Study on Nonlinear Evolution Equation with Blow-up of Solution

Finite Element Analysis of Forming Mode of Dictyostelium Discoideum Chemotaxis Model (Keller-Siegel Model) with COMSOL Multiphysics

Projective Representative

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Abstract

In this paper, the modes of chemotaxis dynamics model that proposed by Keller (Keller, E.F.) and Siegel (Siegel, L.A.) in the early 1970s were analyzed by finite element analysis (FEM) method with COMSOL Multiphyscs. The large-scale calculations were carried out at UV2000. As a result, the range of chemotaxis parameters used in simulation was greatly expanded, and the existence of various modes which cannot be predicted using conventional classical solution were found.

Keywords:　Dictyostelium Discoideum Chemotaxis Model, Keller-Siegel, Finite Element Analysis

Introduction

There are billions of bacteria living in 1 [g] soil, and the species of bacteria may reach 100 million. Dr. Satoshi Omura (The Nobel Prize in Physiology or Medicine 2015) collected bacteria lurking in the soil, continued exploratory research on useful natural organic compounds produced by microorganisms, and found that anthelmintic for animals, the antiparasitic antibiotics "Ivermectin", is also effective for human beings. It is found that the "Ivermectin" is especially effective for onchocerciasis (river blindness, a tropical endemic disease), which is able to prevent the aggravation of symptoms and the infection. Therefore, the exploration of living bacteria in the soil plays an important role in the elucidation of life phenomena, at the same time, also has been attracting attention for the creation of new anti-cancer agents.

Dictyostelium Discoideum is a single-celled ameba that feeds on the bacteria in soil to repeat growth and division as shown Fig.1. Normally the bacteria are uniformly distributed in the soil when a good food chain exists. However, once the food chain is interrupted somewhere, distribution of the

bacteria will no longer be uniform and ameba will fall into starvation.

In this state, the amoeba will secrete acrasin, and the acrasin will attract other amoeba. As a result, a large number of amoeba will aggregate and form a slug-like multicellular body. This will further develop into sporangiophore, the fruiting body, with handle and head over time. The heads of the fruiting bodies will grow into new spores, and these new spores will then separate from the fruiting bodies, sprout, and turn into new single-celled amoebas that feed on the bacteria in soil. The life cycle of dictyostelium discoideum is repeated as described above. The mathematic model that describes the aggregation of ameba is the chemotactic model——Keller - Siegel model[1]. The dictyostelium discoideum is a useful model organism in embryology research because it exists everywhere, and the life cycle is only about 24 hours. Above all, due to the significant feature of chemotaxis, which is similar to that of the white blood cells, it has also become the typical example of chemotactic model

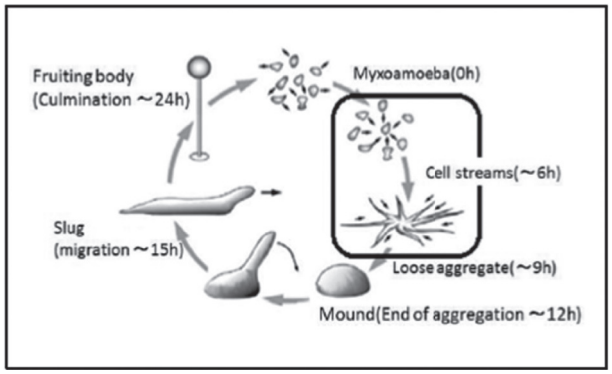

Fig.1 Growth and division of Dictyostelium Discoideum.

In this research, the modes of chemotaxis dynamics model that proposed by Keller (Keller, E.F.) and Siegel (Siegel, L.A.) in the early 1970s were analyzed by finite element analysis (FEM) method with COMSOL Multiphyscs. The large-scale calculations were carried out at UV2000. As a result, the range of chemotaxis parameters used in simulation was greatly expanded, and the existence of various modes which cannot be predicted using conventional classical solution were found.

