Development of Advanced Simulation Methods for Solid Earth Simulations

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Geodynamo: We have developed our simulation code of geodynamo to include length-of-day variation effect. By this, we found that the magnetic energy, convection kinetic energy, and magnetic dipole moment oscillate as the Earth's rotation speed changes. The amplitude of magnetic energy oscillation is much larger than that of rotation speed oscillation in the long period (almost the same as magnetic diffusion time) case. The oscillation phase between magnetic and kinetic energy slides pi radian. Numerical planet: We have developed a simulation code of the Stokes flow system with a free surface under a self gravitating field. The largely deformable surface of a planetary body is represented by a distribution of color function. The gravity potential is used for calculating self-gravitating field. A low viscous material (sticky air) surrounding the planet mimics its free surface motion. As an example of our target problem, we have conducted the core formation simulations in three dimensions. Mantle convection: We are also developing a simulation code of mantle convection which includes the effect of two-phase flow, i.e., molten and solid phases. By assuming the motion of the molten phase as a permeable flow through a solid matrix, we successfully solved a fast migration of molten phase even in the presence of slow (Stokes) flow in a solid phase.

Keywords: Geodynamo simulation, Yin-Yang grid, Length-of-Day variation, Mantle convection, Core formation, Stokes flow, mixed precision arithmetic, Two-phase flow, Permeable flow

1. Geodynamo simulation (Miyagoshi)

The length-of-day variation of rotation speed is observed in the Earth. It possibly affects the outer core convection, magnetohydrodynamic dynamo process, and variation of geomagnetic field. Correlation of geomagnetic field variation with length-of-day variation has been pointed out. For example, the relation between geomagnetic field variation and Milankovitch cycle is a controversial issue [1]. The orbital element of the Earth is known to change with Milankovitch cycle. It could causes variation of solar radiation which relates to the cycle between ice age and interglacial age. Prosperity and decline of continental ice sheet may change the mass distribution of water on the earth and its inertial moment. As a result, the rotational speed of the Earth can change.

However, there are no geodynamo models which take an effect of length-of-day variation into consideration. In this fiscal year, we have constructed the first geodynamo model which includes the length-of-day variation effect, based on the Yin-Yang geodynamo dynamo model [2][3]. The points of the development are as follows. In a momentum equation of magnetohydrodynamic equation, rotation speed in Coriolis force term was uniform. Then, we have added fluctuation of the rotational speed to the Coriolis force term. In addition, a new force term with time differential of rotation speed is newly added to the momentum equation. The length-of-day variation is given by a single frequency sin function with two parameters (amplitude and period of the variation).

As a default, we give the Ekman number, which means the ratio of diffusion force to Coriolis force, as 1.9E-5 (strong rotation) and the Rayleigh number, which means the ratio of buoyancy force to diffusion force, as 1.5E8. Here, both of Prandtl and magnetic Prandtl numbers are unity. First we calculated without rotation speed variation, and found that magnetohydrodynamic dynamo occurred. Then Magnetic energy in the outer core is found to be amplified and saturated several times larger than the kinetic energy. Magnetic dipole moment is also found to be the largest one among other higher moments.

Next, we calculate cases including length-of-day variation effect. For a typical case, we put the period of a variation to be almost equal to the magnetic diffusion time, and the amplitude of a variation to be two percent of the rotation speed. Our simulation results show that the length-of-day variation causes oscillations of magnetic energy, convection kinetic energy, and magnetic dipole moment. These periods are the same as that of rotation speed (magnetic diffusion time). On the other hands, the amplitudes of magnetic energy and magnetic dipole moment oscillation are about one hundred percent, which are quite larger than the amplitude of rotation speed variation (two percent). The amplitude of kinetic energy oscillation is about fifty percent. It is found that the oscillation phase between magnetic and kinetic energy slides pi radian.

We also calculate the case with short period which is one percent of magnetic diffusion time. Different from the long period case, the kinetic energy of convection oscillates rapidly with large amplitude (about one hundred percent), but magnetic energy does not. Magnetic dipole moment oscillates with the same period with the amplitude about several percent.

In the next fiscal year, we would like to search the dependence of oscillation on various rotational frequency and amplitude, and Ekman number, Rayleigh number, and so on.

Especially, we would like to study the case, in which rotational speed varies with several frequencies (not one frequency).

