

# Development of Advanced Simulation Methods for Solid Earth Simulations

Project Representative

Mikito Furuichi

Institute for Research on Earth Evolution, Japan Agency for Marine-Earth Science and Technology

Authors

Mikito Furuichi<sup>\*1</sup>, Akira Kageyama<sup>\*2</sup>, Takehiro Miyagoshi<sup>\*1</sup>, Masanori Kameyama<sup>\*3</sup>,  
Nobuaki Ohno<sup>\*4</sup>, Takashi Nakagawa<sup>\*1</sup>, Kensuke Yokoi<sup>\*5</sup> and Daisuke Nishiura<sup>\*1</sup>

\*1 Institute For Research on Earth Evolution, Japan Agency for Marine-Earth Science and Technology

\*2 Graduate School of System Informatics, Kobe University

\*3 Geodynamics Research Center, Ehime University

\*4 Graduate School of Simulation Studies, University of Hyogo

\*5 Cardiff University

**Geodynamo:** We have performed numerical geodynamo simulations with length-of-day (LOD) variation associated with Milankovitch cycle by using the Yin-Yang geodynamo code which was developed in this project. We mainly investigated the period of twenty thousand years for LOD variation. We found that the LOD variation causes geomagnetic field variation. It strongly affects the fluid flow, the Joule heating, and so on. The change of the Joule heating is important for geomagnetic field variation.

**Numerical planet:** We have developed simulation code for a growing planet with core formation in 3D. In this FY, we have developed the implicit time stepping technique toward thermal convection simulation surrounded by sticky air. Our proposed method which deals with the material transport as a nonlinearity, is found as effective scheme to reduce the computational cost to obtain the numerically stable solution in comparison to the conventional explicit time stepping method.

**Keywords:** Geodynamo simulation, Length-of-Day variation, Yin-Yang grid, Mantle convection, Core formation, Stokes flow, mixed precision arithmetic, implicit time integration, inexact newton like nonlinear solver

## 1. Geodynamo simulation (Miyagoshi)

We have studied geomagnetic field variation associated with the Milankovitch cycle by magnetohydrodynamic (MHD) geodynamo simulations by using the Yin-Yang dynamo code which was developed and used in this project [1][2][3]. The geomagnetic field is not temporally constant and changes with various periods. By recent progresses of paleomagnetism, geomagnetic field variations with the periods of several tens of thousands years have been discovered [4]. For this variation, the relation to climatic changes is pointed out [5]. The relation is as follows. The cycle of ice and non-ice ages arises by the variation of the intensity of solar radiation caused by the Milankovitch cycle. The volumes of continental ice sheets also changes with the cycle. This causes redistribution of water mass on the Earth, thus changes of the inertial momentum of the Earth. It causes rotational speed changes or length-of-day (LOD) variations (Fig.1). The LOD variation may affect geomagnetic field variation through changes of convection of fluid iron in the Earth's outer core. However, there was no geodynamo model with LOD variation. We have investigated the mechanism of

geomagnetic field variation with the period of twenty thousand years, which is close to one of the Milankovitch cycle.

The mechanism of geomagnetic field variation is found by the detail analysis of numerical simulation results. Our results

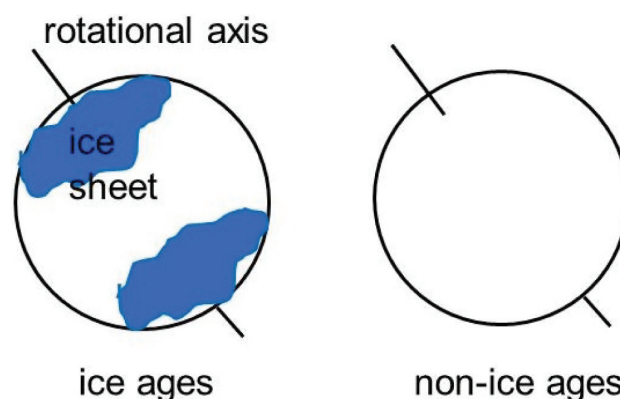


Fig. 1 The schematic picture of the LOD variation by the Milankovitch cycle. In ice ages the water mass concentrates near the polar region by the development of the continental ice sheet while there is no concentration in non-ice ages. This causes the LOD variation.

show that geomagnetic field changes with the amplitude of several tens of percent. We found that the important factor for geomagnetic field variation is the variation of the Joule heating. The LOD variation initially affects the fluid motion in the core. The Joule heating is strongly affected by the change of convection, thus it changes associated with LOD variation. We published the paper for detail results in this fiscal year [6]. The contents of the study were press released from Japan Agency for Marine-Earth Science and Technology (JAMSTEC).

