寒冷圏進捗報告

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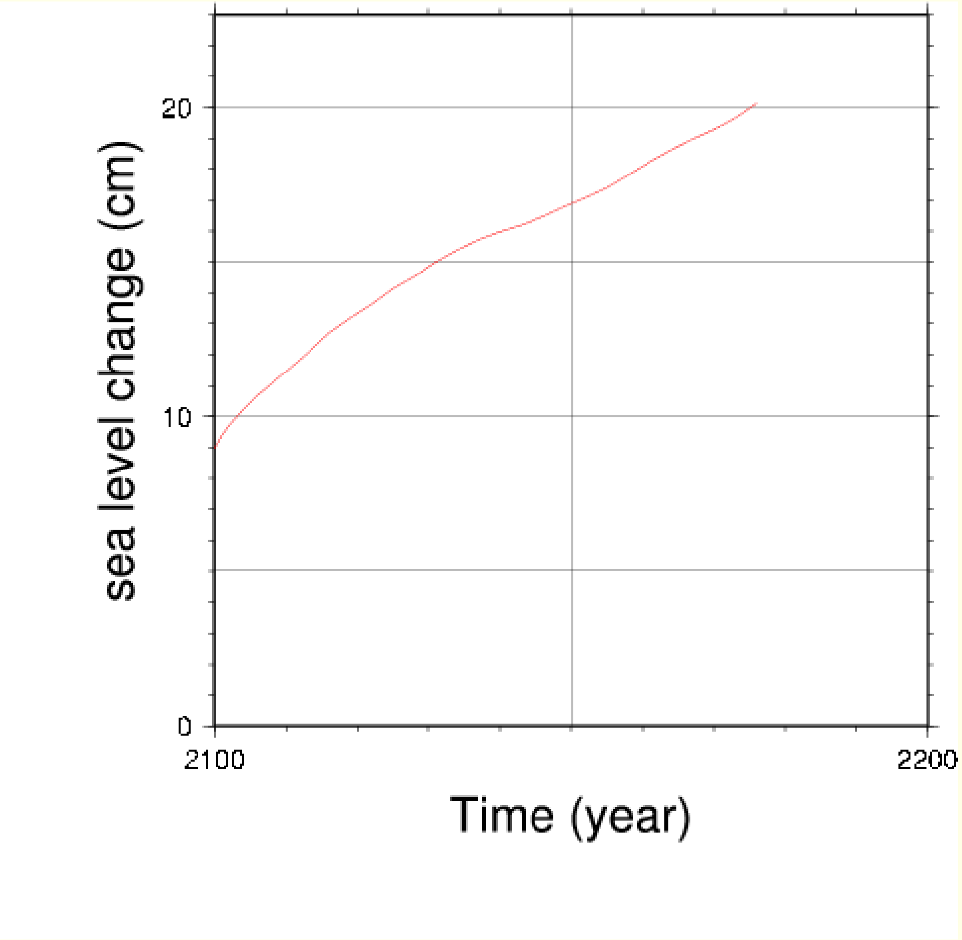
partly based on:

- Saito, F., A. Abe-Ouchi, & H. Blatter (in press): An improved numerical scheme to compute horizontal gradients at the ice-sheet margin: its effect on the simulated ice thickness and temperature. Ann. Glaciol., **46**

K2 + $I_C IES$... (1)

K2 model + $\boldsymbol{I_C IES}$ under fixed A2 scenario from A.D. 2100

$\Longrightarrow$ Several subjects to be considered



Something strange: weaker response than expected

K2 + $I_C IES$... (2)

- Ice sheet topography used in GCM need to be revised.
  - Control experiment required to be re-tuned.
- Simulated thickness near margin is too large to see the effect of global warming
  - How to reduce the error near margin? (today's main topic)
- To find 'effective' experiment for climate/ice-sheet coupling: preparation for the final simulation

Introduction

There are two fundamental problems in ice-sheet modeling:

- Error in numerical solution for mass-balance equation
- Strange behavior due to thermo-mechanical coupling

Since resources are limited, we cannot completely simulate an ice sheet by an numerical model.

⇒ We have to consider in which situation these problems become significant/insignificant.

Error in numerical solution for mass-balance equation (1)

(1) $$\frac{\partial H}{\partial t} = \nabla \cdot \vec{q} + M$$ Mass balance equation

(2) $$\vec{q} = I(T) H^{n+2} \left[\nabla s \cdot \nabla s\right]^{\frac{n-1}{2}} \nabla s$$ flux under SIA

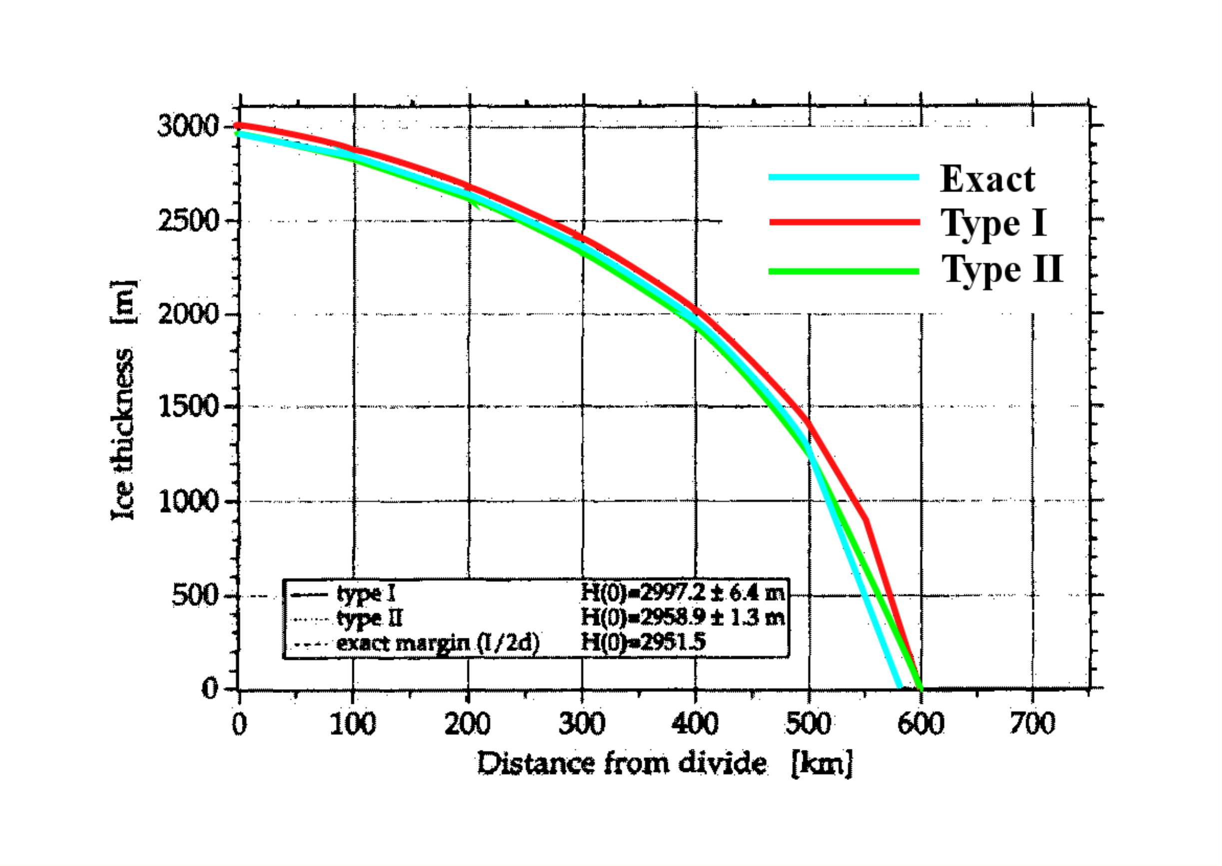
M	surface mass balance (snow, melting $\cdots$)
H	thickness
$I(T)$	temperature-dependent term ($\sim$ softness)
s	surface topography
n	flow parameter ~ 3

$$\nabla s \cdot \nabla s = \left[\frac{\partial s}{\partial x}\right]^2 + \left[\frac{\partial s}{\partial y}\right]^2 = \left[\frac{\partial s}{\partial r}\right]^2$$

$$\Longrightarrow \quad \boxed{\text{flux} \propto [\text{thickness}]^{n+2} \, [\text{surface gradient}]^{n}}$$