Chemotactic change model

The Keller-Siegel model in one-dimensional open finite set I is described as the following equations, where u(x,t) and v(x,t) for (x,t) in $I \times (0, \infty)$ are the cell density of the Dictyostelium Discoideum and the concentration of chemoattactant(cAMP) which the Dictyostelium Discoideum releases, respectively. The boundary conditions set on both ends of I are the Nuemann condition, which means there are no fluxes of the cell and the chemotactant. In this paper, the numerical solutions based on the finite element method was discussed.

$$u_t = u_{xx} - a(uv_x)_x \qquad (x,t) \text{ in } I \times (0,\infty)$$

$$v_t = v_{xx} - \gamma v + \alpha u \qquad (x,t) \text{ in } I \times (0,\infty)$$

$$u_x(L_1,t) = u_x(L_2,t) = v_x(L_1,t) = v_x(L_2,t) = 0 \qquad t \text{ in } (0,\infty)$$

$$u(x,0) = \overline{u}(x),\ v(x,0) = \overline{v}(x) \qquad x \text{ in } I$$

$$\int_0^L \frac{u^n - u^{n-1}}{\delta t}\varphi \, \mathrm{dx} = -\int_0^L u_x^n \varphi' \, \mathrm{dx} + a\int_0^L u^n v_x^n \varphi' \, \mathrm{dx}.$$

$$\int_0^L \frac{v^n - v^{n-1}}{\delta t}\phi \, \mathrm{dx} = -\int_0^L v_x^n \phi' \, \mathrm{dx} + \alpha\int_0^L u^n \phi \, \mathrm{dx} - \gamma\int_0^L v^n \phi \, \mathrm{dx}$$

Usually it is necessary to derive the weak forms of these equations as shown here, but using the advanced software called COMSOL Multiphysics we could get finite-element solutions without deriving the weak form. We only set merely PDE which corresponds to the strong form of the Keller-Siegel equations as shown in Fig.2.

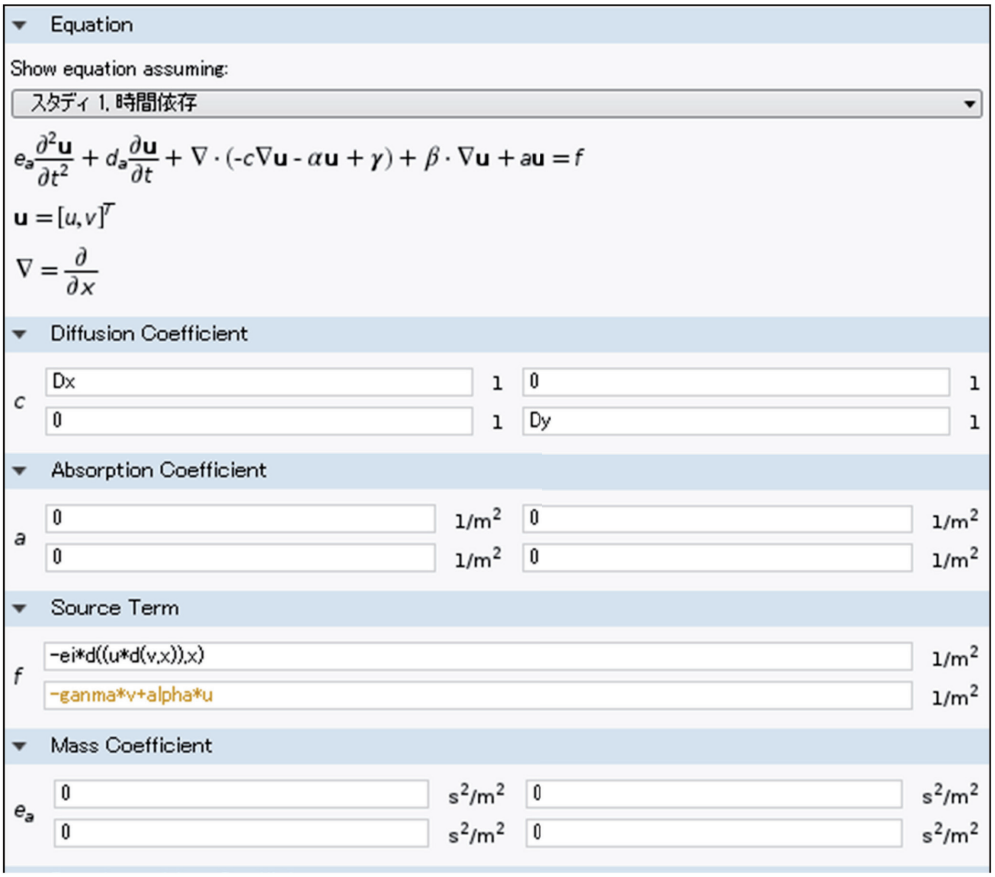

Fig.2 PDE interface of coefficient form included in COMSOL Desktop (GUI of COMSOL Multiphysics).