## 2. Mantle convection simulation (Miyagoshi & Kameyama)

We have studied the mantle convection in super-Earths by the ACuTEMAN code [7-8]. Super-Earths are extra-solar terrestrial massive planets. The mass reaches ten times the Earth's at the maximum. The mantle convection affects the operation of plate tectonics, or the generation of the planet's magnetic field through influences to core convection. These factors are related to the habitability of the super-Earths. The mantle convection also affects the thermal evolution history of super-Earths. Our motivation is to understand the fundamental physical process of mantle convection in super-Earths and get keys to understand these problems.

One of the important factors for mantle convection in super-Earths is a strong adiabatic compression effect. The size of super-Earths is of course considerably large than that of the Earth, which means the adiabatic compression effect and the stratification of the density are important. The intensity of the adiabatic compression effect is measured by the dissipation number  $D_i = \alpha g d / C_p$ , where  $\alpha$  is the thermal expansivity,  $g$  is the gravity,  $d$  is the thickness of the mantle, and  $C_p$  is the specific heat in constant pressure, respectively. In the Boussinesq approximation which is often used in models of the mantle convection for the Earth, the adiabatic compression effect is neglected and  $D_i$  is regarded as zero. At the surface of massive super-Earths with ten times the Earth's mass, the  $D_i$  reaches about 5. We take 5 for the  $D_i$ .

Figure 2 shows one of the numerical simulation results. In this simulation, the Rayleigh number is  $10^{10}$  and the

temperature-dependent viscosity contrast between the top and the bottom is  $10^3$ . One of the important results is that the activity of hot plumes is totally lowered while that of cold plumes remains strong. The reason is as follows. The adiabatic compression effect depends on the original temperature of plumes. The buoyancy force of hot plumes is weakened because of the increment of the adiabatic temperature gradient by the strong adiabatic compression effect. In contrast, the negative buoyancy of cold plumes is hardly weakened because the adiabatic compression effect depends on the original temperature of plumes. In addition, by the formation of the thick lithosphere due to the high viscosity contrast, the temperature of the isothermal core increases. This reduces the difference of the temperature between hot plumes and ambient materials so the buoyancy force of hot plumes is more and more lowered. The result is published in this fiscal year [9].

## 3. Core formation simulation: numerical planet (Furuichi)

We have been developed a numerical code for simulating global sinking of the dense metal-rich material over long time scale during the planetary core formation process. Our code named "Nplat", solves the Stokes flow motion with free surface under a self-gravitation by using the sophisticated algorithms designed for ES2 [10-13]. With this code, we can investigate the dynamical change of the internal structure owing to the compositional density anomaly during the core formation. Such dynamics might generate thermal and compositional heterogeneity on the CMB. The conventional explicit time stepping algorithm however is difficult for simulating a thermal evolution because a numerical oscillation problem arises in solving free surface motion.

To investigate an efficient solution strategy for such free surface thermal convection problem, we have developed the implicit time integration schemes. We deal with the material transport given by tracer markers as the nonlinearity coupled to the Stokes flow equations. As the nonlinear solver, we employ the inexact newton-like nonlinear solver implemented with Jacobian free newton Krylov (JFNK) and Picard preconditioner.

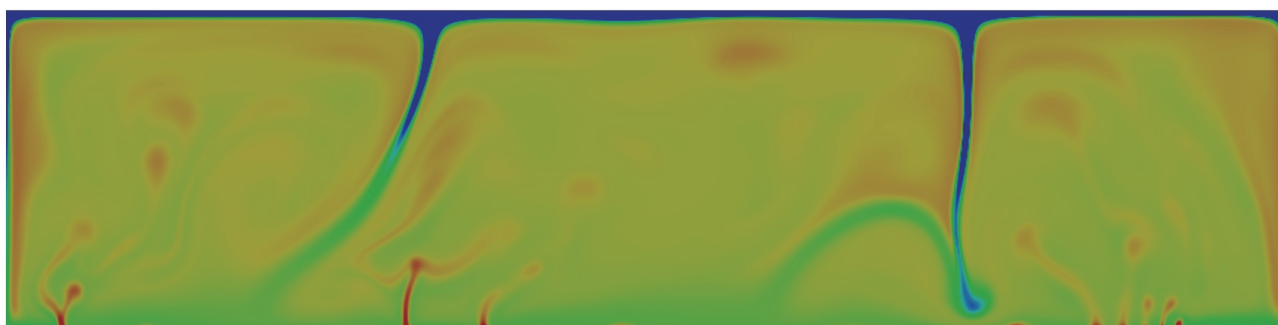


Fig. 2 One of the numerical simulation results for mantle convection in super-Earths. The color shows the potential temperature distribution. Red is large and blue is small, respectively.