Error in numerical solution for mass-balance equation (2)

Comparison between the exact solution and numerical one



Huybrechts et al. (1996)

Steady state of an ideal ice sheet under simple boundary condition (which can be analytically solved)

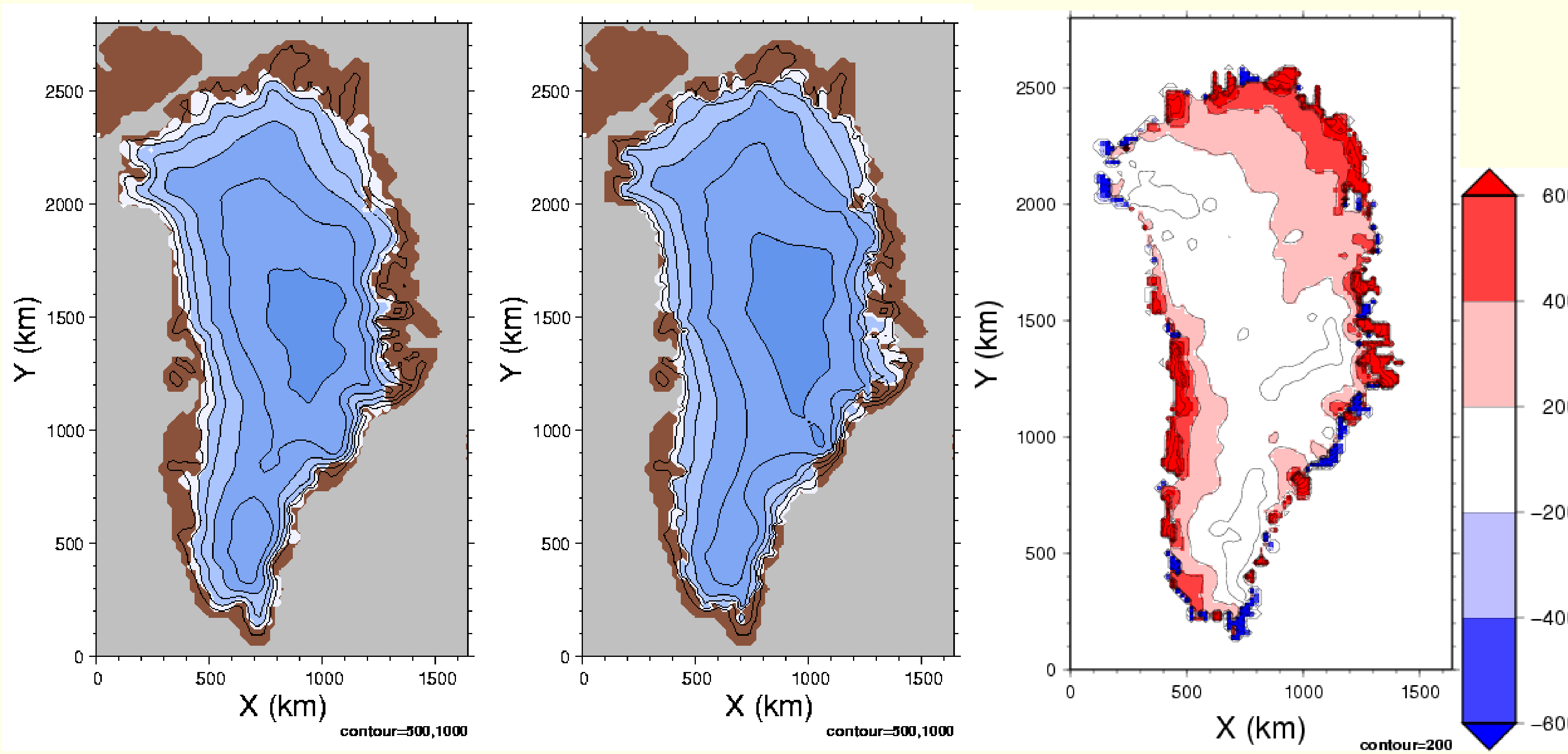
Overestimation of the thickness near the margin

due to a pure numerical reason.

Error in numerical solution for mass-balance equation (3)

Example: Simulation of present Greenland ice sheet

Obs. Siml. Diff.



Error in numerical solution for mass-balance equation (4)

The error become significant in (e.g.) global warming simulation

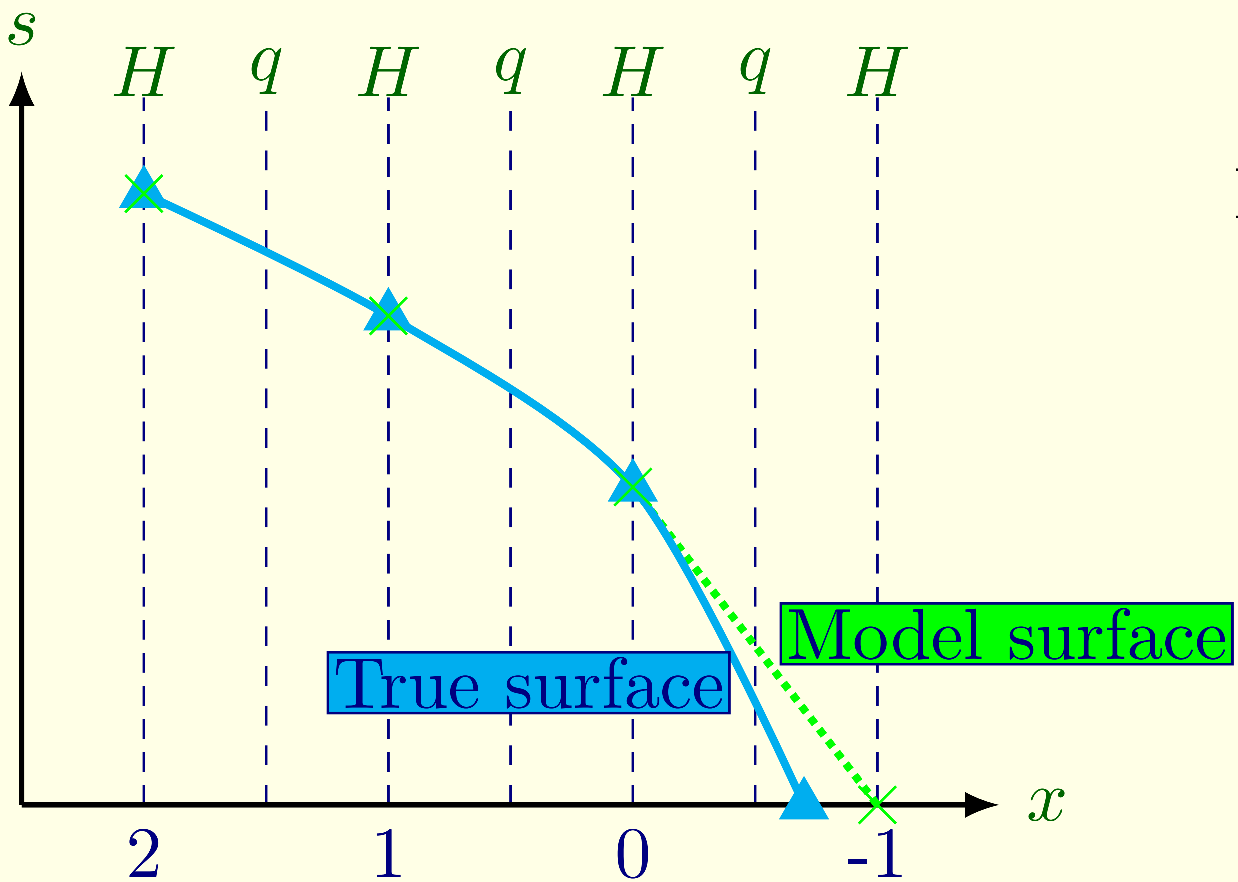
- Melting occurs mainly near the margin
  $\rightarrow$ influence on simulated response to increase in melting
- Bias to increase in melting
  500 m overestimation $\simeq -3$ K bias in temperature

There are (of course) other reasons for margin overestimation

- Transient behavior (history of climate forcing)
- Uncertain processes (basal sliding, $\cdots$)
- Ignored processes (ice stream, microphysics,$\cdots$)

Error in numerical solution for mass-balance equation (5)

Reasons: Exact margin position cannot be well reproduced by the finite grid system. — $q = q\left(H, \dfrac{\partial s}{\partial x}\right)$