Results

As shown below, with using UV2000, we have extended the chemotactic parameter 'a' up to 5, which appears as the coefficient of the nonlinear terms in the u- equations. If the chemotactic parameter 'a' is very small, e.g. a=0.01[3) 4)], we can get the analytical prediction. In this case, the time evolution is very simple and diffusive. When 'a' is increased from 0.1 to 5 as shown in Fig.3, it is found that each time evolution has been changed and various modes appear, which cannot be predicted analytically.

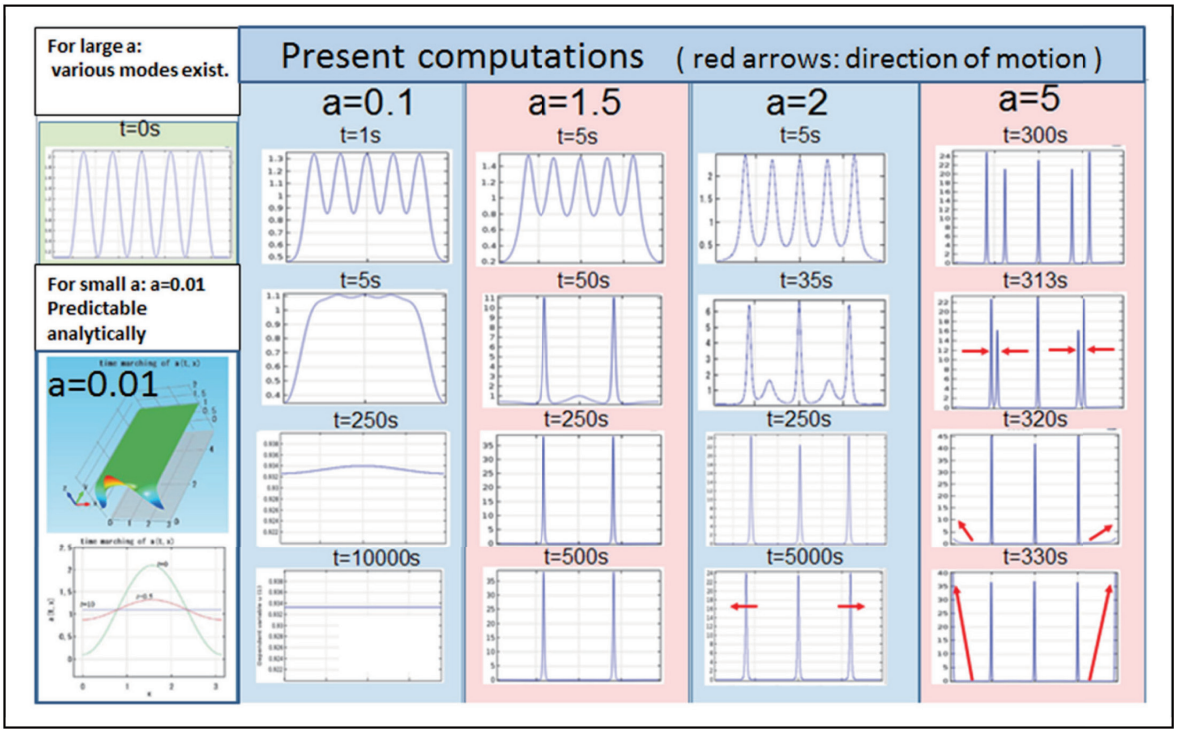

Fig.3 Various modes of Keller-Siegel model predicted by the present computation; Red arrows included note the direction of movement.

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