In order to evaluate the performance by our solver, we solved thermal evolution problem surrounded by sticky air layers of Fig. 3a. Fig. 4a and Fig. 4b show the averaged velocity of the middle layer by the explicit and implicit time stepping methods, respectively. The explicit method requires the small time step size ( $CFL \geq 5.0e-3$ ) to obtain the numerically stable solution, but implicit method is found to be unconditionally stable for time step size. Fig. 3b shows the computational cost and load balance

by each solution method to reach  $t = 1.0e7$ . With the implicit time stepping technique (Imp), we could reduce 60% of CPU time compared with explicit method (Explicit). We also propose the semi-implicit method (Semi-imp) which uses grid based advection method to reduce the cost to evaluate the nonlinear residual. Our proposed semi-implicit method successfully reduces 90% of CPU time by the conventional explicit method.

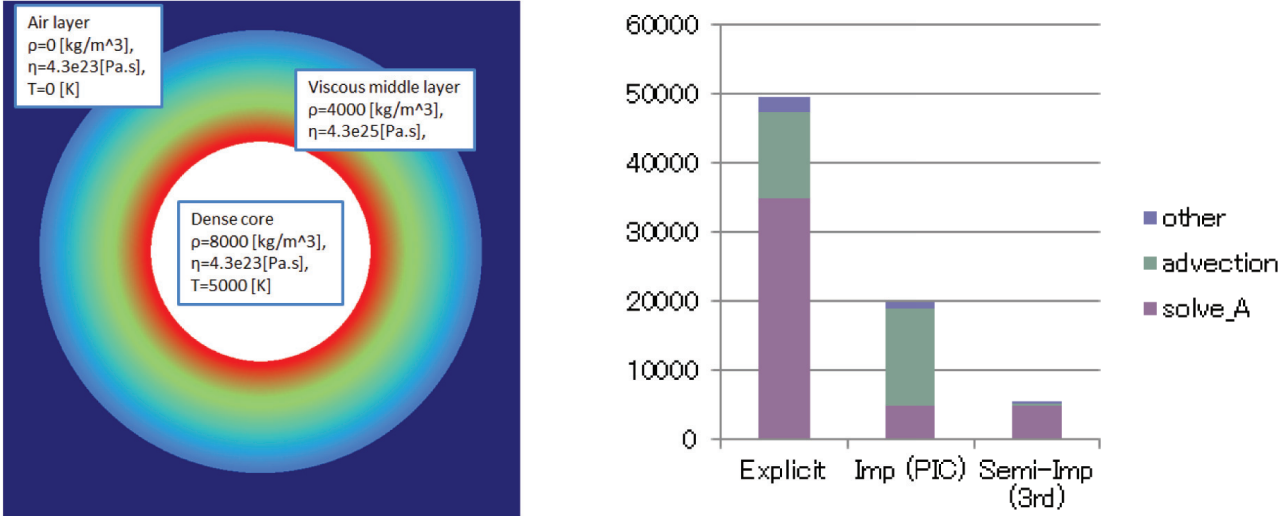


Fig. 3 Left:(a) initial temperature profile of three layers. Right:(b) CPU time and load balance to solve free surface thermal convection. Explicit, Imp (PIC) and Semi-imp (3rd) stand for the explicit, full implicit and semi-implicit time integration method, respectively.