Remedies

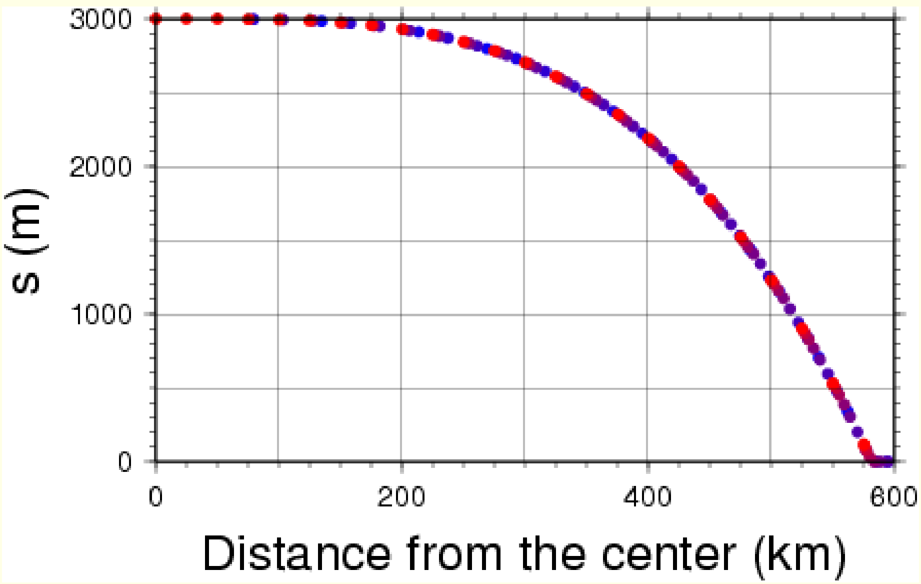
- Higher resolution
- Higher-order finite-difference
- Asymmetric scheme (not use zero-thickness grids)

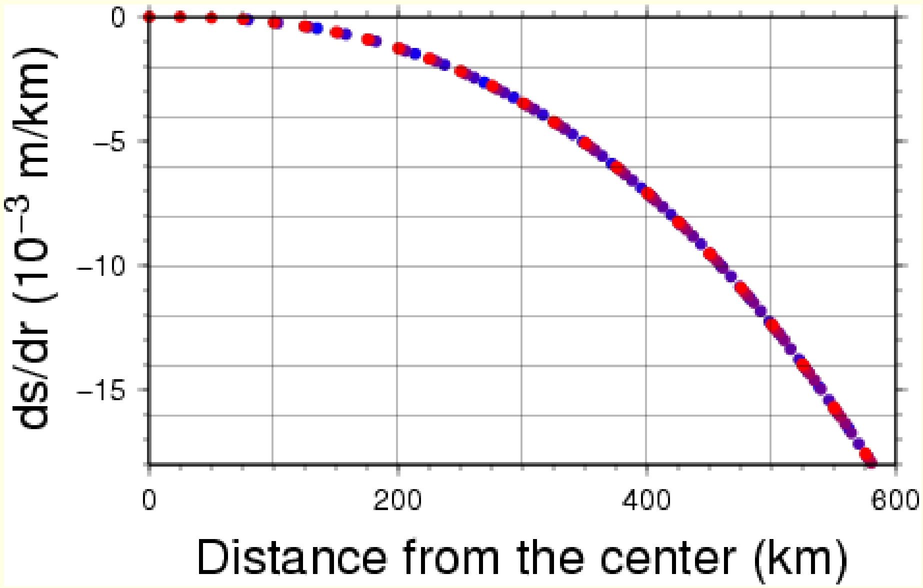
Error in numerical solution for mass-balance equation (6)

Surface gradient in an ideal analytical ice sheet

$$(3) \quad s = H_0 \left[1 - \left(\frac{r}{L_0} \right)^{3.5} \right], \quad r = \sqrt{x^2 + y^2}$$

$$(4) \quad H_0 = 3000\mathrm{m}, L_0 = 581.5\mathrm{km}, \Delta x = 25\mathrm{km}$$

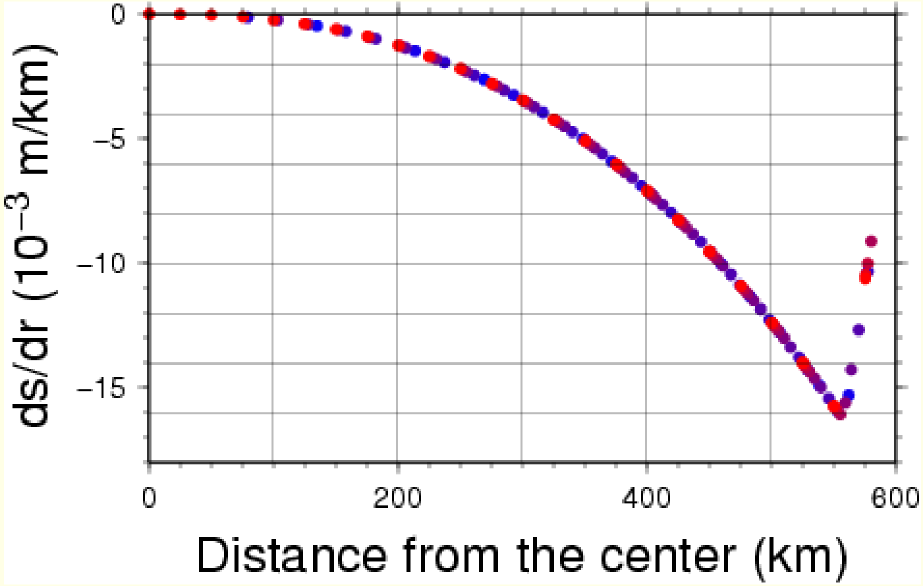




Error in numerical solution for mass-balance equation (7)

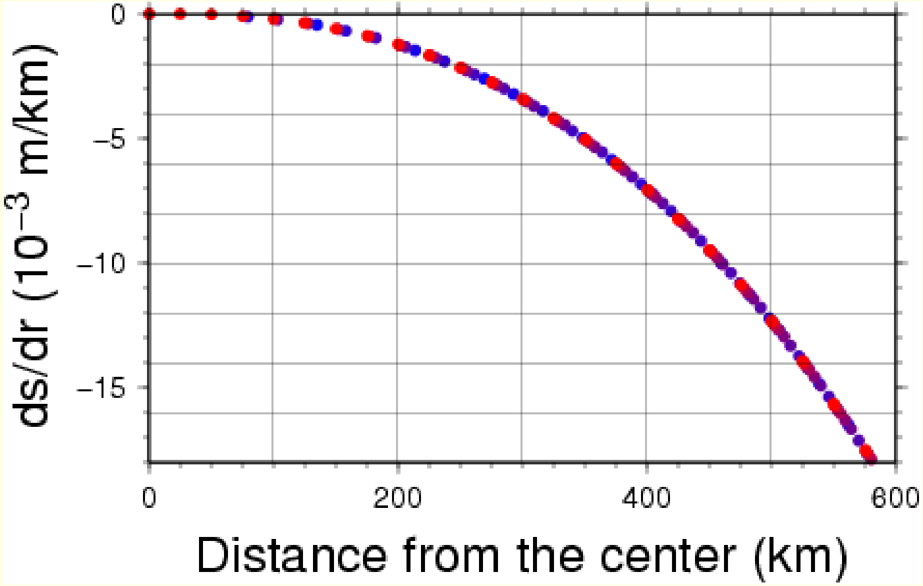
Surface gradient computed by finite difference schemes

