル数 Pr を 0.1、エクマン数 Ek を 3×10^{-6}、球殻の内径外径比 χ を 0.85、修正レイリー数 Ra を 0.05 とした。境界条件は、温度固定、応力無し条件である。初期には回転系での静止状態にランダムな温度擾乱を加えた。7500 回転まで時間積分したところ、Heimpel and Aurnou (2007) の結果と整合的な赤道ジェットと中高緯度の縞状構造が出現した（図 1 左）。しかしながら、さらに時間積分を進めると、中高緯度が全体に西風加速され縞状構造が消滅していき（図 1 中央）、12800 回転にいたると南北中高緯度に幅広の 1 本の帯状流が両半球にそれぞれ出現した（図 1 右）。中高緯度の幅広な帯状流の形成は、対流運動で駆動された回転軸方向に一様な 2 次元渦が地形性ロスビー波として外側へと伝播し、東風角運動量を抜き去ることで西風加速が生じていると考えられる。ただし、中高緯度の幅広ジェットが出現した状態でもいまだに運動エネルギーが増加しつづけているので、統計的定常状態を得るためにさらなる時間積分が必要である。長時間積分で中高緯度の縞状構造が消滅したことは、木星・土星の帯状流が惑星深部の対流運動により直接的に生成されてはいないことを示唆するかもしれない。

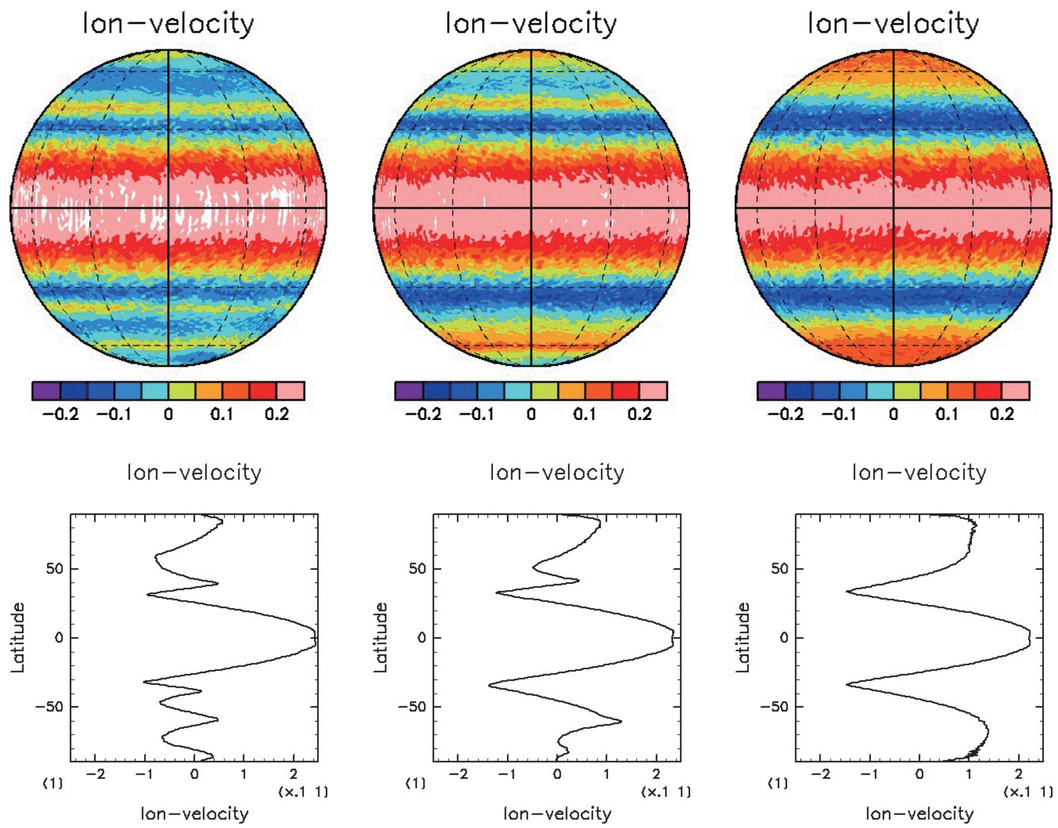

図 1 表面帯状流（上）と平均帯状流（下）のスナップショット。左から t=47630、64030、80430。Ra=0.05、Pr=0.1、Ek=3x10^{-6}、χ =0.85 の場合。

キーワード:木星, 土星, 縞状構造, 赤道順行流, ロスビー波

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