2. Development of numerical planet simulation code (Furuichi)

The simulation of the Stokes flow system is an interesting challenge in the field of computational geodynamics because it is relevant to the numerical study of the mantle dynamics that plays an essential role in the Earth's long-time scale thermal evolution. In this FY, we have developed a new solution code of the Stokes flow, which deals with a motion of free surface under a self-gravitation (Fig. 1) on ES2. Our code aims to simulate the long-time scale evolution of solid earth system with realistic boundary (surface) conditions, and is a part of NEMS (Numerical Earth Model Suites) project.

For the numerical treatment of free surface and self-gravitation, we employ a 'spherical Cartesian' approach, in which the surface of the deformable planetary body is defined by the color functions in the Cartesian geometry. In addition, a sticky air, which has very small viscosity with zero density, is introduced around the planetary object to mimic the free surface of the planet. As for a transport of color function, we use the CIP-CSLR method, which is the low diffusive advection
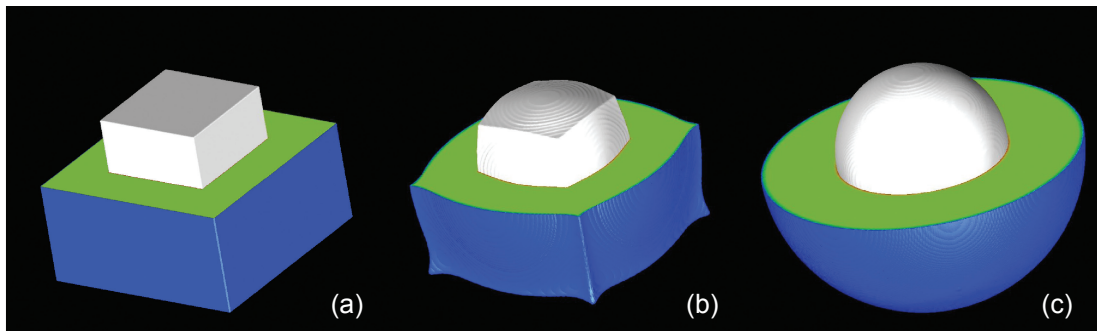

Fig. 1 Snapshot of the simulated evolution of two layered system with free surface in self-gravitating field (a) initial state; (b) intermediate state; (c) steady state layered sphere. Outer half cropped isovolume and inner white isosurface represent color function of $s = 1$ and $s = 2$ respectively. Two layer have the same isoviscous property and density, surrounded by the sticky air.
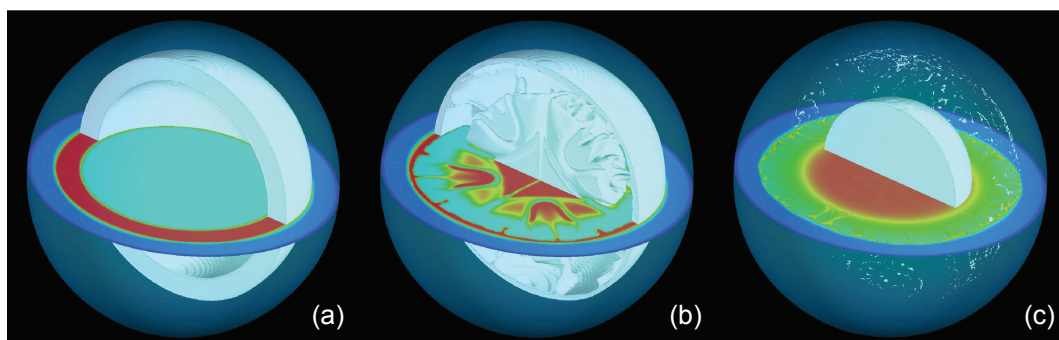

Fig. 2 Snapshots of the core formation simulation with three layered model in soft core scenario (a) initial state; (b) intermediate RT instability mode; (c) resultant layered sphere. Outer most semi-transparent isosurface and half cropped white isosurface represent mass density at $\rho = 0.2$, and color function at $\Phi_2 = 0.8$ respectively. The color on ortho plane shows mass density. Property of each layer are given by ($\eta_{air} = 10^{-3}, \rho_{air} = 0.0$), ($\eta_1 = 10^0, \rho_1 = 0.83$), ($\eta_2 = 10^{-3}, \rho_2 = 1.67$) and ($\eta_3 = 10^{-3}, \rho_3 = 1.0$).