Central scheme



$$\left.\frac{\partial s}{\partial x}\right|_0 = f(s_{-1}, s_0, s_1)$$

Assymetric scheme



$$\left.\frac{\partial s}{\partial x}\right|_0 = f(s_0, s_1, s_2)$$

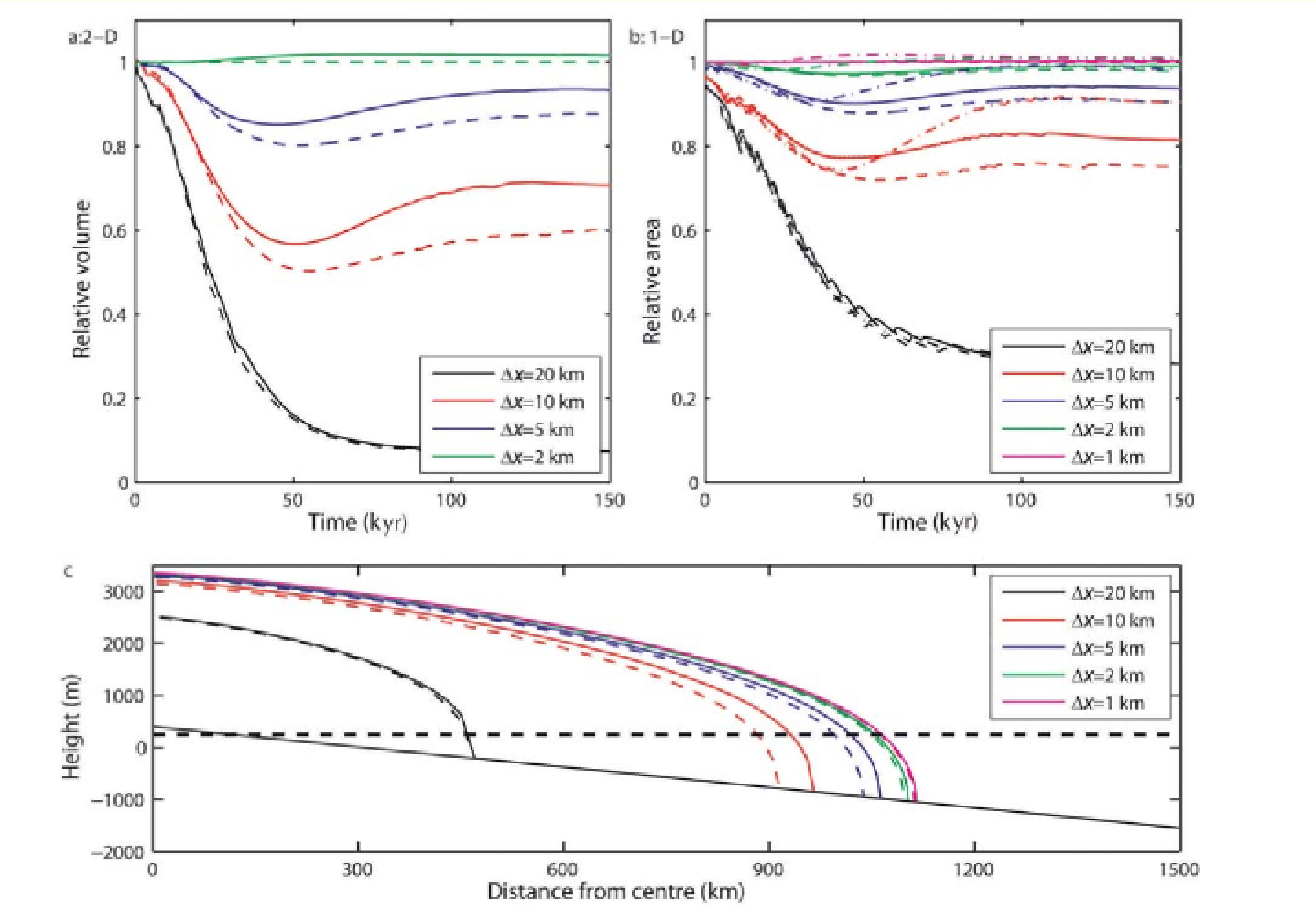
Error in numerical solution for mass-balance equation (8)

Remedies and application

- Hindmarsh & Le Meur (2001); Vieli & Payne (2005)
  - Two-dimension, “moving grid” approach (scaled by length)
  - 5th order asymmetric scheme
- van den Berg et al. (2006)
  - Three-dimesnion
  - *slope correction formula*: asymmetric scheme on $\vec{q}$
  - Albedo-elevation feedback included
  - Influence on transient behavior, steady-state is discussed
  - Application on isothermal case

Error in numerical solution for mass-balance equation (9)

Effect of slope correction formula in van den Berg et al. (2006)



Volume	Area
Cross-section	

Solid: corrected

Dashed: not corrected

Error in numerical solution for mass-balance equation (10)

van den Berg et al. (2006)

- Error in computed flux at the margin significantly affect the transient simulation as well as steady-states
- Albedo-elevation feedback enhances the error
  - Higher-elevation $\rightarrow$ Smaller melting
- Drawback: 2nd order asymmetric scheme $\boxed{\text{only on } \vec{q}}$
  - $\nabla \cdot \vec{q}$ is still by a symmetric scheme
- How much in thermodynamicaly $\boxed{\text{coupled}}$ cases?
  - Enhanced? Reduced?

Numerical improvement on gradient at the margin (1)

$$(5) \quad \frac{\partial H}{\partial t} = \nabla \cdot \vec{q} + M$$ Mass balance equation

$$(6) \quad \vec{q} = I(T) H^{n+2} \left[\nabla s \cdot \nabla s\right]^{\frac{n-1}{2}} \nabla s$$ flux under SIA

Asymmetric difference scheme is adopted both on computation of $\vec{q}$ and $\nabla \cdot \vec{q}$.

To present its effect on steady-state result

- Special focus: thermodynamial coupling
- Albedo-elevation feedback excluded

Numerical improvement on gradient at the margin (2)

$$(7)\qquad \frac{\partial H}{\partial t} = -\nabla \cdot \vec{q} + M \;,$$

$$(8)\qquad \frac{\partial H}{\partial t} = -\nabla \cdot (D \nabla H) - \nabla \cdot (D \nabla b) + M \;,$$

$$(9)\qquad \vec{q} = D \nabla (H + b) \;, D = D(H, \nabla H, I(T))$$

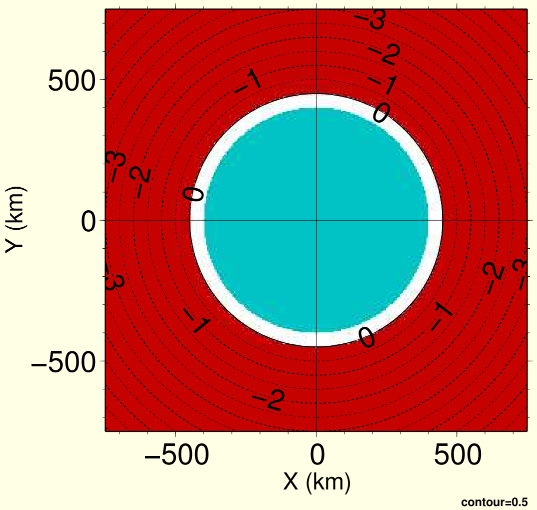
$$(10)\qquad \begin{cases} \left.\dfrac{\partial q}{\partial x}\right|_0 = \dfrac{9q_{\frac{1}{2}} - 8q_0 - q_{\frac{3}{2}}}{3\Delta x} & \text{margin grid} \;, \\ \left.\dfrac{\partial q}{\partial x}\right|_i = \dfrac{q_{i+\frac{1}{2}} - q_{i-\frac{1}{2}}}{\Delta x} & \text{interior grid} \;, \end{cases}$$

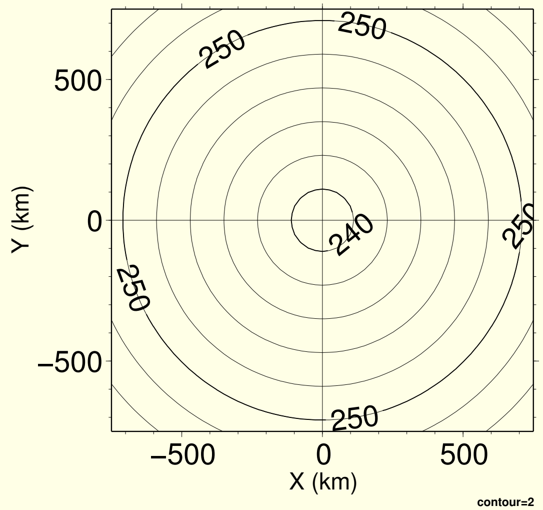
$$(11)\qquad q_0 = D_0 \left.\frac{\partial H}{\partial x}\right|_0 = D_0 \frac{4H_1 - H_2 - 3H_0}{2\Delta x} \;.$$

Numerical improvement on gradient at the margin (3)

Boundary conditions— Payne et al. (2000)

- Modelling an ice sheet under radially symmetric boundary
- No basal sliding / Flat bedrock topography
- Surface mass balance and temperature are fixed