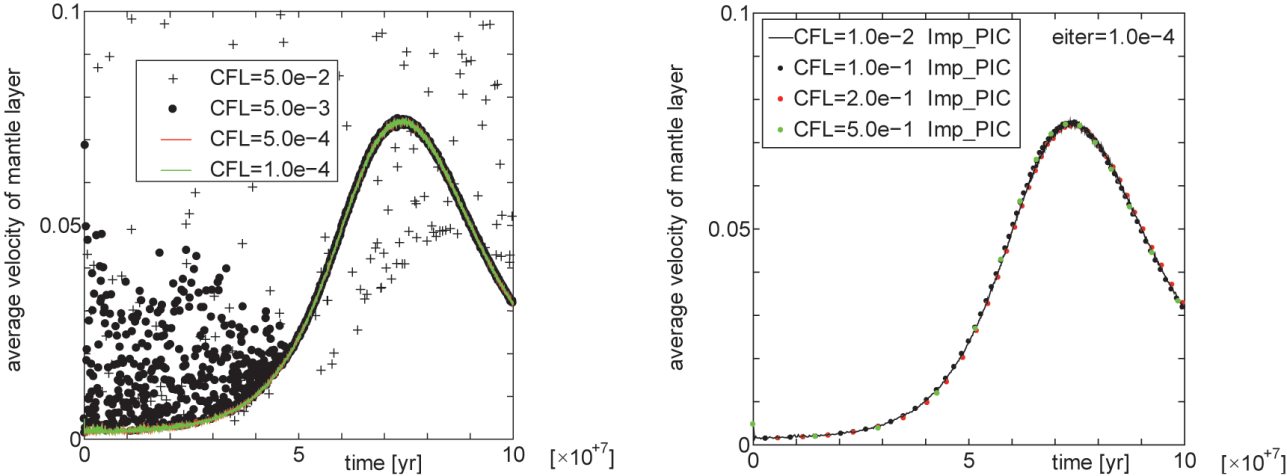


Fig. 4 Plot for averaged velocity of middle layer in free surface thermal convection test. Left :(a) Explicit time stepping method. Right :(b) Implicit material transport method

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# 先端的固体地球科学シミュレーションコードの開発

課題責任者

古市 幹人 海洋研究開発機構 地球内部ダイナミクス領域

著者

古市 幹人<sup>\*1</sup>, 陰山 聡<sup>\*2</sup>, 宮腰 剛広<sup>\*1</sup>, 亀山 真典<sup>\*3</sup>, 大野 暢亮<sup>\*4</sup>, 中川 貴司<sup>\*1</sup>,  
横井 研介<sup>\*5</sup>, 西浦 泰介<sup>\*1</sup>

\*1 海洋研究開発機構 地球内部ダイナミクス領域

\*2 神戸大学 大学院システム情報学研究科

\*3 愛媛大学 地球深部ダイナミクス研究センター

\*4 兵庫県立大学大学院 シミュレーション学研究科

\*5 カーディフ大学

本プロジェクトでは、地球シミュレータ2を活用した先端的なシミュレーションコードを開発することにより、惑星の形成過程から地球ダイナモの生成並びにマントル対流の駆動といった固体地球科学ダイナミクスの諸問題の解決に取り組んでいる。さらにコード開発で得られた知見は積極的に他分野でも応用する事を目指している。

ダイナモ：本プロジェクトで開発された Yin-Yang ダイナモコードを用いて、気候変動による氷期-間氷期サイクルに伴う自転速度変動を考慮した地球ダイナモシミュレーションを行った。ミランコビッチサイクルの一つに極めて近い2万年周期の自転速度変動を考慮した計算を行った結果、コア内の経度方向の流れが強く変動する事、ジュール散逸の変動が磁場変動に重要な役割を果たしている事等が分かった。得られた成果を“Physical Review Letters”誌に成果を発表し、海洋研究開発機構よりプレスリリースも行われた。

マントル：本プロジェクトで開発された ACuTE 法を用いて、スーパーアースのマントル対流の数値シミュレーションを行った。その結果強い断熱圧縮の効果によりコア-マントル境界からのホットプルームがかなり弱められる事、対照的に惑星表面からのコールドプルームはあまり弱められない事等が分かった。得られた成果を“Astrophysical Journal Letters”誌に発表した。

数値惑星：コア形成時のグローバルスケールの金属層の落ち込みを再現する3次元シミュレーションコードを開発している。本年度は、隕石衝突イベントにより地球内部に持ち込まれる熱を変形する表面形状と共に追跡するための、ソルバーの改良を行った。具体的には移流解を陰的に取り扱うための大規模非線形ソルバーを開発し、時間ステップを用いて計算コストを大幅に削減する事に成功した。

キーワード:地球ダイナモ, インヤン格子, 地球回転変動, マントル対流, コア形成, ストークス流れ, 陰解法,  
大規模非線形ソルバー