method based on a fully fixed Eulerian mesh and suitable for a computation on the vector processors [4-5]. Self-gravitation is obtained by solving the Poisson equation of gravity potential. The ill-conditioned Stokes flow problems due to sticky air and rheological modeling are solved by the preconditioned Krylov subspace method in a fully coupled approach of the velocity-pressure system. By using a strong Schur complement preconditioner with mixed precision arithmetic utilizing the double-double method [6-7], our newly developed solver is robust to the large viscosity jump. In order to get scalability against an increasing problem size, our iterative Stokes flow solver also involves the Geometric multigrid (GMG) process as preconditioner, and its computational cost is dominant in the overall performance of the solver. Therefore, in order to achieve efficient parallel performance on ES2, careful implementation of GMG method with agglomeration technique is required [7].

Our simulation could successfully reproduce the evolution of Rayleigh-Taylor instability of spherical metal layer (Fig. 1) with various model parameters. This simulation code allows one to tackle the controversial issue of the overturn process in the Earth's core formation, between the central planetary embryo and surrounding iron layer (Fig. 2).

3. Development of mantle convection simulation code (Kameyama)

In this project, we are also developing a new simulation code of mantle convection, based on our own code named "ACuTEMan" [8-10], which includes the effect of two-phase flow, i.e., the presence and/or migration of molten materials in a solid mantle. Through the temporal evolution of the terrestrial planets, two-phase flow is considered to play crucial roles in the development of thermal and chemical structures of the interior by, for example, an intrusion of molten liquid iron into an unmolten (silicate) protocore in the earliest stage of the planets. Therefore, it is one of the most important directions of the studies of solid Earth sciences to establish numerical techniques of large-scale simulations of mantle convection incorporating the effects of two-phase flow.

In this FY, we concentrated our effort to the construction of the numerical techniques of material transport in two-phase system. In particular, we focused on an efficient scheme of motion of molten (liquid) phase, which is expected to occur much faster than that of highly viscous (solid) phase. Here, we assumed that the motion of the liquid phase takes place as a permeable flow through a solid matrix. Note that the validity of the above technique in solid Earth sciences has been already demonstrated by one-dimensional numerical modeling of fluid migration during a shear deformation of fault zone [11]. In addition, we extended the technique to the problems where a flow occurs simultaneously in both the molten (liquid) and unmolten (solid) phases, by assuming that the motion of solid phase is driven by the balance between the viscous resistance

and the buoyancy force coming from the mixture of two phases. By conducting several preliminary calculations with varying permeability and/or density gap between the phases, we have confirmed that our numerical technique successfully solve the motion of molten phase even in the presence of flow in solid phase.

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先端的固体地球科学シミュレーションコードの開発

プロジェクト責任者

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　我々は、地球ダイナモやマントル対流をはじめとする地球内部のダイナミクスの全体像を理解するために、地球シミュレータを駆使した大規模計算機シミュレーション研究を行っている。そして、そのために必要となる大規模並列計算手法や基本数値アルゴリズムの独自開発に積極的に取り組んでいる。

　ダイナモ：地球自転速度変動の効果を考慮した地球ダイナモシミュレーションコードを開発した。典型ケースとして自転速度が磁場散逸時間の周期で変動する場合、磁気エネルギー、磁場ダイポール成分共に約 2 倍の振動が生じた。これは自転速度変動の振幅（自転速度の 2%）よりもはるかに大きかった。外核の磁気エネルギーと運動エネルギーは丁度 180 度逆位相で変動した。

　数値惑星：本年度は、自己重力下での自由境界表面をともなうストークス流れを解くシミュレーションコードを開発した。そして、地球の中心核（コア）の形成過程を 3 次元において再現することに成功した。

　マントル：固体マントル中で溶融相（液相）の運動を取り扱う手法の開発を行った。予備的な計算の結果、開発した手法は固相のストークス流と液相の浸透流の双方を同時に、かつ適切に取り扱いできることを確認した。

キーワード：地球ダイナモ, インヤン格子, 地球回転変動, マントル対流, 非弾性流体近似, コア形成, ストークス流れ, 2 相流, 浸透流