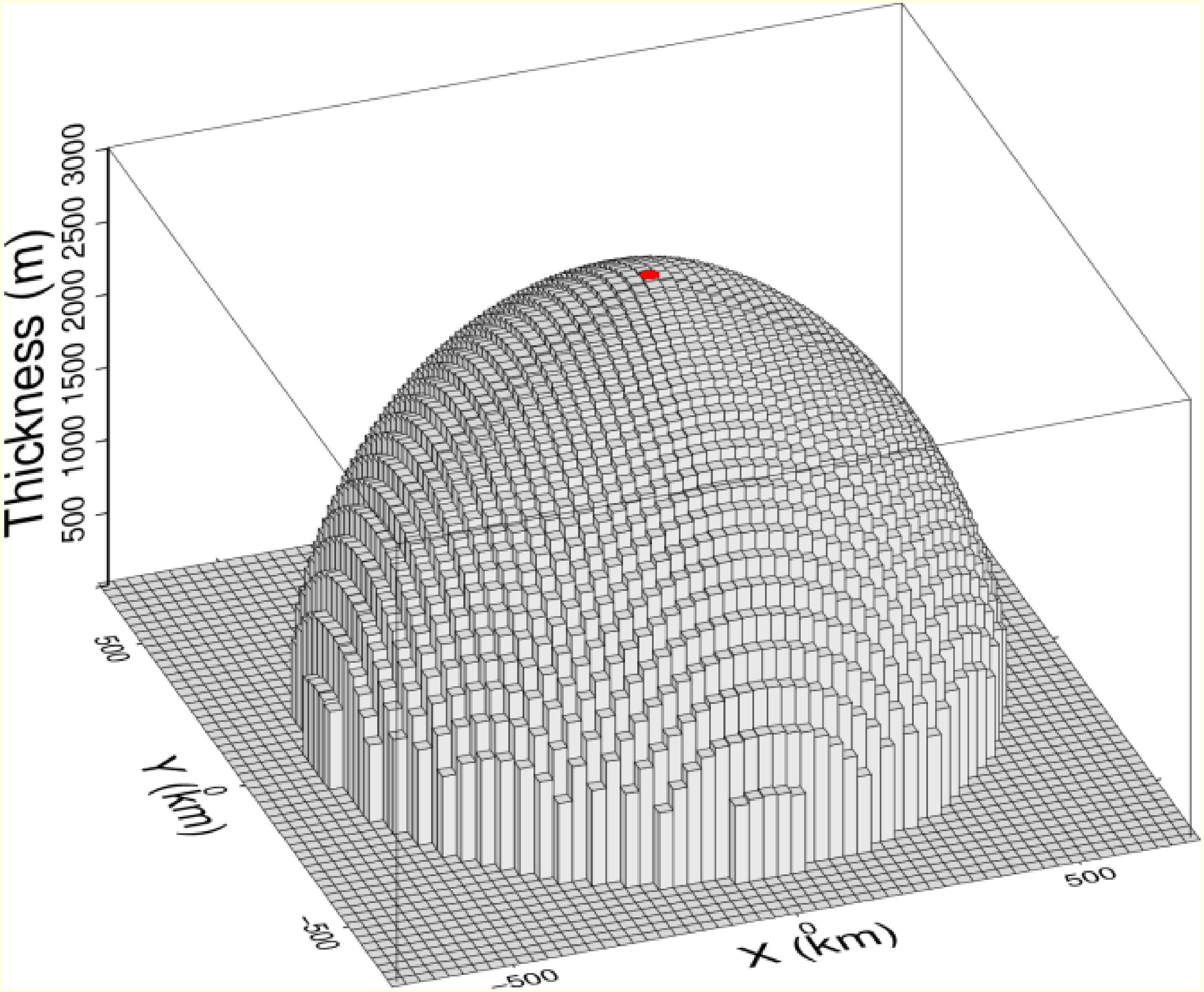
Left:

Surface mass balance (m/yr)

Right:

Surface temperature (K)

Numerical improvement on gradient at the margin (4)



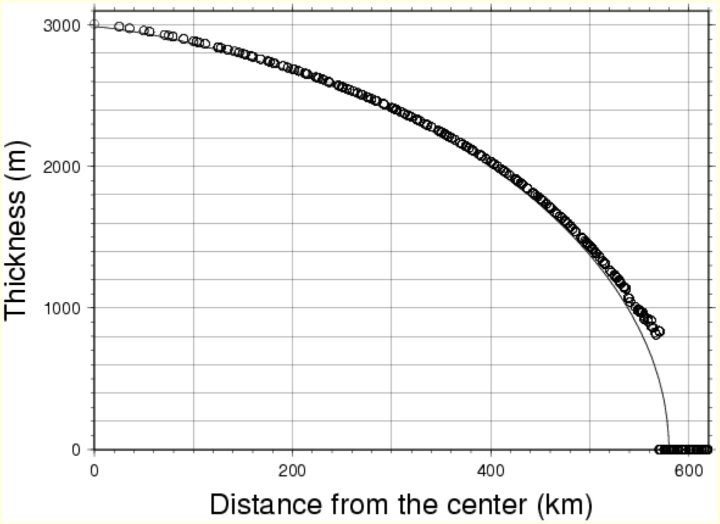
Simulated ice sheet thickness

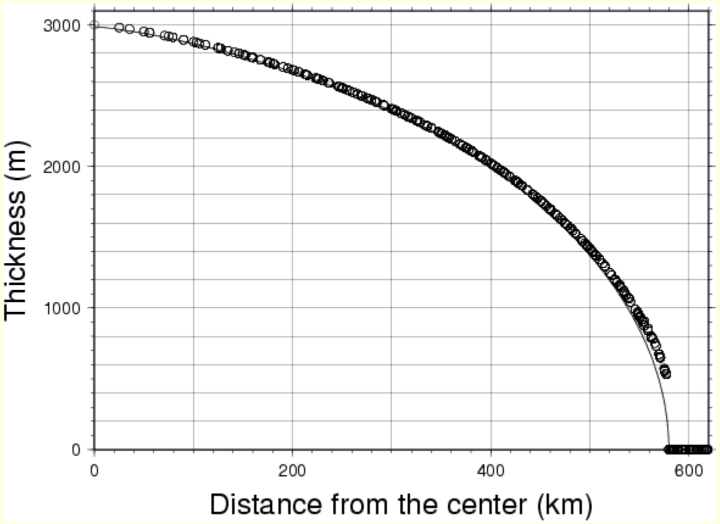
Numerical improvement on gradient at the margin (5)

Uncoupled case (constant $I(T)$)

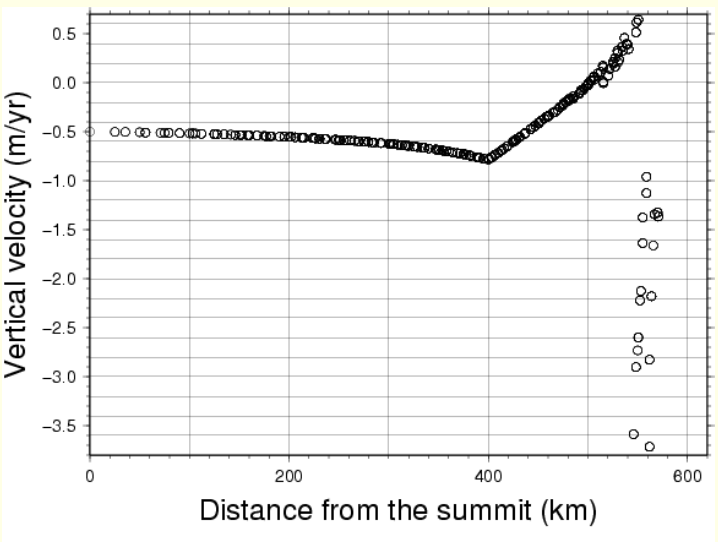
Old scheme

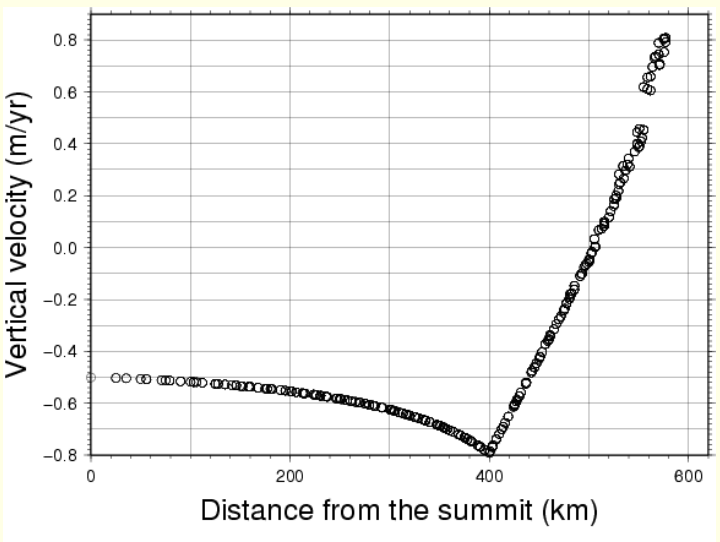
New scheme





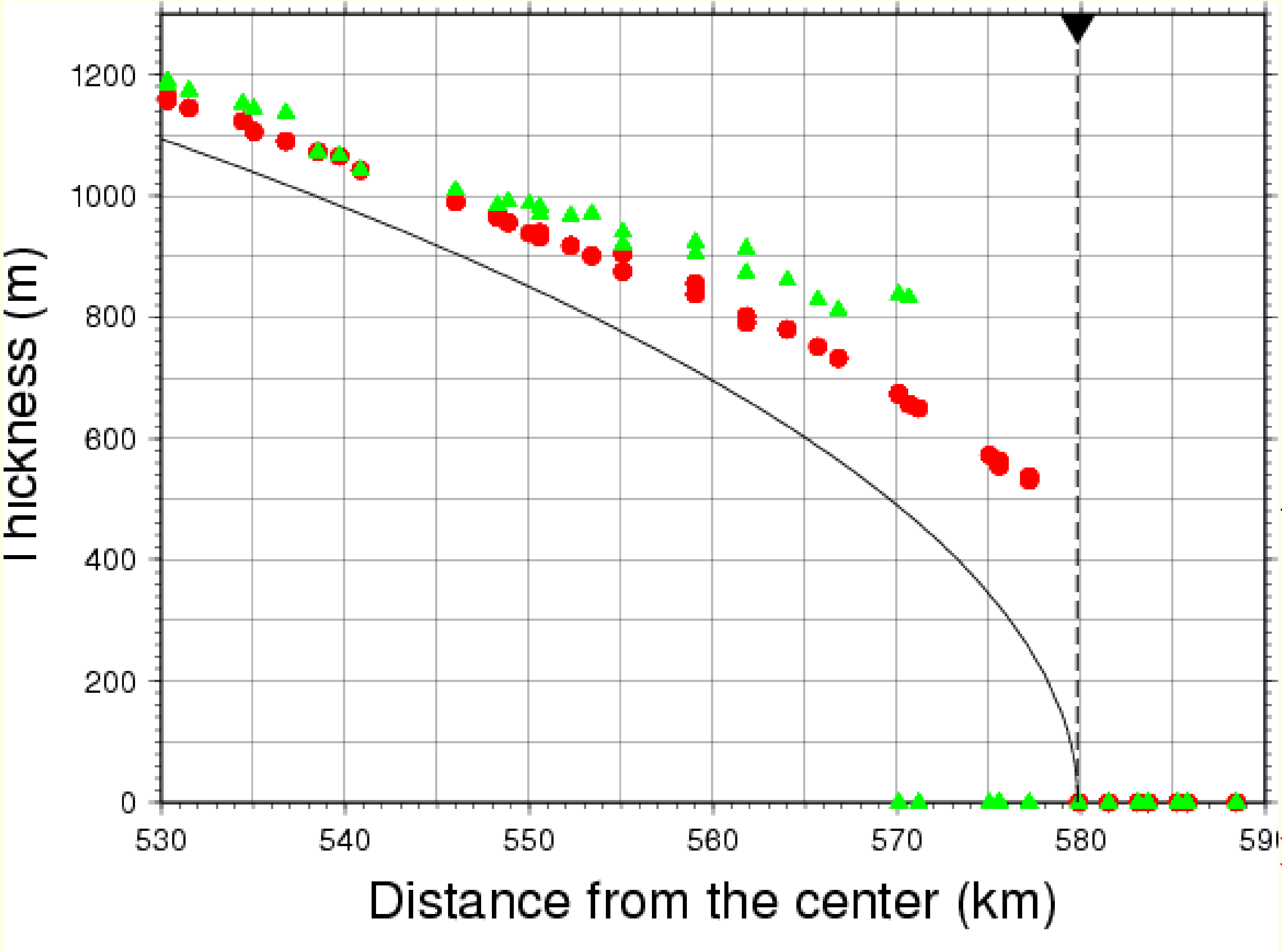
H





w_{surface}

Numerical improvement on gradient at the margin (6)



Uncoupled case
(Zoom up)

red = New

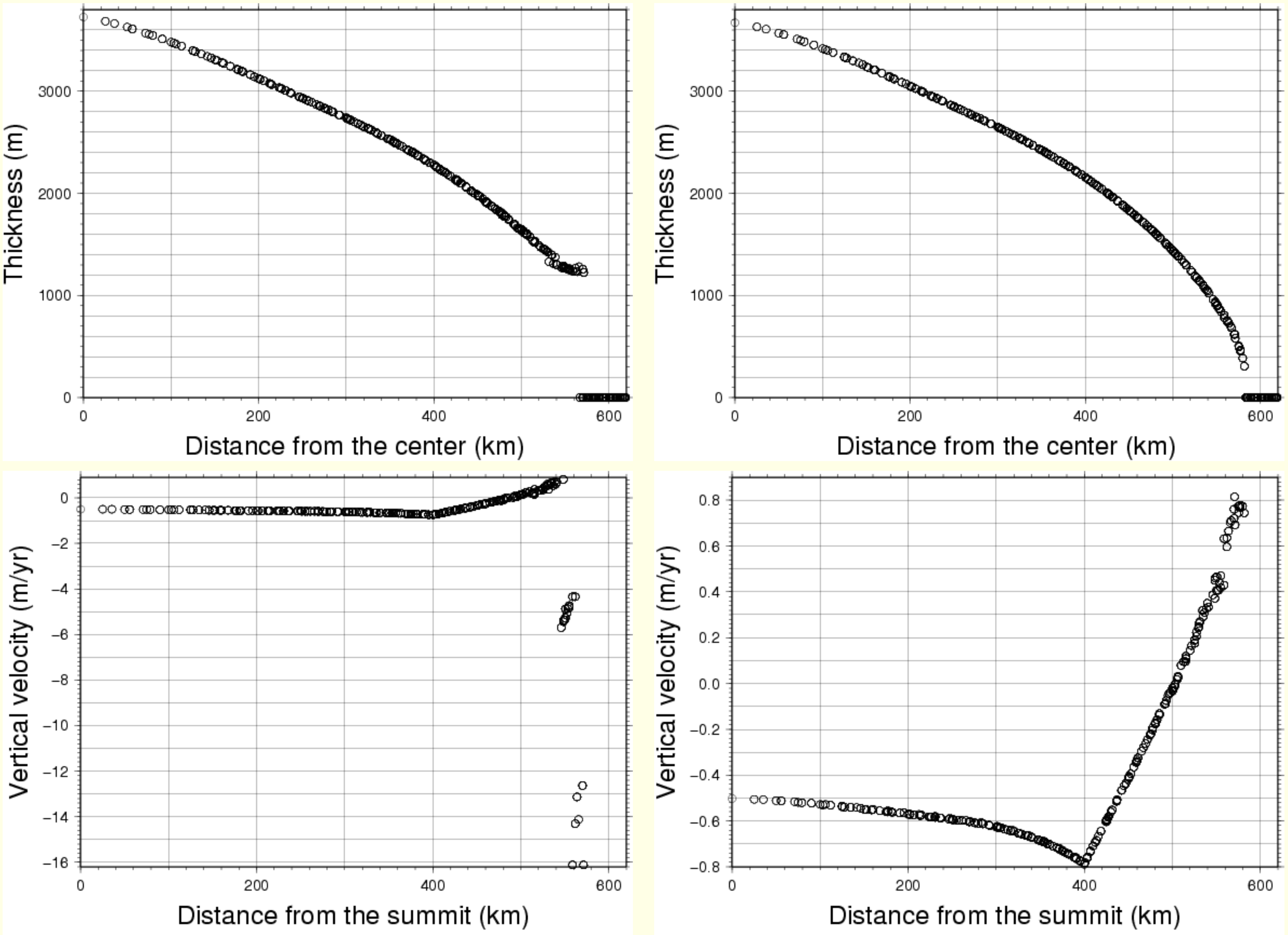
green = Old

- Still surface is overestimated
- Less *special* direction

Numerical improvement on gradient at the margin (7)

Thermodynamically coupled case

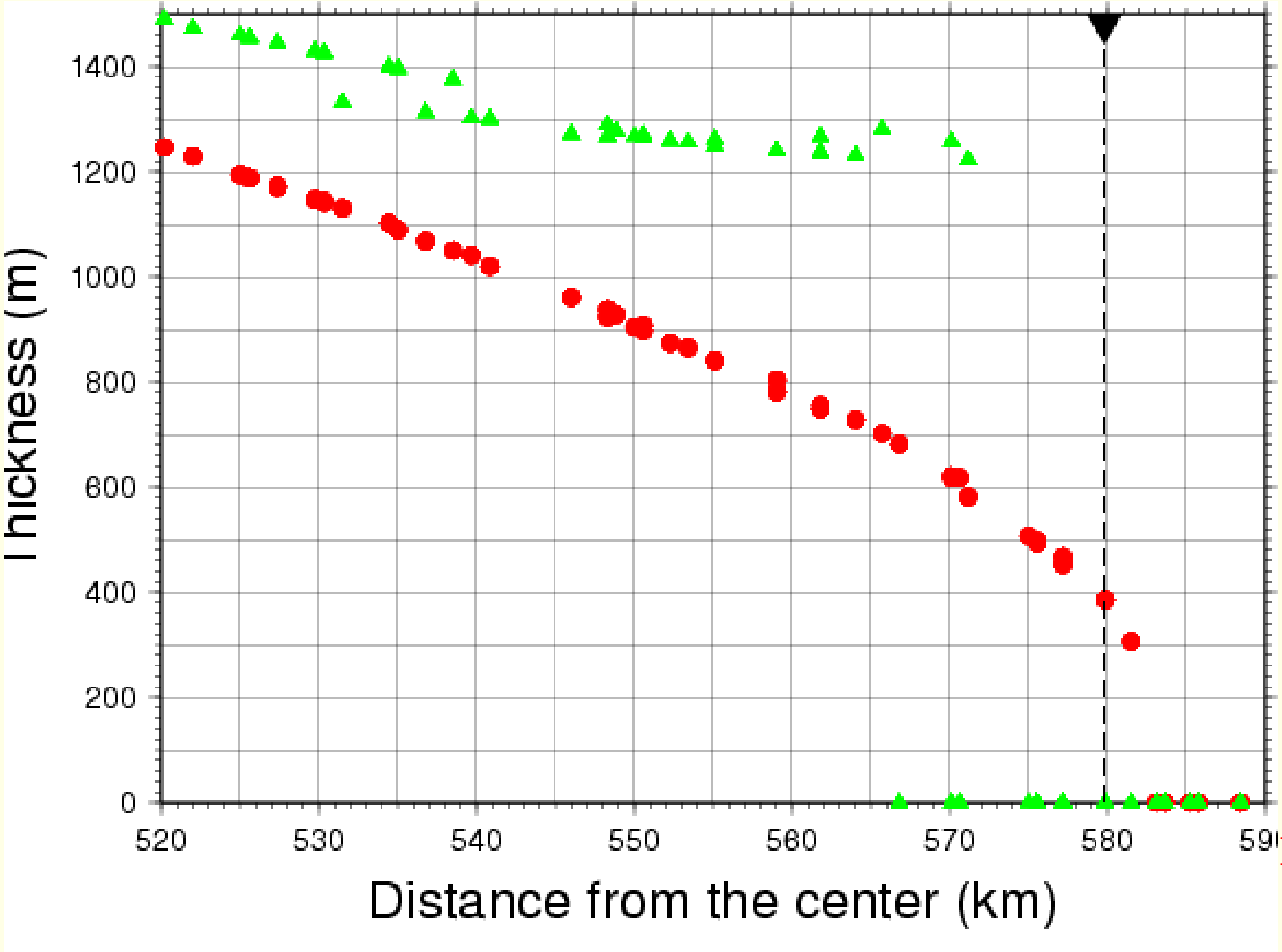
Old scheme | New scheme



H

w_{surface}

Numerical improvement on gradient at the margin (8)



Coupled case
(Zoom up)

red = New
green = Old

- 500 m difference at most
- Less *special* direction

Conclusion

Numerical improvement on computation of mass balance and flux at the margin is presented.

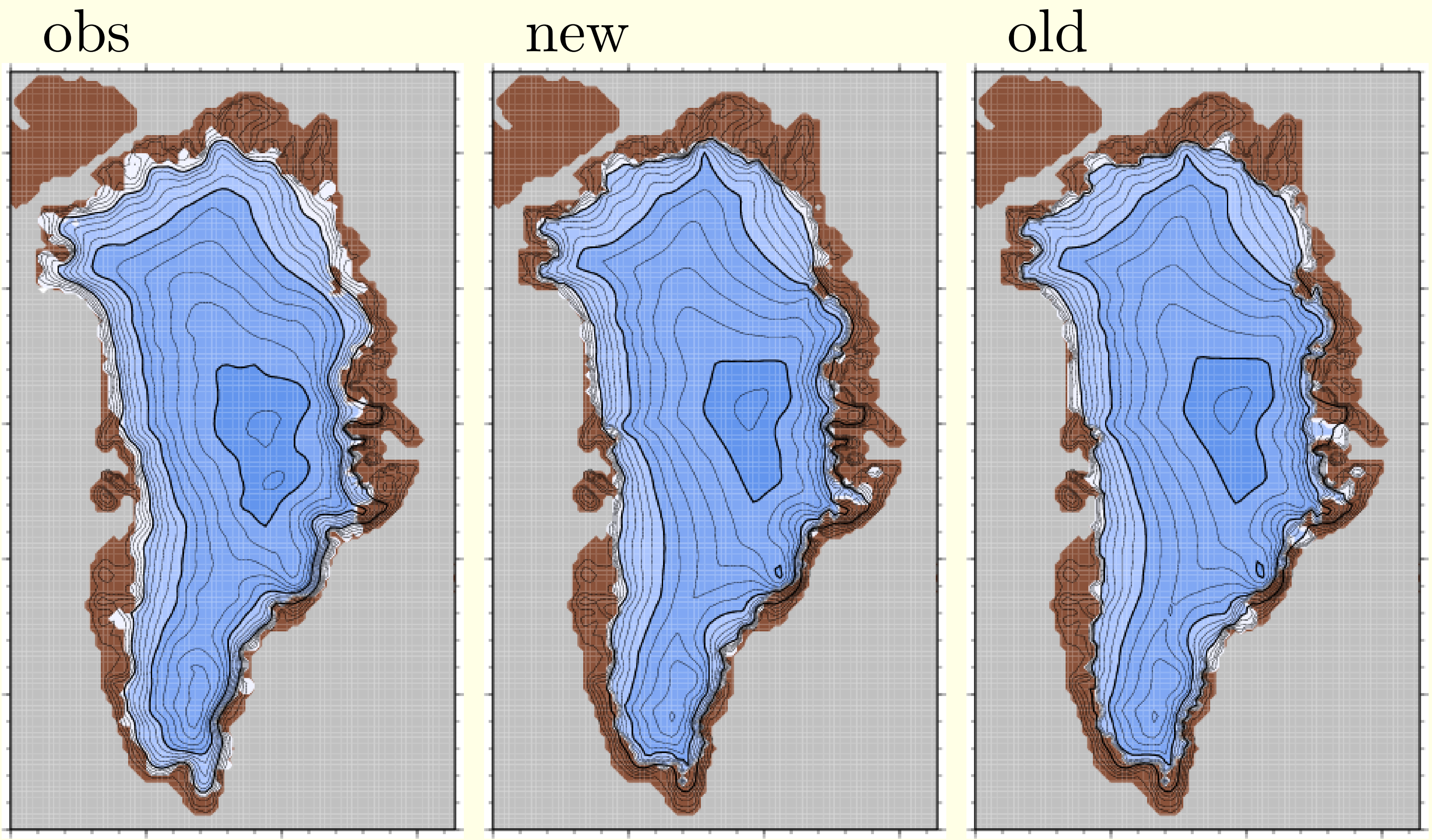
- It helps to reduce the error near the margin
- Asymmetry at the margin is also reduced.
- Error was enhanced in the thermodynamically coupled case
- Thickness at the margin is still overestimated by ~ 400 m

Perspectives (1)

- Transient behavior may be affected
  — Advancing/retreating time scales.
- Realistic ice sheet?
- Higher-order difference scheme is expected to further reduce the overestimation

Perspectives (2)

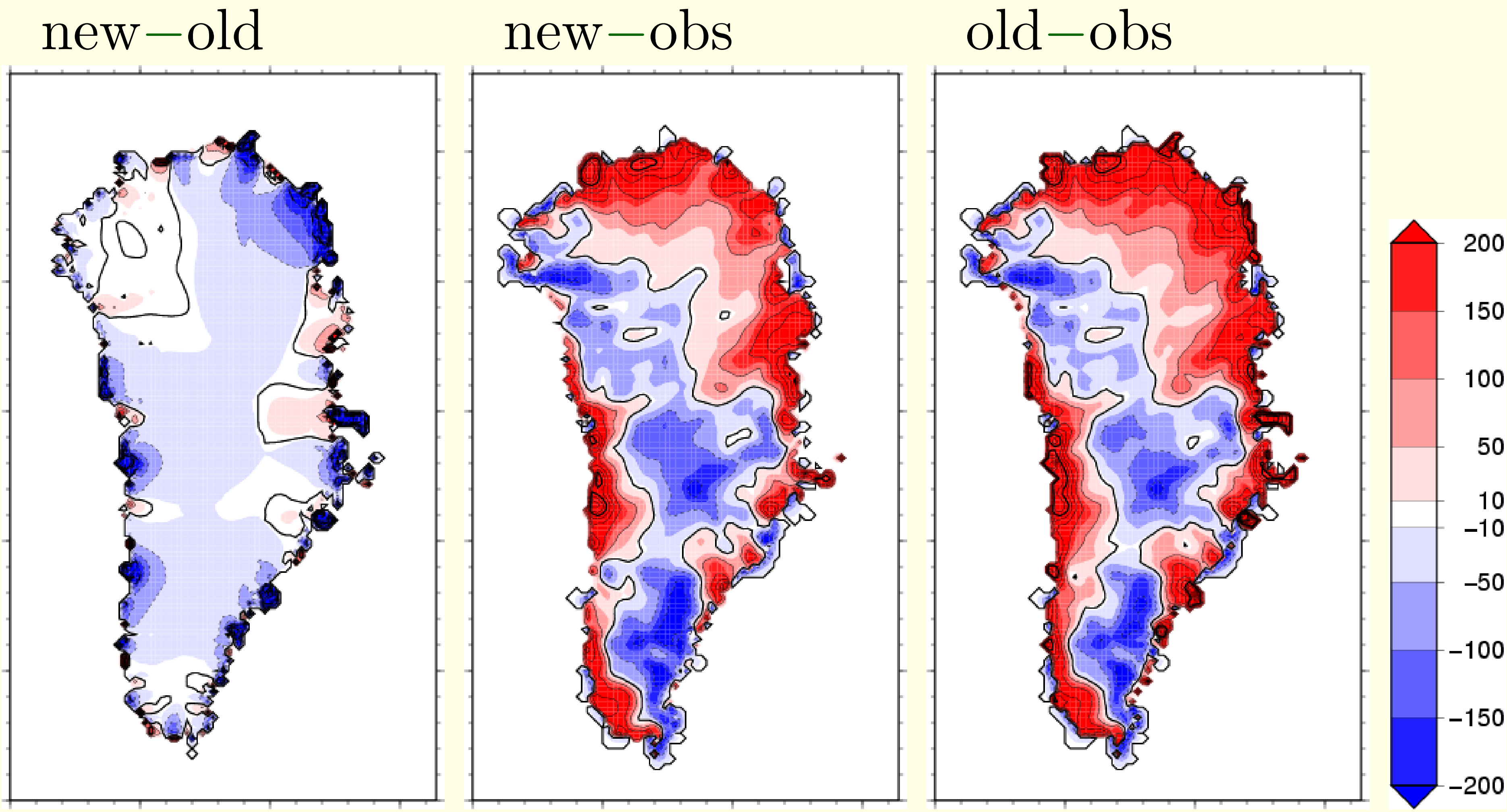
Influence on simulated Greenland ice sheet by the new scheme



present steady-states

Perspectives (3)

Influence on simulated Greenland ice sheet by the new scheme



contour=50, 200 m   100, 500 m   100, 500 m

Tuning again using the new scheme (1)

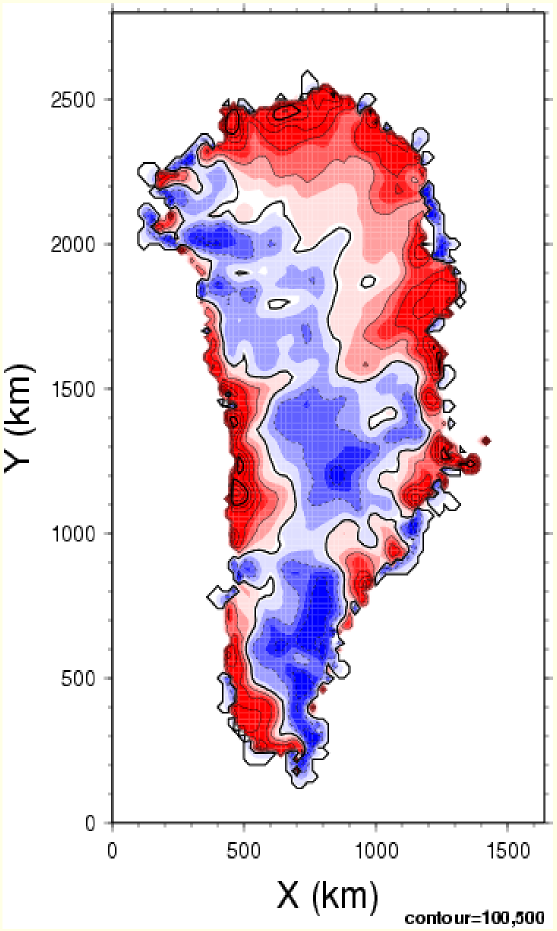
Uncertain parameters:

- Enhancement factor (ice stiffness) (E)
- Geothermal heat flux (Γ)
- Basal sliding process
- Higher order dynamics such as *ice stream*

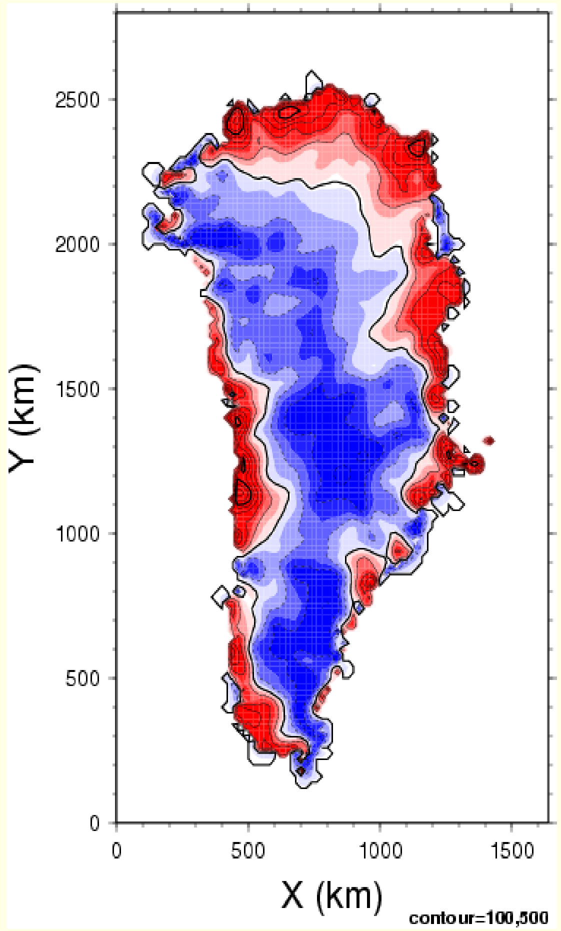
Tuning again using the new scheme (2)

Examples:

$E = 3$



$E = 5$



$E = 3, \Gamma = 2$